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Temperature Factor for the Accurate Calculation of LV Cables

MIGUEL ANGEL RODRÍGUEZ POZUETA,
LUIS FERNANDO MANTILLA PEÑALBA, JOSÉ RAMÓN LANDERAS DÍAZ
Departamento de Ingeniería Eléctrica y Energética
Universidad de Cantabria
E.T.S.I.I. y T., Avda. los Castros, s/n. 39005 Santander
SPAIN

Abstract: In the present article the voltage drop in a low voltage isolated cable is analyzed by the authors. This analysis considers the effect of resistivity variation with the temperature; which, as well, depends on the current. It makes to them to define the 'temperature factor'.

The authors demonstrate that the voltage drop due to the cable reactance cannot be neglected when temperature effects are taken into account.

Finally, it is shown two fast, simple and systematic methods to calculate the cross-section of a LV isolated cable. These methods consider the cable real temperature and reactance. One of them is analytical and the other uses the tables of per unit voltage drop provided by manufacturers and standards.

Key-Words: isolated cables, voltage drop, temperature, low voltage, resistivity.

1 Introduction

The second annex of the 'Technical Guide to the Application of the Low Voltage Electrotechnical Regulations' [5], published by the Spanish Ministry of Science and Technology, is devoted to the calculation of LV cables. Henceforth, this guide will be denoted as 'The Guide'.

The Guide points out that the calculation of voltage drop in an isolated cable using the resistivity of a conductor at 20ºC, as is recommended in some texts, may lead to the underestimation of this value and to the selection of an excessively small cross-section. For this reason, The Guide recommends the use of the maximum temperature withstood by the cable in voltage drop calculations. In this case, it is possible to be excessively cautious and to select an unnecessarily large cross-section.

If a more accurate calculation is required, The Guide provides a fairly accurate method for determining the temperature of a cable in order to obtain its resistivity from it.

References [2], [3] and [12] present a summary of the calculation of a cable cross-section taking into account its temperature following the recommendations of The Guide. [12] presents a first approximation to the coefficient which we have termed 'temperature factor' to take into account the effect of the temperature on the resistivity of the cable.

The present paper describes a detailed and systematic study of the temperature factor, establishing the equations which allow it to be obtained rapidly from the current flowing through the cable.

This paper also includes a detailed analysis of the per unit voltage drop tables provided by The Guide [5] and the cable manufacturers [4]. The importance that voltage drop through reactance can have is determined and the most appropriate resistivity, ρ, and per unit reactance, x, values for use in the calculations are determined.

Finally, two methods are presented for the rapid, simple and systematic calculation of an LV cable cross-section taking into account both its temperature and its reactance. One of them is analytical and the other is based on the per unit voltage drop tables.

2 Influence of Temperature on Voltage Drop

The voltage drop of an LV cable can be calculated quite accurately as the sum of the voltage drops, e_R and e_X, caused by its resistance and its reactance, respectively. The first of these is normally the most important and depends on the resistivity of the conductors; which, in turn, varies linearly with the temperature. The relationships that permit obtaining e_R and e_X are the following:

$$e_R = \frac{K_F \rho}{S} L I \cos\varphi = \frac{K'_F \rho L}{S U} P \qquad (1)$$

$$e_X = K_F L x I \sin\varphi = \frac{K'_F L x}{U} Q \qquad (2)$$

In these expressions sin φ is positive for inductive loads and negative for capacitive loads. On the other

hand, the coefficient K_F is equal to 2 in single-phase lines and $\sqrt{3}$ in balanced three-phase lines. Moreover the coefficient K'_F is equal to 2 in single-phase lines and 1 in balanced three-phase lines.

Let us remind that a conductor's resistivity varies with the temperature according to this rule:

$$\rho = \rho_{20}\left(1 + \alpha\left(\theta - 20\right)\right) \qquad (3)$$

ρ resistivity at temperature θ (Ω mm^2/m)

ρ_{20} resistivity at 20ºC (Ω mm^2/m)

α coefficient of resistivity variation with temperature (ºC^{-1})

Let us call $\theta_{máx}$ the maximum temperature withstood by a cable, which will arise when the current $I_{máx}$ is flowing through it if the environmental temperature is θ_0. The resistivity values at these two temperatures are $\rho_{máx}$ and ρ_0, respectively.

