# A NEW OPTIMAL PLANNING BASED METHOD FOR REACTIVE POWER PRICING IN POWER SYSTEMS

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Abstract- Today, in view of the fact that power industry is progressing towards competitiveness and restructuring or deregulation, the presence of algorithms to calculate price of energy and ancillary services in a network is appreciated more necessary than before. The most important of these services is reactive power supply in the power network because voltage profile and stability, system losses and secure transmission of power from production to consumption side, have direct relationship to how reactive power is supplied through the network. Hence, using the appropriate pricing ways and algorithms for reactive power is of great importance. The main goal of this paper is showing a way on the basis of optimizing different functions used in optimal planning of reactive power to help calculate the cost of reactive power consumed in the network. The results of the proposed method have been given for a five-bus sample network in different cases of the objective function.

Keywords- Independent System Operator, Reactive Power Pricing, Power Factor, Optimal Power Flow

#### I. Introduction

An efficient transmission network plays a crucial role in growing power markets around the world. In the last five years generating capacity grew enormously; however, transmission investment has been declining for many years. Many countries have already adopted competitive market programs where an Independent System Operator (ISO) or Regional Transmission Organization (RTO) is responsible for scheduling and dispatching generators on regional networks, implementing a market-based mechanism for allocating scarce transmission capacity [1].

The direct current (DC) system model is a common approximation for estimating spot market prices under a constrained network. The DC load model is insufficient since it ignores reactive power effects on the production of real power, line congestion and voltage constraints.

Hogan in Reference [2] created a separate price mechanism for reactive power in order to stimulate its production with a purpose of satisfying voltage constraints. Later, Kahn and Baldick in [3] demonstrated that although Hogan's pricing example for reactive power yielded a Pareto improving (more efficient) dispatch, it was not a solution of the formal optimal power flow problem. After 1994, the theoretical discussion of reactive power pricing shifted into the engineering literature, where it focused on how marginal reactive power should be determined and priced. Hao [4], [5] explored the technical and economic

issues of determining reactive power structures, and designed a practical solution for managing reactive power services. Reference [6] discussed auction design for ancillary services.

A great deal of engineering researches centered on the technical side of the solution algorithm. Weber in [7] modified standard optimal power flow (OPF) analysis to simulate real and reactive power prices. Gil [8] proposed a theoretical approach of marginal cost pricing for reactive services. Alvarado [9] suggested marginal cost pricing for dynamic reactive power. These studies emphasize the important role of reactive power in the efficient production and distribution of electricity. They conclude first, that a DC approximation is not sufficient to mimic power flows in a congested network; and second, that reactive power output itself is costly and creates network congestion.

This paper presents an alternative pricing mechanism where the prices of real and reactive power are separated. In addition, separating real and reactive power bids in Iran's network is practical. In paper, a simple five bus network OPF solution is presented as a starting point. For OPF solution, four objective functions have been considered and new method has been presented for reactive power pricing in Iran's network.

## II. Definition of objective functions in optimal reactive power planning

Optimal reactive power planning by means of capacitor placement in power systems, aims at three main objectives as follows:

- Voltage profile improvement
- Power loss decrease
- Reduction of reactive power compensation and compensation cost.

Therefore to mathematically express the issue of reactive power planning, we will deal with three quantities of real power losses  $(P_{Loss})$ ,total absolute value of generated reactive power or compensators consumption  $(Q_{ini})$  and total absolute value of voltage variations of buses from ideal values ( $\Delta |V_i|$ ). Generally objective function can depend on one, two or three of mentioned quantities and considering the fact that reduction of energy loss is a very important factor in optimal control of reactive power, we will use the quantity of  $(P_{Loss})$  for definition of all objective functions. So objective function can be expressed in one of the four following formulas:

$$F_1 = P_{loss} \tag{1}$$

$$F_2 = (\alpha_p.P_{loss})^{\beta_p} + (\alpha_v.\Delta|V|)^{\beta_V}$$
(2)

