Prony-RBFNN Approach for Tuning Power System Stabilizer

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Abstract:- This paper presents a combined approach based on Prony analysis and radial basis function neural network for monitoring small signal stability and parameter tuning of power system stabilizer. Prony analysis method is used to estimate the modal components of low frequency oscillations associated with synchronous generators. In this method, the measured (simulated) time-domain signal is decomposed into damped sinusoids with four parameters per mode: amplitude, frequency, damping and phase angle. Once the local mode responsible for poor damping of the low frequency oscillations is identified, its damping factor and damped frequency are used to predict the parameters of the stabilizer using a radial basis function neural network. The tests results show that Prony analysis-neural network technique can be effectively applied in small signal stability analysis and power system stabilizer design.

Key-Words:- Power system stability and control, identification, Prony analysis, RBF, simulation

1 Introduction

Small signal stability analysis is concerned with the dynamic behavior of power systems following small perturbation from operating points. Its main objective is to predict and monitor the poorly damped low frequency oscillations resulting from rotor oscillations. The most critical types of these oscillations are the local-mode and interarea-mode oscillations[1-4]. The former occurs between one machine and the rest of the system and is between 0.5 to 2 Hz. The later occurs between interconnected machines and is between 0.1 and 0.5 Hz. The stability of these oscillations is of vital concern and essential power system planning, operation and control. For secure power system operation, the operators need fast and efficient computational tools to allow online stability assessment. This paper is concerned in stability assessment of local mode oscillations.

Traditionally, small signal stability analysis studies of power systems are carried out in frequency domain using modal analysis method [2-5]. This method implies estimation of the characteristic modes of a linearized model of the system. It requires first load flow analysis, linearization of the power system around the operating point, developing a state-space model of the power system, then computing the eigenvalues, eigenvectors, and participation factors [5]. Although eigenvalue analysis is powerful, however, it is not suitable for online application in power system operation, as it requires significantly large computational efforts. An alternative method to avoid the computational burden is to use online modal identification techniques that can quickly assess the stability of the power system on the basis of data samples obtained by measurements and automatically provides new estimates as new data samples are received. In these techniques, the characteristic modes of the dynamic system are determined from the dynamic behavior obtained either by online measurement, or by computer simulation of the linearized model [6-8]. Once the modes are obtained, the specific electromechanical mode $(e^{\lambda it})$ that provides the largest contribution to the low frequency oscillation is identified and then it can be used for tuning the parameters of a conventional power system stabilizer (PSS) using conventional phase compensation [1,2], adaptive or neural network (NN) technique [9-11]. Recently, neural networks have been applied in many areas of power systems including identification and control of nonlinear systems and tuning of PSSs for their high computational speed, generalization and learning ability [9-11].

This paper presents an online signal processing technique for monitoring the small signal stability based on Prony analysis [6-8]. It is a technique for modeling sampled data as a linear combination of exponentially damped sinusoids. In this paper, Prony analysis is applied to the speed signal of a synchronous generator connected to an infinite bus system in order to determine the amplitude, frequency, damping, and phase angle of the modal contents of the speed signal and to identify the local mode responsible for poorly damped low frequency oscillations. Furthermore, a radial basis function neural network (RBFNN) that has been trained offline is used to predict the parameters of a conventional phase lead-lag power system stabilizer. Radial basis function neural network is used for its advantages of rapid training, generality and simplicity over feedforward backpropagation neural network [12]. A block schematic of the proposed scheme is shown in Fig.1.



Fig.1: Block schematic of Prony-RBF PSS scheme

2 Description of the Study System

To study the stability of local mode oscillations associated with a single generator or plant, the single machine infinite bus system (SMIBS), as shown in Fig. 2, is used. The generator is represented by a third-order machine model and is equipped with automatic voltage regulator (AVR) and conventional PSS [2].