When the current I is flowing through a cable, it has a temperature θ and a resistivity ρ. In these conditions, the **temperature factor** is defined with the following equation:

$$k_\theta = \frac{\rho}{\rho_{máx}} \qquad (4)$$

The Guide establishes that the increase in temperature of a cable with respect to the environment is proportional to the square of the current flowing through it. From this, we have deduced the following equation:

$$k_\theta = k_{\theta 0} + \left(1 - k_{\theta 0}\right) \cdot \left(\frac{I}{I_{máx}}\right)^2 \qquad (5)$$

$k_{\theta 0}$ is the temperature factor at environmental temperature θ_0 and, according to (3); it can be obtained by this formula:

$$k_{\theta 0} = \frac{1 + \alpha \cdot \left(\theta_0 - 20\right)}{1 + \alpha \cdot \left(\theta_{máx} - 20\right)} \qquad (6)$$

Since the coefficients α of variation of resistivity with temperature of copper and aluminium are practically identical (α = 0.0039ºC^{-1}), the same k_θ values can be used for both materials.

With the Spanish standardized values of $\theta_{máx}$ and θ_0, the temperature factor takes the values shown in Table 1. Let us notice that the temperature factor varies between 0.8 and 1. The column of table 1 that corresponds to a null value of the current shows the values of $k_{\theta 0}$.

Table 1: Temperature factor (if α = 0.0039ºC^{-1})

θ_0	$\theta_{máx}$	I/I$_{máx}$														
		0	0.1	0.2	0.3	0.4	0.5	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1
25ºC	70ºC	0.853	0.855	0.859	0.866	0.877	0.890	0.906	0.915	0.925	0.936	0.947	0.959	0.972	0.986	1
	90ºC	0.801	0.803	0.809	0.819	0.833	0.851	0.873	0.885	0.898	0.913	0.928	0.945	0.962	0.981	1
40ºC	70ºC	0.902	0.903	0.906	0.911	0.918	0.927	0.937	0.943	0.950	0.957	0.965	0.973	0.981	0.990	1
	90ºC	0.847	0.848	0.853	0.861	0.871	0.885	0.902	0.912	0.922	0.933	0.945	0.957	0.971	0.985	1

3 Impedance Factor

The **impedance factor k_Z** is this quotient (see [11]):

$$k_Z = \frac{e}{e_R} \qquad (7)$$

In the calculation of the cross-section of a cable we will use this nomenclature:

S' cross-section of the conductor that gives rise to the voltage drop e when its reactance is neglected and its resistivity is $\rho_{máx}$.

S" cross-section of the conductor that produces the same voltage drop e when its resistivity is $\rho_{máx}$ and its reactive voltage drop e_X is taken into account.

S"' cross-section of the conductor that produces the voltage drop e when its reactive voltage drop e_X is taken into account and its real resistivity at temperature θ is considered.

Then we get:

$$S'' = k_Z \cdot S' \qquad (8)$$

$$S''' = k_\theta \cdot S'' \qquad (9)$$

Although the value of k_Z for a given cable varies with its temperature, when this parameter is calculated not for a given cross-section, but rather to obtain a predefined voltage drop, it turns out that k_Z is independent of the temperature. In this case, k_Z can be obtained from the cross-section S' by means of the formula (10):

$$k_Z = \frac{1}{1 - \frac{x / 1000}{\rho_{máx}} S' \operatorname{tg} \varphi} \qquad (10)$$

4 Per Unit Voltage Drop Tables

In The Guide [5] and in manufactures' catalogues [10] there are per unit voltage drop tables e_u; that is, voltage drop per current and length unit. The manufacturers give their values of e_u only at the temperature $\theta_{máx}$ while The Guide provides values of e_u at various temperatures. The effects of both resistance and reactance are considered in these values.

Analysing these tables, the values of the most suitable parameters of resistivity ρ and per unit reactance x (reactance per length unit) for the voltage drop calculation can be determined for every cross-section. An example of these parameters it is shown in the Table 2.

Table 2: Values of ρ and x
(Cables of copper and 0.6/1 kV at 90ºC)

S	The Guide		Manufacturer	
	ρ	x	ρ	x
mm^2	$\Omega\ mm^2/m$	Ω/km	$\Omega\ mm^2/m$	Ω/km
1.5	0.0231	0.165	0.0229	0.154
2.5	0.0236	0.137	0.0230	0.139
4	0.0235	0.129	0.0230	0.127
6	0.0236	0.125	0.0233	0.114
10	0.0233	0.106	0.0231	0.106
16	0.0235	0.097	0.0232	0.108
25	0.0232	0.094	0.0229	0.094
35	0.0234	0.091	0.0232	0.087
50	0.0247	0.090	0.0245	0.087
70	0.0239	0.087	0.0238	0.085
95	0.0234	0.087	0.0230	0.090
120	0.0234	0.083	0.0236	0.085
150	0.0237	0.082	0.0234	0.090
185	0.0234	0.083	0.0235	0.081
240	0.0231	0.080	0.0236	0.081

