Proceedings of the 5th WSEAS Int. Conf. on Applied Electromagnetics, Wireless and Optical Communications, Corfu, Greece, August 23-25, 2005 (pp31-36)

A new approach in modelling EM Wave Propagation in Troposphere using Finite Elements

Kamran Arshad, F. A. Katsriku and Aboubaker Lasabae
School of Computing Science
Middlesex University, London, UK

I. ABSTRACT

A finite element based beam propagation method for the analysis and study of electromagnetic wave propagation in the troposphere is presented. An efficient one dimensional form of the helmholtz equation with height dependent refractivity is used to investigate ducting phenomenon. Vertical tropospheric profile characteristics are assigned to each element through the discritization process and the solution is made to advanced using the marching algorithm.

II. INTRODUCTION

Numerous method are available for predicting electromagnetic wave propagation in the atmosphere, e.g., geometric optics, physical optics, normal mode analysis, and combinations of the above [1]. However, the presence of vertical refractivity stratification in the atmosphere complicates the application of some of these methods. To model refractivity variations in the horizontal as well as vertical direction, geometric optics, coupled-mode analysis, or hybrid methods have been employed [2]- [4].

In the past, emphasis was given to geometrical optics techniques. These methods [1] provide a general geometrical description of ray families, propagating through the troposphere. They are based on the discrimination of the medium into sufficiently small segments, with a linearly varying modified refractivity index. In each segment, the radiowave propagating angle is calculated using either the Snells law or its generalized form, if the Earths curvature is considered. Ray tracing methods present many disadvantages; for example the radiowave frequency is not accounted for and it is not always clear whether the ray is trapped by the specific duct structure [5]. An alternative approach for tropospheric propagation modeling was developed by Baumgartner [6] and later extended [7], [8]. This method, is known as Waveguide Model or Coupled Mode method. The main disadvantage of coupled mode technique lie in the complexity of algorithms and the large computational demands, especially when higher frequencies and complicated ducting profiles are involved [22].

One of the most reliable and widely used techniques is the Parabolic Equation (PE) Method, initially developed for the study of underwater acoustics problems and later on extended to tropospheric propagation [9]. The PE is based on the solution of the two dimensional Helmholtz equation in the paraxial limit with refractive index profile $n(x, y, z)$. The calculations may take into account the radius of the Earth and terrain effects whereas the polarization of the propagating radiowaves is implemented on the surface boundary conditions. Models based on the parabolic approximation of the wave equation have been used extensively for modelling refractive effects on tropospheric propagation [9], [10], in last decade. The biggest advantage to using the PE method is that it gives a full-wave solution for the field in the presence of range-dependent environments.

Various methods for the solution of the PE have been developed and presented in the literature. Two of the most popular are based on the finite-difference techniques [11], and the split-step Fourier algorithm [12]. Other models for propagation over terrain have also been developed and presented [13]- [15]. The most efficient algorithm seems to be the Split Step Solution which employs the Fast Fourier Transform (FFT) to advance the solution over small range steps. The algorithm has been widely used in many applications [9]. More specifically, Barrios [16] treated horizontally inhomogeneous environments and a terrain model respectively. Craig and Levy [10] applied the Split Step Solution to assess radar performance under multipath and ducting conditions. Split-step methods are extremely attractive in literature but they lack flexibility for boundary modelling [17]. Further, if variations of refractive index with height are fast, error will be more, because in split-step method, error depends on the height variations of refractive index [17].

A method to model tropospheric electromagnetic wave propagation where the refractivity is a function of height is presented using finite element method (FEM). The versatility and accuracy of the FEM as compared with other methods in the solution of EM wave propagation problems in certain areas as photonic device design has been well established [20], [21]. In the present work, we extend the FEM to the solution of parabolic equation in the troposphere. In contrast to the finite difference time domain (FDTD) method which gives the field distribution at nodes, the FEM gives the field within the whole domain and allows fast varying domain profiles to be modelled through the material properties of the elements. The solution is then advanced in the propagation direction using the marching algorithm. Vertical tropospheric profile characteristics are assigned to every mesh element, while solution advances in small variable range steps, each excited by solution of the previous step.