Fig.2: Single Machine Infinite Bus System

In small signal stability simulation, the power system model is linearized at a particular operating point to obtain the linearized system model given in the state-space form

$$\begin{aligned} \mathbf{x} &= A\mathbf{x} + B\mathbf{u} \\ \mathbf{y} &= C\mathbf{x} + D\mathbf{u} \end{aligned} \tag{1}$$

where $\mathbf{x} = \begin{bmatrix} \Delta \omega & \Delta \delta & \Delta e'_q & \Delta E_{fd} & \Delta x_5 & \Delta u_{PSS} \end{bmatrix}^T$, $\mathbf{y} = \begin{bmatrix} \Delta \omega \end{bmatrix}$ and $\mathbf{u} = \begin{bmatrix} \Delta T_m & \Delta V_{ref} \end{bmatrix}^T$; Δ denotes the perturbation of the states, inputs and output from their operating values. The linearized model of the SMIBS can be derived with the aid of the wellknown Phillips-Heffron block diagram [2]; relating the pertinent variables such as electrical torque, speed, rotor angle, terminal voltage, field voltage and internal voltage as shown in Fig.3. The parameters of the linearized model K_1 - K_6 are function of operating conditions. Analysis and calculations of the parameters of the SMIBS are illustrated in details in [2]. The small signal stability response in terms of the change in the rotor speed $\Delta \omega$ following a small change in the mechanical torque ΔT_m or the reference voltage ΔV_{ref} can be simulated with the aid of the block diagram of the SMIBS or the state-space model of (1). Dynamic data for the generators and excitation system used in the study, and the matrices of the state-space model, constructed from typical machine parameters at specific operating point, are given in [2].



Fig.3: Phillips-Heffron block diagram of SMIBS

3 Analysis of Small Signal Stability 3.1 Modal Analysis

When the power system experiences a small disturbance as a result of small changes of loads, the system will be driven to an initial state $\mathbf{x}(t_o) = \mathbf{x}_o$ at time $t_o = 0$. Then, if the input is removed at $t = t_o$, the system respond according to the state equations

$$\begin{aligned} \mathbf{x} &= A\mathbf{x} \\ \mathbf{y} &= C\mathbf{x} \end{aligned} \tag{2}$$

The state equations of the linearized model given in (2) can be used to determine the eigenvalues λ_i of the system matrix A, where $\lambda_i = \sigma_i \pm j\omega_i$ are the distinct eigenvalues with a corresponding set of right and left eigenvectors U_i and V_i , respectively; σ_i is the damping factor and ω_i is the damped angular frequency. The right and left eigenvectors are orthogonal, and are usually scaled to be orthonormal. The state equations of (2) can be expressed in terms of modal variables by using the modal transformation $\mathbf{x} = \mathbf{U}\mathbf{z}$, which leads to

$$\boldsymbol{z} = \boldsymbol{V}_i \boldsymbol{A} \boldsymbol{U}_i \boldsymbol{z} = \boldsymbol{\Lambda} \boldsymbol{z} \tag{3}$$

where Λ =diag(λ_i) [13]. Following small disturbance, the dynamic response of the system states can be

described as a linear summation of various modes of oscillations

$$\mathbf{x}(t) = \sum_{i=1}^{n} U_i \left(V_i \mathbf{x}_o \right) e^{\lambda_i t}$$
(4)

The number of the characteristic modes $e^{\lambda_i t}$ equals to the number of states of the linearized power system model. Real eigenvalues indicate modes, which are aperiodic. Complex eigenvalues indicate modes, which are oscillatory. For a complex eigenvalue $\lambda_i = \sigma_i \pm j\omega_i$, the amplitude of the mode varies with as $e^{\sigma_i t}$ and frequency of the oscillation, $f = \omega/2\pi$. The damping ratio, ζ_i , is defined as

$$\varsigma_i = \frac{-\sigma_i}{\sqrt{\sigma_i^2 + \omega_i^2}} \tag{5}$$

For a single output, the system response y(t) can be computed as

$$y(t) = \sum_{i=1}^{n} B_i e^{\lambda_i t} = \sum_{i=1}^{n} A_i e^{\sigma_i t} \cos(2\pi f_i t + \phi_i)$$
(6)

where A_{i} , σ_{i} , f_{i} and ϕ_{i} are the i^{th} mode amplitude, damping factor, frequency, and phase angle, respectively, and *n* is the number of modes.