Table 3: Resistivity ρ

	ρ ($\Omega\ mm^2/m$)		
	20ºC	70ºC	90ºC
Copper	0.019	0.022	0.024
Aluminium	0.030	0.036	0.039

Examining the results that have been obtained it is concluded that the best calculations are done when the resistivity ρ takes the values of the Table 3. Also it is verified that x varies little with cross-sections, especially if they are greater than 16 mm^2. In addition, x is not influenced by temperature and its value for three-phase cables is slightly bigger than the ones for groups of three one-phase cables. To calculate e_X, in default of more accurate values, it will be utilized 0.086 Ω/km as a mean value of the per unit reactance x.

The error made by underestimating the cable reactance has been calculated and Table 4 has been drawn up to indicate the cross-sections above which this error begins to be important (bigger than 6%).

Table 4: Cross-sections S' (in mm^2) above which the calculation error becomes bigger than 6 % if e_X is neglected

	Cos φ			
	0.8	0.85	0.9	0.95
Copper	25	25	35	50
Aluminium	35	50	70	95

It is deduced that, except for power factors very close to the unity, the error caused by underestimating the reactance may be of the same order of magnitude or even greater than that caused by the use of the resistivity $\rho_{máx}$. Hence, if the aim is to be as accurate as possible in the voltage drop calculation taking into account the temperature, the cable reactance should also be taken into account.

5 Cross-Section Calculation Methods

On the basis of the previous paragraphs, we have developed two methods for determining the cross-section of a cable taking into account the effects of both reactance and temperature on the voltage drop.

The first of these is analytical and is based on the use of the temperature and impedance factors. First, the cross-section S' is calculated. To do this the formula (1) is used supposing that e_R has the same value that e (e_X is neglected). Then the impedance factor k_Z can be determined from (10) and S" from (9). Next the temperature factor is obtained from (5) using the current $I_{máx\_}$. This is the maximum current that can be withstood by the cross-section S_, the standardized one immediately inferior to S". Now, conditions (11), based on the relationship (9), are successively checked. In this way it can be deduced whether S_ is the cross-section to be used or whether it should be the next-highest standardized one (S_+).

$$S\_ / S'' \le 0{,}8 \rightarrow S = S_+ \quad (11a)$$

$$S\_ / S'' \le k_{\theta 0} \rightarrow S = S_+ \quad (11b)$$

$$S\_ / S'' \le k_{\theta} \rightarrow S = S_+ \quad (11c)$$

$$S\_ / S'' > k_{\theta} \rightarrow S = S\_ \quad (11d)$$

Condition (11a) is based on the fact that the minimum possible value for k_θ and $k_{\theta 0}$ is 0.8 (table 1).

The second method uses manufacturers' per unit voltage drop tables, which give only the values of e_u at the temperature $\theta_{máx}$. From these, the voltage drop

at temperature θ can be obtained by means of the temperature factor. In effect, if we know the values of e_u (per unit voltage drop at the temperature $\theta_{máx}$ when the power factor is cos φ ($e_u = e_{u\varphi,\ \theta máx}$)) and e_{u1} (per unit voltage drop at the temperature $\theta_{máx}$ when the power factor value is the unity ($e_{u1} = e_{u1,\ \theta máx}$)), its per unit voltage drop $e_{u,\theta}$ for the same power factor cos φ at the temperature θ ($e_{u,\theta} = e_{u\varphi,\ \theta}$) can be obtained thus:

$$e_{u,\theta} = e_u - \left((1 - k_\theta) \cdot e_{u1} \cdot \cos\varphi\right) \qquad (12)$$

Normally, although the value of the power factor is not among the tabulated values, one could carry out a sufficient approximation to the value of its per unit voltage drop by fast interpolation of the values of the table. When it is desired a more accurate calculation of e_u or when the power factor is capacitive, the formula (13) can be used:

$$e_{u\varphi} = e_{u1} \cos\varphi + \frac{\left(e_{u\varphi'} - e_{u1} \cos\varphi'\right)}{\sin\varphi'} \sin\varphi \qquad (13)$$

In this formula the per unit voltage drop $e_{u\varphi}$ of a cable for a power factor cos φ is obtained from the tabulated values $e_{u\varphi'}$ (for the power factor cos φ') and e_{u1} (for the unity power factor). All per unit voltage drops of expression (13) are at the same temperature.

This expression also allows to obtain e_{u1} if the values of e_u that appear in the table are $e_{u\varphi}$ and $e_{u\varphi'}$, that correspond with two power factors (cos φ and cos φ'); no one of them is for the unity power factor.