$$F_3 = (\alpha_p.P_{loss})^{\beta_p} + (\alpha_q.Q_{inj})^{\beta_q}$$
(3)

$$F_4 = (\alpha_P.P_{loss})^{\beta_P} + (\alpha_q.Q_{inj})^{\beta_q} + (\alpha_v.\Delta|V|)^{\beta_V}$$
(4)

where, coefficients  $\alpha_q$  ,  $\alpha_v$  and  $\alpha_p$  are positive real numbers and  $\beta_q$ ,  $\beta_v$  and  $\beta_n$  are positive even numbers. These six coefficients determine the weighting factor of quantities to specify the value of objective function. For example if in objective function  $F_2$ , the values of  $\beta_{\nu}$  and  $\alpha_{\nu}$  are more than  $\beta_{p}$  and  $\alpha_{p}$ , it means that voltage profile improvement is more important than reduction. Also by coefficients  $\beta_q$ ,  $\beta_v$  and  $\beta_p$  and changing coefficients  $\alpha_q$ ,  $\alpha_v$  and  $\alpha_p$ ; it can be made a kind of balance among quantities  $P_{Loss}$ ,  $\Delta |V_i|$  and  $Q_{ini}$ . For instance if the cost of 10 KVAr of capacitor placements is assumed approximately equivalent to 1 kW loss in the line, the objective function  $F_3 = (10P_{Loss})^2 + Q_{inj}^2$  can be used to reduce the losses and capacitor placement related costs.

$$P_{ij} = G_{ij} (V_i - V_j)^2 = G_{ij} [V_i^2 + V_j^2 - 2V_i V_j \cos(\delta_{ij})]$$
 5)  

$$Q_{ij} = B_{ij} (V_i - V_j)^2 = B_{ij} [V_i^2 + V_j^2 - 2V_i V_j \cos(\delta_{ij})]$$
 (6)  

$$P_{loos} = \sum_{i} \sum_{j} P_{ij}, Q_{loss} = \sum_{i} \sum_{j} Q_{ij} + \sum_{j} Y_i (V_i)^2$$
 (7)  

$$i = 1, 2, ..., n$$
 
$$j = i + 1, ..., n$$
  

$$\Delta |V| = \sum_{i} |(1 - V_i)|$$
 
$$i = 2, ..., n$$
 (8)

(9)

 $Q_{inj} = \sum_{i=1}^{n} (InjectedQ(i)) \qquad i = 2,...,n$ where,  $P_{ij}$  and  $Q_{ij}$  are the active and reactive power losses between two buses i and j;  $V_i$  is voltage of bus i,  $Y_{ci}$  is related admittance for each bus;  $G_{ii}$ ,  $B_{ij}$  are the matrix members of susceptance and conductance; and n is the number of network buses.  $\Delta |V_i|$  is voltage difference, Q(i) is injected VAr in  $i^{nth}$  bus and  $Q_{inj}$  shows the total injected VAr into network buses. Injected Q(i) is a discrete alternative, i.e. the VAr injection into  $i^{nth}$  bus is accomplished through

## III. Pricing of reactive power based on consumption power factor

choosing from available values of capacitor banks.

As it is known, the rate of reactive power consumed by a consumer is conversely related to its power factor and big consumers with low power factor cause to inject reactive power (positive or negative) into their own networks. So a considerable capacity of devices, transformers, cables and overhead transmission lines, is occupied by reactive power. Each country in order to force its consumers for keeping an allowed power factor, establishes a set of regulations to penalize those consumers having low power factor.

Both allowed power factor and the ways to calculate penalty, vary in different countries of the world. This different view arises from the fact that allowed power factor in each country is determined considering the network status including its technical and economic matters. That is reason why the ways to calculate penalties vary due to different conditions of the networks in different countries. Since attitudes towards consumers having power factor lower than permitted level vary, the reactive power pricing could be different in countries.