Next, an analysis is performed to find the specific electromechanical mode that provides the largest contribution to the low frequency oscillation. In modal analysis, the electromechanical mode is identified by analyzing the right and left eigenvectors in conjunction with the participation factors [5]. The participation factors provide a measure of association between the state variables and the oscillatory modes.

3.2 Prony Analysis

Prony analysis is a technique for modeling sampled data of an exponentially damped signal as a linear combination of damped sinusoids [6-8]. It gives an optimal fit to the measured signal in the sense of the least-squared error technique (LSE). If N samples of the response v(t) is recorded as $v(k\Delta t) = v(k), k=0, 2,$..., N-1, then y(k) can be expressed as a linear combination of *n* distinct modes

$$y(k) \cong \sum_{i=1}^{n} B_i e^{\lambda_i k \Delta t} = \sum_{i=1}^{n} B_i z_i^k$$
(7)

where $z_i = e^{\lambda_i \Delta t}$, and Δt is the sampling time. The *n* distinct eigenvalues λ_i 's and amplitudes B_i 's can be identified using the three-step Prony analysis as follows:

1. Construct a linear prediction model (LPM) $y(k) = a_1 y(k-1) + a_2 y(k-2) + \dots + a_n y(k-n)$ (8)

Repeating (8) (*N*-*n*) times to form the LPM

$\begin{bmatrix} y(n) \end{bmatrix}$	$\int y(n-1)$	y(n-2)	 <i>y</i> (0)	$\begin{bmatrix} a_1 \end{bmatrix}$
y(n+1)	<i>y</i> (<i>n</i>)	y(n-1)	 <i>y</i> (1)	a_2
:	:	:	 :	:
y(N-1)	y(N-2)	y(N-3)	 y(N-n-1)	a_n
or				

(9)

The least-square estimate of A can be obtained using the psuedo inverse of matrix Φ

$$\mathbf{A} = \mathbf{\Phi}^{\dagger} \mathbf{Y} = (\mathbf{\Phi}^{\mathrm{T}} \mathbf{\Phi})^{-1} \mathbf{\Phi}^{\mathrm{T}} \mathbf{Y}$$
(10)

Υ=ΦΑ

2. Find the roots (eigenvalues) of the characteristic polynomial associated with the LPM of step 1

$$z^{n} - a_{1}z^{n-1} - a_{2}z^{n-2} - \dots - a_{n} = 0$$
(11)

where $\lambda_i = \log(z_i/\Delta t) = \sigma_i \pm j\omega_i$

3. Estimate the amplitude and phase angle of each mode obtained in step 2.

$$\begin{bmatrix} y(0) \\ y(1) \\ \vdots \\ y(N-1) \end{bmatrix} = \begin{bmatrix} z_1^0 & z_2^0 & \dots & z_n^0 \\ z_1^1 & z_2^1 & \dots & z_n^1 \\ \vdots & \vdots & \dots & \vdots \\ z_1^{N-1} & z_2^{N-1} & \dots & z_n^{N-1} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{bmatrix}$$
(12)

or

 $\mathbf{Y} = \mathbf{A}\mathbf{B}$ The least-square estimate of **B** can be obtained as **B** =

$$\mathbf{\Lambda}^{\mathsf{T}}\mathbf{Y} = (\mathbf{\Lambda}^{\mathsf{T}}\mathbf{\Lambda})^{\mathsf{T}}\mathbf{\Lambda}^{\mathsf{T}}\mathbf{Y}$$
(13)

The degree of fitness of the Prony LPM (12) to the simulated (measured) low frequency oscillation (6) can be measured in terms of the signal-to-noise-ratio (SNR) defined as [6]

$$SNR = 20 \log \left(\frac{\|y(k)\|}{\|y(k) - \hat{y}(k)\|} \right)$$
(14)

where ||y(k)|| is the second norm of the measured signal $||y(k) - \hat{y}(k)||$ is the norm of the error signal between the measured and estimated signals. For perfect fitting the number of samples N and the number of modes *n* are varied until the $SNR \ge 40 \text{ dB}$.