Also, it is necessary to keep in mind that, in order to apply a table of per unit voltage drops e_u corresponding with three-phase lines for the calculation of a one-phase line, it is necessary to multiply the tabulated values by 1.155. This value is the quotient between the respective values of K_F for one and three-phase lines, respectively ($1.155 = 2 / \sqrt{3}$).

In the following sections there are a more detailed explication of these two methods using examples.

6 Analytical Method Example

We want to calculate the cross-section of a 400 V three-phase line. Its load is 145 kW, its power factor is 0.9 and its length is 40 m. Spanish regulations [6] establish an admissible voltage drop equal to 0.5% (e(%) = 0.5).

The line will be made with a group of one-phase cables of copper and 0.6/1kV whose insulating material is reticulated polyethylene. These cables will be put inside a buried tube.

As they are buried cables the environmental temperature θ_0 is 25ºC.

The reticulated polyethylene has a maximum admissible temperature, $\theta_{máx}$, equal of 90ºC.

The current is calculated thus:

$$I = \frac{P}{\sqrt{3}\ U \cos\varphi} = \frac{145\,000}{\sqrt{3} \cdot 400 \cdot 0.9} = 232.5\ A$$

The admissible voltage drop e is:

$$e = \frac{e(\%)}{100} U = \frac{0.5}{100} 400 = 2\ V$$

S' is calculated from the relationship (1), taking to e_R the value of e obtained previously (2 V) and to the resistivity the value indicated in table 3 for a maximum temperature, $\theta_{máx}$, equal to 90ºC ($\rho_{máx}$ = 0.024 Ω mm²/m). This is a three-phase line, therefore the coefficient K_F is equal to $\sqrt{3}$:

$$S' = \frac{\sqrt{3} \cdot 0.024}{2} \cdot 40 \cdot 232.5 \cdot 0.9 = 174\ mm^2$$

Next, it is analyzed the convenience of including the reactance voltage drop effect. For it the value of S' is compared with the ones in table 4. In our example S' is greater than 35 mm², then the reactance effect must be considered.

From (9) and (10) we obtain these results (if x = 0.086 Ω/km):

$$k_Z = \frac{1}{1 - \frac{0.086 / 1000}{0.024} \cdot 174 \cdot 0.484} = 1.43$$

$$S'' = k_Z \cdot S' = 1.43 \cdot 174 = 248.8\ mm^2$$

The maximum current $I_{máx\_}$ withstood by a cable of cross-section S_ can be determined from the table 52 N1 of standard [8] or from the table 7 of standard [7]. Moreover, it is necessary to take into account the factor of 0.8 (for group of cables in a same tube) recommended in the ITC-BT-07 [6]. We obtain a value of S_ equal to 240 mm² and a value of $I_{máx\_}$ equal to 440 A (= 550 x 0.8). This result also can be obtained by means of the table A of The Guide's section BT-14 [5].

If the current I (232.5 A) that flows through the cable was bigger than $I_{máx\_}$, thermal criterion would be the most severe. Then we would chose the smaller

standardized cross-section that would be able to withstand the current of 232.5 A.

But in this example the current I has a value lower than $I_{máx\_}$. So, in this case it is deduced that voltage drop criterion is the most severe and the calculation continues checking the conditions (11):

$$\frac{S\_}{S''} = \frac{240}{248.8} = 0.965$$

$$k_{\theta 0} = \frac{1 + 0.0039 \cdot (25 - 20)}{1 + 0.0039 \cdot (90 - 20)} = 0.801$$

$$\frac{I}{I_{máx\_}} = \frac{232.5}{440} = 0.528$$

$$k_{\theta} = 0.801 + \left((1 - 0.801) \cdot 0.528^2 \right) = 0.856$$

These temperature factors could also have been obtained from the table 1.

As the quotient S/S" is bigger than the temperature factor k_θ, the cross-section to be used will be S_. Therefore, cables of 240 mm^2 will be utilized.

Finally, it is necessary to check that the chosen cross-section also verifies the short-circuit current criterion: when a short-circuit takes place, the cross-section of a cable must be able to withstand, during the protection time, the high current that is going to flow through it.

7 Per Unit Voltage Drop Method

We want to calculate the cross-section of a 230 V one-phase line. Its load is 10868 W, its power factor is 0.9 and its length is 10 m. Spanish regulations [6] establish an admissible voltage drop equal to 1% (e(%) = 1).

The line will be made with cables of aluminium and 0.6/1kV whose insulating material is ethylene-propylene. These cables will be put together inside a tube that has superficial installation.