The remainder of the paper is organized as follow. In section 3, brief description of refractivity and ducting in troposphere is presented. Finite Element formulation of PE in troposphere is explained in section 4 and the boundary conditions used in simulations are given in section 5. Finally, results and discussions are given in section 6. Section 7 concludes the paper.

III. Refractivity and Tropospheric Ducting

An important physical property of the troposphere is its refractive index. For an absorbing medium, a complex refractive index is defined as,

$$n = c\ \frac{k}{w} - i\tilde{k} \tag{1}$$

where k is the wave number, w is the angular speed and $\tilde{k}$ is the extinction coefficient. In a dielectric material such as glass, none of the light is absorbed and therefore $\tilde{k} = 0$. Near the earth surface its value is close to unity and hence a more practical value, the refractivity is defined, and it may be calculated through the observation of pressure, temperature and humidity and is given as [18],

$$N = (n-1).10^6 = \frac{77.6p}{T} - \frac{5.6e}{T} + \frac{3.75.10^5 e}{T^2} \tag{2}$$

where p is the total pressure in mbar, e is the water vapor pressure and T is the temperature in Kelvin.

The surface refractivity gradient is the difference in refractive index between the surface and a given altitude, is used in characterizing various refractive conditions such as Subrefraction, Standard and Trapping. Tropospheric ducting occurs when the gradient $\frac{dN}{dz} < -157$, allows only waves above the cut off point to propagate [17]. Main cause of ducting is abrupt changes in refractive index of the medium. In section 5, we consider different refractivity profiles and analyze ducting in troposphere.

IV. Finite Element Formulation of Parabolic Equation in Troposphere

The analysis to follow starts from a parabolic equation of the form [23],

$$\{\partial_z^2 + j2k\partial_x + k^2[n^2(x,z) - 1]\}u(x,z) = 0 \tag{3}$$

where,

$$n^2(x,z) = \frac{\epsilon(x,z)}{\epsilon_0}$$

ϵ_0 is the permittivity of free space and k is the wave number. Further, notation $\partial_x \equiv \frac{\partial}{\partial x}$ and $\partial_z \equiv \frac{\partial}{\partial z}$ has also been introduced.

From equation 3, applying the FEM over the domain $z_{min} \leq z \leq z_{max}$ will yield the following matrix equation,

$$j2k[B]\frac{\partial\{u\}}{\partial x} + ([A] - k^2[B])\{u\} = 0 \tag{4}$$

where,

$$[A] = \sum_e \int [k^2 n^2 \{N\}\{N\}^T - \{N_x\}\{N_x\}^T] dz \tag{5}$$

$$[B] = \sum_e \int [\{N\}\{N\}^T] dz \tag{6}$$

The element shape functions are given as follows:

$$\{N_x\}\{N_x\}^T = j2k\frac{h}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \tag{7}$$

$$\{N\}\{N\}^T = \frac{-1}{h}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + \frac{k^2(n^2-1)h}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \tag{8}$$

where h is the element height and is given as $h = z_2 - z_1$.

Equation 4 may be solved using a marching algorithm where each successive step excites the solution. Such a procedure will yield a matrix equation of the form:

$$j2k[B]\frac{u(x,z) - u(x-\delta x,z)}{\delta z} + ([A] - k^2[B]) \\ [\theta\{u(x-\delta x,z)\} + (1-\theta)\{u(x,z)\}] = 0 \tag{9}$$

Theta θ is an artificial parameter, and different values of theta yields different solutions. For crank nicholson scheme, value of theta is 0.5 and hence the equation becomes,

$$[j2k[B]\delta x[B] + \frac{1}{2}[A] - 2k^2[B]]u(x-\delta x,z) \\ = [j2k[B]\delta x[B] - \frac{1}{2}[A] + 2k^2[B]]u(x,z) \tag{10}$$

The Crank Nicholson scheme is unconditionally stable and the global truncation error is $O(\delta x)^2$ [24].