4 Power System Stabilizer Design

Originally, the low frequency oscillations problem is tackled by applying PSS, which provides a supplementary excitation control signal to enhance the damping of the poorly damped low frequency oscillations. The conventional design using a leadlag compensator was investigated on a linearized model of a single machine infinite bus system (SMIBS)[1]. A stabilizing signal derived from generator speed, frequency or power is admitted to the reference input of the automatic voltage regulator (AVR) so that an electrical torque component in phase with speed variation is created to increase system damping. Most utility companies,

because of its simple structure, design and implementation, have adopted this type of design.

PSS typically is designed based on linear control theory using the concept of phase compensation [1,2]. The parameters are determined based on a linearized model of the power system around a nominal operating point where they can provide optimum damping performance of low frequency oscillations. Phase compensation is accomplished by adjusting the PSS parameters to provide a appropriate phase lead to compensate for the phase lags through the generator, AVR and excitation system over a wide frequency range (0.1-2.0 Hz) of low frequency oscillations such that the PSS provides torque changes ΔT_e in phase with speed changes $\Delta \omega$. Tuning should be performed when system configuration and operating conditions result in the least damping [2]. Moreover, a good tuning scheme is required to achieve robust performance over a wide range of operating conditions by tuning the PSS parameters according to online identified damping factor and damped frequency of the poorly damped local mode. In this paper, the Prony analysis method is adopted to identify the poorly damped local mode, and a RBFNN is used to predict the parameters of the conventional PSS.

5 Radial Basis Function Network

The radial basis function neural network (RBFNN) comprises one of the most used feedforward neural network [12,14]. Figure 4 illustrates a RBFNN comprising an input layer with k nodes, hidden layer with h neurons and output layer with m neurons. The RBFNN consists of only one hidden layer of radial basis functions or neurons. At the input of each neuron, the distance between the neuron center and the input vector is calculated. The output of the neuron is then, formed by applying the basis function to this distance. The RBFNN output is formed by a weighted linear sum of the hidden neuron outputs and the unity bias

$$O_m(x^p) = \sum_{j=1}^n w_{mj} \phi_j(x^p) + w_o$$
(15)

where, $\phi_j(\mathbf{x}^p)$ is strictly positive radial symmetric function (kernel) with a unique maximum at its center c_j and which drops off rapidly to zero away from the center. The process of determining the weights is called training or learning process. In training, the network the parameters are adjusted so that the training data fits the network output (15) such that an error measure (the difference between the target and the predicted outputs of the network) is minimized. A sum squared-error (SSE) function is commonly used [14]



Fig.4: Radial basis function neural network

6 Simulation Results

6.1 Prony Analysis Results

The eigenvalues of the linearized power system described by (2) at certain operating conditions are shown in Table 1. The system has 6 modes corresponding to the number of system states. The negative values of the damping factors indicate that the system is stable. Table 1 shows the presence of low frequency electromechanical mode (f = 0.6956 Hz) with a relatively low damping factor (ratio).

Mode	$\lambda_i = \sigma_i \pm j\omega_i$	f_i (Hz)	$\zeta_i(\%)$	
1	-18.6795	0	100.00	
2	-4.5910+j7.4215	1.1812	52.61	
3	-4.5910- <i>j</i> 7.4215	1.1812	52.61	
4	-1.1664+ <i>j</i> 4.3705	0.6956	25.79	
5	-1.1664- <i>j</i> 4.3705	0.6956	25.79	

0

100.00

Table1: Eigenvalues of the Linearized System

-0.2015

6

In creating the LPM in step 1 of Prony analysis, values for the number of samples N and number of modes *n* and are needed, where $n_{\text{max}} = N/2$ [6]. Those two numbers should be chosen such that the LPM fits the measured data as perfectly as possible. The number of samples depends on the sampling frequency $(f_s = 1/\Delta t)$ and the length of the data window T_{wind} . In this study, T_{wind} was set at 5s. For a specific sampling frequency, the number of modes nis varied until perfect fitting is observed. The effect of number of modes n and number of samples N on the fitness of the Prony LPM in terms of the SNR is examined. Both numbers are varied until perfect fit is obtained. Table 2 shows the appropriate numbers of *n* and *N* that give $a SNR \ge 40$ dB. Table 3 gives the frequency and damping ratio for the modes with frequencies in the range of 0.1-2 Hz for different values of *n* with N = 500 ($f_s = 100$ Hz). It can be seen from Table 3 that exact estimates of the frequency and the damping ratio were obtained with n = 40.