Table 5: Per unit voltage drops at 90ºC for groups of 3 single-phase cables of aluminium and 0.6/1 kV

eu (V/A. km)					
S	Cos φ		S	Cos φ	
(mm^2)	0.8	1	(mm^2)	0.8	1
16	3.42	4.15	120	0.53	0.55
25	2.19	2.62	150	0.45	0.45
35	1.60	1.89	185	0.37	0.36
50	1.21	1.39	240	0.30	0.27
70	0.86	0.97	300	0.26	0.22
95	0.65	0.70	400	0.22	0.17

As they are not buried cables the environmental temperature θ_0 is 40ºC.

The ethylene-propylene has a maximum admissible temperature, $\theta_{máx}$, equal of 90ºC.

Manufacturer's catalogue [10] has Table 5. This table gives per unit voltage drops at 90ºC for three-phase lines made with three one-phase cables of aluminium.

Power factor is 0.9, then:

$$\cos \varphi = 0.9 \rightarrow \sin \varphi = 0.434$$

The current is calculated thus:

$$I = \frac{P}{U \cos \varphi} = \frac{10868}{230 \cdot 0.9} = 52.5 \text{ A}$$

The admissible voltage drop e is:

$$e = \frac{e(\%)}{100} U = \frac{1}{100} 230 = 2.3 \text{ V}$$

This is a one-phase line but we will use a table for three-phase lines. Then in the calculations we must use this per unit voltage drop (three-phase equivalent):

$$e_u = \frac{1}{1.155} \cdot \frac{2.3}{52.5 \cdot 10} =$$

$$= 3.79 \cdot 10^{-3} \text{ V / A, m} = 3.79 \text{ V / A, km}$$

Consulting the table 5 this is obtained:

$$S\_ = 16 \text{ mm}^2$$

$e_{u0.8\_} = 3.42$ V/A, km $e_{u1\_} = 4.15$ V/ A, km

By means of the relationship (13) we obtain the per unit voltage drop $e_{u\_}$ at 90ºC and power factor 0.9:

$$e_{u\_} = 4.15 \cdot 0.9 + \frac{(3.42 - 4.15 \cdot 0.8)}{0.6} 0.434 =$$

$$= 3.81 \text{ V / A, km}$$

The table 52-C2 of the standard [8] shows that an aluminium cable of 16 mm^2 withstands a current $I_{máx\_}$ equal to 72 A. As the current I that flows through the cable is 52.5 A, smaller than $I_{máx\_}$; it is deduced that in this case the voltage drop criterion is the most severe and the calculation will be continued using this criterion. Otherwise, the section of the cable would be obtained by the thermal criterion choosing the smallest standard cross-section that was able to withstand the current of 52. 5 A.

Now we determine this quotient:

$$\frac{I}{I_{máx\_}} = \frac{52.5}{72} = 0.73$$

Then, according to the table 1, the temperature factor k_θ is 0.93.

Next, we calculate the per unit voltage drop $e_{u,\theta\_}$ when the aluminium cable of section $S\_$ is at temperature θ and power factor 0.9. To do this the formula (12) is used:

$$e_{u,\theta\_} = 3.81 - \left((1 - 0.93) \cdot 4.15 \cdot 0.9\right) = \\ = 3.55 \text{ V / A, km}$$

As $e_{u,\theta\_}$ is smaller than e_u, the cross-section to be utilized will be $S\_$; that is, 16 mm^2. Otherwise it would be necessary to choose the cross-section S_+ (25 mm^2).

Finally, it will be necessary to check if this cross-section of 16 mm^2 also verifies the short-circuit current criterion.

The definitive voltage drop e_{def} that is produced in this line of 16 mm^2 is obtained thus:

$$e_{def} = 1.155 \cdot 10 \cdot 52.5 \cdot 3.55 \cdot 10^{-3} = 2.15 \text{ V}$$

The factor the 10^{-3} is because length is measured in meters and $e_{u,\theta\_}$ is measured in V/A, km. The 1.155 factor appears because this is a one-phase line and $e_{u,\theta\_}$ has been calculated by means of a table for three-phase lines.

8 Conclusion

The temperature factor proves to be an ideal procedure for assessing the effects of temperature on the voltage drop in LV isolated cables.

If the aim is to obtain an accurate calculation of the voltage drop incorporating the influence of temperature, the reactance should also be included. This can be done quite simply by using the impedance factor.

Adequate values for ρ and x can be deduced from The Guide's per unit voltage drop tables. It is shown that x hardly varies with the conductor cross-section. These tables give the error caused by underestimating the effects of reactance on the cable.

Finally, two procedures are presented for calculating the cross-section of a cable taking into account its temperature and reactance.

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