Suppose that we intend to price reactive power on the basis of active energy consumed. In this case, reactive power price consumed by consumers who have power factor lower than the allowed power factor is determined as follows:

First, the bill of monthly active energy consumed by consumer is calculated through the relationship:

$$BIL = D.CD + E.CE \tag{10}$$

where, D is consumer demand (kW), CD is demand cost per unit (\$/kW), E is monthly consumed electrical energy (kWh) and CE is average cost of electrical energy per unit (\$/kWh). In such conditions, a bad consumer has to pay the reactive power cost equal to:

$$Cost = BIL * (\frac{\cos \phi_p}{\cos \phi_a} - 1)$$
 (11)

where, Cost is cost of consumed reactive power,  $\cos\phi_p$  is allowed power factor and  $\cos\phi_a$  is average monthly power factor. In fact, in this kind of pricing, in order to encourage consumers to increase their own power factor to reach to an allowed level, only the reactive load consumed by consumers who have lower power factor than the allowed one is included in their bills.

## IV. Suggested method for reactive power pricing

Assume that, reactive power pricing in a network is performed according to method introduced in the previous section. In the described method, pricing manner forces the customer to increase its power factor to reach the allowed level so that reduction of reactive current in the lines results in decrease of conductors losses in the network of the consumer and certainly bus voltages approach 1 p.u.

Now suppose a customer whose power factor is lower than the allowed power factor. In order to pay less reactive power cost or nothing, this consumer tries to reach to the allowed level by means of placing a specified capacitor. But it should be seen that whether Independent System Operator (ISO) who has the choice of control and operation of the transmission network, has achieved his goal through capacitor placement; that is minimizing the network lines losses and controlling the bus voltages or not? It is possible that the capacitor placed in the common bus with lower power factor, could not optimize the objective functions or cost introduced in the section 2. In fact in the new method of

reactive power pricing, the aim of capacitor placement in a bus with having power factor lower than the allowed level is optimization of objective functions not necessarily improve the common power factor to the allowed level. The reason is that the optimization of an objective function, depending on which one it is, can contribute to obtain the best results in terms of losses(objective function # 1), losses and bus voltages (objective function # 2), losses and compensation costs (objective function # 3), losses and voltages of buses and also, compensation costs (objective function # 4) throughout the network; besides reduction of lines losses and control of voltages of buses is now better realized in comparison with the case when capacitor placement is merely done for increasing common power factor. Considering the points mentioned above, the cost of consumed reactive power for each bus can be calculated by the following relation:

$$Cost = BIL * \left( \frac{Q_{\min F_i(x)} - Q_{\cos \phi_p}}{Q_{\min F_i(x)}} \right) \quad i = 1, ..., 4$$
 (12)

where,  $Q_{\min F_i(x)}$  is the capacitor placed in the related bus in order to optimize desired objective function and  $Q_{\cos\phi_p}$  is the placed capacitor in the bus which helps common power factor reach to allowed level.

In this kind of pricing, depending on the network conditions and the fact that which of the three parameters of lines losses, bus voltages and compensation costs in control and operation of transmission network is more important for Independent System Operator. Any of the four objective functions already described can be used as a basis to calculate the cost of reactive power consumed. Further, it is feasible to use different objective functions in a particular network according to their conditions in different months or seasons of the year or the hours of a day to make different pricings, e.g. in peak load hours as a critical condition in the network, objective function #2 can be considered to optimize line losses and bus voltages, whereas at base load hours in the network, objective function # 4 could be used as the best among the four objective functions which yields the best results in terms of losses, bus voltages and compensation costs. According to available information, using the objective function number 4, the cost of 10 kVAr

capacitor is approximately assumed to be equal to 1 kW of losses in the line. So for the discussed objective function, the expression related to losses is ten times as much as compared with expression related to injected capacitor reactive power. By changing coefficient of  $\Delta |V_i|$  i.e.  $\alpha_V$  in objective function, the factor of 100 has leaded to the best result.

## V. Case Study

In this section results of proposed pricing method are shown on a sample network with 5 buses (Fig. 1). Tables 1 to 4 indicates parameters of loads, lines and generators for the sample system of Fig1. The capacitor is placed at bus 4. In this paper the cost of reactive power paid by load of the bus 4 has been studied.