Table 2: Effect of the numbers *n* and *N* on the *SNR*

#	SNR (dB)			
Modes	N=50	N=100	N=250	N=500
п	$f_{\rm s}$ =10 Hz	$f_{\rm s}$ =20 Hz	$f_{\rm s}$ =50 Hz	$f_{\rm s}$ =100 Hz
5	55.3404	22.1111	-	-
10	147.31	97.8158	4.0790	5.245
20	-	113.3780	53.7273	3.764
30	-	109.8464	88.4492	5.60
40	-	132.4904	95.2527	41.508

Table3: Frequency and Damping Ratio of the Modes, N = 500, $f_s = 100$ Hz

# of	Mode # 4		Mode # 6	
Modes	f(Hz)	ζ(%)	f(Hz)	ζ(%)
<i>n</i> = 10	0.9279	23.7103	-	-
<i>n</i> = 20	0.9764	30.6954	-	-
<i>n</i> = 30	0.6688	21.2078	1.1952	52.2701
<i>n</i> = 40	0.6956	25.7859	1.1812	52.6084
Exact	0.6956	25.7859	1.1812	52.6084

Estimates of the damping factor, frequency, amplitude, and phase angle of the modes with frequencies between -3Hz and 3 Hz for n = 40 and N = 500 are given in Table 4. The \pm frequencies indicate the presence of complex conjugate modes.

Table 4: Estimates of the Modal Components of the LPM Modes, $f \le 2$ Hz, n = 40, N = 500

Mode	σ_{i}	f_I	Amplitude	ϕ
Order		(Hz)	В	(rad)
3	-0.2015	0	0.0171	1.0440
4	-1.1664	0.6956	2.5426	-0.2756
5	-1.1664	-0.6956	2.5426	0.2756
6	-4.5910	1.1812	1.0475	2.4239
7	-4.5910	-1.1812	1.0376	-2.4022
8	-18.6795	0	0.9978	-3.0869

Examining the results of Table 4, one can see that the exact modes of the system, in particular the low frequency electromechanical mode with the lowest damping factor, were perfectly identified using Prony LPM. Similar results were obtained at lower number of modes *n* and samples *N* in which the *SNR* ≥ 40 dB.

Figure 5 shows a Prony fit to the rotor speed response for a 0.2 pu pulse torque disturbance for 20 ms. The estimated response represents a linear combination of an *n* number of damped sinusoids. For n = 20, Prony fit is shown in Fig.5-a. This is corresponding to a *SNR*=3.764 dB. When *n*

increased to 40, the *SNR* becomes 41.508 dB indicating a perfect curve fit as shown in Fig.5-b.



Fig.5-a: Prony fit of the rotor speed, n = 20, N = 500



Fig.5-b: Prony fit of the rotor speed, n = 40, N = 500

6.2 RBFNN Results

In this study, a RBFNN is adopted to predict the power system stabilizer parameters. The input layer has two neurons for the generator's local mode damping factor σ and damped frequency ω_d .

The output layer has two neurons for the PSS gain K_c and the time constant T_1 , as shown in Fig.1. For the RBFNN, the number of neurons in the hiddenlayer is determined by the learning procedure to reach a satisfactory error value, and is equal to the number of training epochs [14]. Different values of widths of the RBFs were examined. A width value of 1.0 was found good.