Assuming $x = \delta x$, the quantity $u(x-\delta x,z) = u(0,z)$ corresponds to the initial field generated at the transmitting side. We choose gaussian beam patterns, because they have excellent numerical properties as well as provides a good representation for paraboloid dish antennas [17]. In these simulations a gaussian shaped field is used and defined as [19],

$$\phi(z) = e^{-\frac{(z-H_0)^2}{k_f}} \tag{11}$$

where H_0 is the altitude of transmitting antenna and k_f is the coefficient which determines the beam width. Equation (10) is a recursive, one dimensional form of the parabolic equation. For each range step, it can be directly solved using finite element method and the resulting solution is introduced as an excitation to the equation of the next step.

V. Boundary Conditions

The solution of equation (10) requires the application of boundary conditions at the starting height, $z = z_{min}$,which in fact is the Earths surface, and at the maximum altitude considered, $z = z_{max}$. An artificial boundary condition has to be applied at the upper limit, to reduce the effects of any possible reflections and allow for the propagation of the signal. Therefore, a first or second order absorbing boundary condition is applied [22], combined with the z_{max} extension.

The entrance boundary conditions are expressed by the equation [23]:

$$\partial_z u|_{z=0} + \alpha u = 0, \tag{12}$$

where

$$\alpha_v = \frac{jk}{\epsilon_e}, \quad \alpha_h = jk\sqrt{\epsilon_e}$$

for horizontal and vertical polarization respectively. ϵ_e is the complex relative permittivity of the medium. For a perfectly conducting surface, equation (12) reduced to $\partial_z u = 0$ for vertical polarization and $u = 0$ for horizontal polarization.

VI. Results and Discussions

To demonstrate the efficacy of the present approach a number of simulations were carried out with various frequencies and media profiles. In all of the simulations the following assumptions were made, a transmitting antenna of height $H_0 = 150$ above sea level with a beam width $k_f = 11$ and vertical polarization of the propagating wave. The ground was also assumed to be perfectly conducting. A simple absorbing boundary conditions is used to terminate the grid.

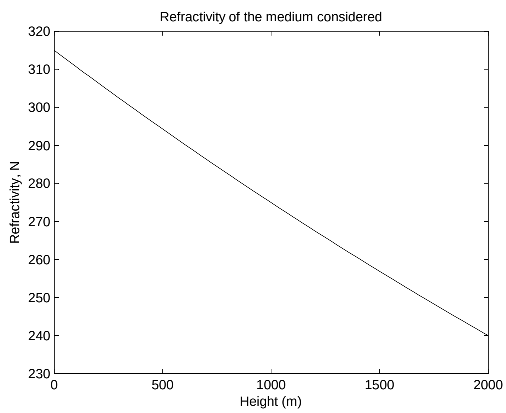

Fig. 1. Standard Atmospheric Profile

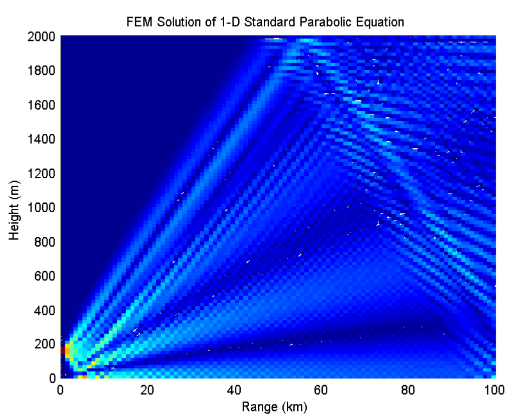

Fig. 2. Coverage diagram with out ABC

Figure 1 shows the dependence of refractivity on height under standard atmospheric conditions as defined by the International Telecommunications Union (ITU)