In order to compare pricing results based on the allowed power factor and the new method, two equations (10) and (12) are respectively used. In the two equations, BIL value has been equally considered in perunit. For more examination two cases are proposed for the network loads:

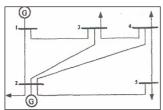


Fig 1: Five bus network

Table 1: Line parameters of 5-bus network

From	To	R	X	$Y_c/2$
1	2	0.02	0.06	0.030
1	3	0.08	0.24	0.025
2	3	0.06	0.18	0.020
2	4	0.06	0.18	0.020
2	5	0.04	0.12	0.150
3	4	0.01	0.03	0.010
4	5	0.08	0.24	0.025

**Table 2: Generators information** 

Gen.#	$P_{\rm max}$ (MW)	$P_{\min}$ (MW)
$G_1$	125	20
$G_2$	125	20

Table 3: Load information of each bus (Case 1)

Bus #	Active Load	Reactive Load
2	0.20	0.065
3	0.45	0.140
4	0.40	0.190
5	0.60	0.190

Table 4: Load information of each bus (Case 2)

Bus #	Active Load	Reactive Load
2	0.20	0.095
3	0.45	0.210
4	0.40	0.300
5	0.60	0.290

Case 1: The power factor of all loads joined to buses is 0.95, but the load of bus 4 has a power factor of 0.9 and needs a capacitor of 5.853 MVAr to improve power factor to allowed level of 0.95.

Case 2: The power factor of all loads is 0.9, but the load of bus 4 has a power factor of 0.8 and needs a capacitor of 11 MVAr to improve its power factor to allowed level of 0.9.

Information related to network loads in both cases has been respectively given in tables 3 and 4.

In the proposed pricing method, any of the four objective functions mentioned in section 2 can be used as a pricing base; the simulation shows that, the required capacitor in bus 4 for optimizing each function and accordingly the cost of reactive power consumed by joined load subject to bus 4 can be calculated using two pricing methods. The summary of pricing results from two ways and both cases, also the needed capacitor values are included in tables 5, 6, 7, and 8. These tables also contain the percentage of loss reductions of the network according to different capacitors choosed optimally due to the objective functions in comparison with the condition without any capacitor placed on bus #4. Moreover the load flow results for each case with different objective functions have been presented in tables 9 and 10.

### VI. Conclusion

In this paper a new optimal planning based method for reactive power pricing has been proposed. Also, for the purpose of capacitor placement in the network, some appropriate objective functions have been introduced to be used in different working conditions of the system. Studies on a sample network show that reactive power pricing based on the proposed method provides better results in comparison with the power factor method which is currently utilized in some countries.

For future studies, some relationship between active and reactive power may be found and then active and reactive power pricing can be done together.

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Table 5. Reactive power cost with PF method and proposed method (Case 1)

		Donativa navvan saat	Reactiv	e power cost	with proposed	method	Loss
		Reactive power cost with PF method	$F_1$	$F_2$	$F_3$	$F_4$	Reduction (%)
	unt of capacitor for ing objective function 5		36.887	49.748	6.870	49.748	-
	0	0.0556	1	1	1	1	0
	5.853	0	0.8413	0.8823	0.1737	0.8823	1.83
Amount of	36.887	0.0407	0	0.2585	4.3693	0.2585	6.14
capacitor at bus #4	49.748	0.1983	0.3486	0	6.2413	0	5.44
	6.870	0	0.8137	0.8619	0	0.8619	2.11
	49.748	0.1983	0.3486	0	6.2413	0	5.44

Table 6. Reactive power cost with PF method and proposed method (Case 2)