Figures 6 and 7 show the target and predicted values of K_c and T_1 obtained by the RBFNN during the training and testing phases. A Training set of 400 input-output patterns representing the dynamic behavior of the SMIBS over wide range of loading conditions (P_e : from 0.05 to 1.0 pu in steps of 0.05 pu and Q_e : from -0.45 to 0.50 pu in steps of 0.05 pu) were used to train the RBFNN. After careful training using "newrb" training function [14], the network reached to a satisfactory SSE of 7.06×10^{-4} after 15 epochs of iterations.

In Fig. 6, it can be seen that the RBFNN successfully and smoothly predicted K_c and T_1 . Likewise, Fig. 7 shows the performance of the proposed RBFNN during the testing phase. A set of 100 input-output patterns, different from the training patterns, was used to test the generalization capability and robust performance of the proposed RBFNN.



Fig.7-b: Target and predicted values of T_1

7 Conclusion

A combined Prony analysis and radial basis function technique for monitoring small signal stability and tuning of PSS parameters is presented. The Prony method has the ability to accurately predict the modal components of modes existing in a measured (simulated) signal. Compared with the eigenvalues analysis, Prony analysis can be implemented easily with any arbitrary degree of complexity of the power system under study. In addition, a RBFNN was trained to predict the PSS parameters using the damping factor and damped frequency of the low frequency mode that were identified by Prony method. The results of the case study show that Prony-RBFNN based approach is reliable, efficient and convenient for monitoring small signal stability and tuning of PSS parameters.

References:

- [1] F.P. DeMello and C. Concordia, Concepts of synchronous machine stability as affected by excitation control, *IEEE Trans. PAS* Vol.88, No.5, 1969, pp. 316-329.
- [2] Y.N.Yu, *Electric Power System Dynamics*, Academic Press, NY, 1983.
- [3] N.Martins, Efficient Eigenvalue and Frequency Response Methods Applied to Power System Small Signal Stability Studies, *IEEE Trans. PWRS*, Vol.1, No.1, 1986, pp. 217-226.
- [4] N. Uchida and T. Nagao, A New Eigen-Analysis Method of Steady-State Stability Studies for Large Power Systems: S Matrix Method, *IEEE Trans. PWRS* Vol.3, No.3, 1988, pp. 706-714.
- [5] Y.Y. Hsu and C.L. Chen, Identification of Optimum Location for Stabilizer Applications using Participation Factors, *IEE Proc.*, *Pt. C*, Vol. 134, No.3, 1987, pp. 238-244.
- [6] J.F. Hauer, Application of Prony Analysis to the Determination of Modal Content and Equivalent Models for Measured Power System Response, *IEEE Trans. PWRS* Vol.6, No.3, 1991, pp. 1062-1068.
- [7] D.J. Trudnowski, J.R. Smith, T.A. Short, and D.A. Piere, An Application of Prony Methods in PSS Design for Multimachine Systems, *IEEE Trans. PWRS* Vol.6, No.1, 1991, pp. 118-126.
- [8] J.H. Hong and J.K. Park, A Time-Domain Approach to Transmission Network Equivalents Via Prony Analysis for Electromagnetic Transient Analysis, *IEEE Trans. PWRS* Vol.10, No.4, 1995, pp. 1789-1797.
- [9] Y.Y. Hsu and C.L. Chen, Tuning of power system stabilizers using an artificial neural network, *IEEE Trans. EC.* Vol.64, 1991, pp. 612-619.
- [10] R. Segal.R, M.L. Kotari, and S. Madnani, Radial basis function network adaptive power system stabilizer, *IEEE Trans. PWRS.* Vol.15, No.2, 2000, pp. 722-727.
- [11] Y. Zhang, G.P. Chen, O.P. Malik, and G.S. Hope, An artificial neural network based adaptive power system stabilizer, *IEEE Trans. EC.* Vol.8, No.1, 1993, pp. 71-77.
- [12] S. Haykin, Neural networks a comprehensive foundation, 2nd Ed., Macmillan Co., NY, 1999.
- [13] T. Kailth, *Linear Systems*, Prentice-Hall, NJ, 1980.
- [14] H. Demuth and M. Beale, *Neural network* toolbox user's guide for use with MATLAB, 2002.