		D 4	Reacti	ve power cost	with proposed	method	
		Reactive power cost with PF method	$F_1$	$F_2$	$F_3$	$F_4$	
Amount of capacitor for optimizing objective function		11	56.996	69.791	11.637	69.791	-
	0	0.125	1	1	1	1	0
	11	0	0.8070	0.8424	0.0574	0.8424	4.79
Amount of	56.996	0.0869	0	0.1833	3.8978	0.1833	13.35
capacitor at bus #4	69.791	0.2695	0.2245	0	4.9973	0	12.74
	11.637	0	0.7958	0.8332	0	0.8332	5.03
	69.791	0.2696	0.2245	0	4.9973	0	12.74

Table 7. Values of objective functions (Case1)

Table 7. Values of objective functions (Case)								
	Amount of		Value of objective function				0	
	capacitor	$F_1$	$F_2$	$F_3$	$F_4$	$P_{Loss}$ (kW)	$Q_{Loss}$ (kVAr)	
Without capacitor	0	0.2563	314.8112	0.25628	314.812	5062.45	893.41	
With capacitor for PF improvement	5.853	0.2469	268.6525	0.2504	268.6559	4969.87	573.41	
	3.6887	0.2358	-	-	-	4751.63	-301.63	
Amount of capacitor at bus	49.748	-	43.7154	-	•	4787	-285.11	
#4	6.870	-	-	0.2503	·	4955.53	-523.04	
	49.748	-	-	-	268.6559	4787	-285.11	
Amount of capacitor for optimizing objective function	-	3.6887	49.748	6.870	49.748	-	-	

Table 8. Valuse of objective functions (Case2)

			Value of object			D	0
	Amount of capacitor	$F_1$	$F_2$	$F_3$	$F_4$	$P_{Loss}$ (kW)	$Q_{Loss}$ (kVAr)
Without capacitor	0	0.3238	592.0547	0.3238	592.0574	5690.46	2964.81
With capacitor for PF improvement	11	0.2935	471.8689	0.3056	471.8810	5417.87	2066.23
	5.6996	0.2431	-	-	-	4930.74	276.69
Amount of capacitor at bus	6.9791	-	71.9208	-	-	4965.61	292.62
#4	11.637	-	-	0.3056	-	5404.05	2020.13
	6.9791	-	-	-	72.4079	4965.61	292.62
Amount of capacitor for optimizing objective function	-	3.6887	49.748	6.870	49.748	-	-

Table 9: Load flow results for each objective function (Case 1)

Objective function	Bus No.	ig Vig  (p.u.)	$\angle \theta$ (Degree)	Objective function	Bus No.	ig Vig  (p.u.)	$\angle \theta$ (Degree)
	1	1.05	0		1	1.05	0
	2	1	-0.42	$F_3$	2	1	-0.43
$F_1$	3	0.9994	-3.54		3	0.9673	-3.19
1	4	0.9920	-3.86		4	0.9623	-3.38
	5	0.9566	-4.01		5	0.9463	-3.89
	1	1.05	0		1	1.05	0
	2	1	-0.42		2	1	-0.42
$F_2$	3	0.9967	-3.64	$F_{4}$	3	0.9967	-3.64
	4	1	-3.99	·	4	1	-3.99
	5	0.9593	-4.04		5	0.9593	-4.04

Table 10: Load flow results for each objective function (Case 2)

Table 10. Load now results for each objective function (Case 2)								
Objective function	Bus No.	$\left V\right $ (p.u.)	$\angle \theta$ (Degree)	Objective function	Bus No.	V  (p.u.)	$\angle \theta$ (Degree)	
	1	1.05	0		1	1.05	0	
	2	1	-0.41		2	1	-0.42	
$F_1$	3	0.9920	-3.56	$F_3$	3	0.9769	-3.33	
1	4	0.9919	-3.84		4	0.9724	-3.53	
	5	0.9651	-4.14		5	0.9584	-4.06	
	1	1.05	0		1	1.05	0	
	2	1	-0.41		2	1	-0.41	
$F_2$	3	0.9983	-3.66	$F_{4}$	3	0.9983	-3.66	
	4	1	-3.97	_	4	1	-3.97	
	5	0.9679	-4.17		5	0.9679	-4.17	