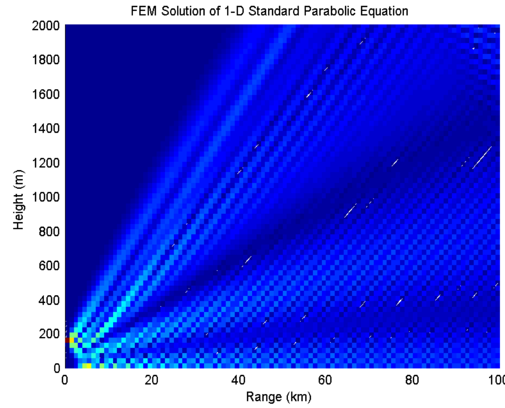

Fig. 3. Coverage diagram with ABC

Figure 2 and 3 presents the coverage diagram of transmitting antenna at 100MHz for standard atmospheric conditions [25]. As can be seen from figure 2 without the implementation of any boundary condition the waves get reflected from the upper boundary. In figure 3 when an absorning boundary condition is implemented these reflections are eliminated.

One advantage of the FEM over other methods such as the ray tracing techniques, is that one can incorporate frequency through the wave number $k = \frac{2\pi}{\lambda}$ in the simulations. Figure 4 shows coverage diagram of transmitting antenna at 200MHz, it can be quite clearly seen, that this diagram is different from that of figure 3 at 100MHz. A ray tracing technique would have produced exactly the same results.

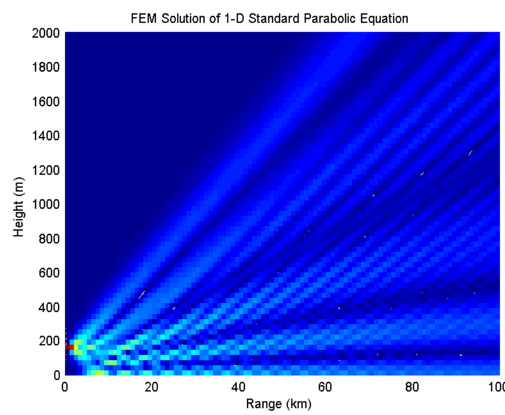

Fig. 4. Coverage diagram with of antenna with transmission at 200MHz

From the above discussion, it can be seen easily that waves propagates undisturbed through the tropospheric medium. From figure 5 to figure 13, a bilinear surface ducting profile is included, starting from the sea level to an altitude of $300m$. Standard atmospheric conditions over this altitude were also assumed, while the waveguide intensity was set to increase by step of -1N-units/m in each diagram. Coverage diagrams along with refractivity profile is shown. This illustrates the trapping mechanism and it is clear that as ducting intensity increases, a greater amount of the propagating energy is

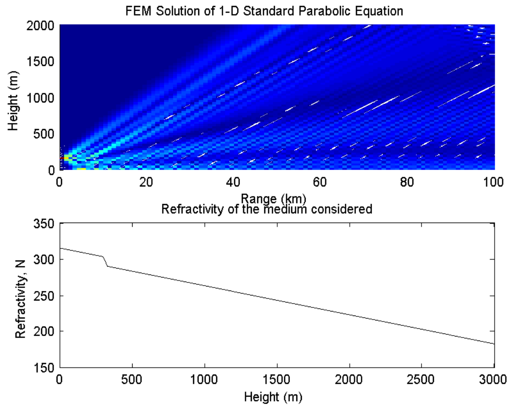

Fig. 5. Coverage diagrams at different waveguide intensity profiles

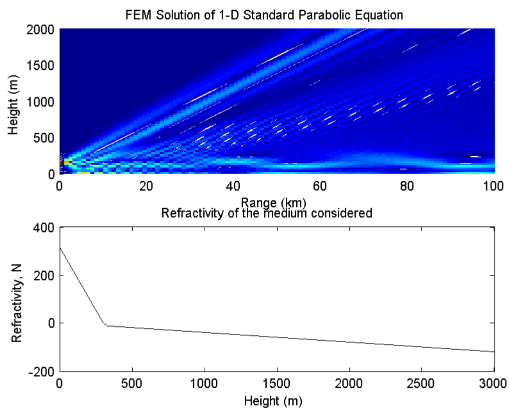

Fig. 6. Coverage diagrams at different waveguide intensity profiles

restricted inside the duct region. Especially in extraordinary profile, shown in figure 13, the waves are almost completely trapped between the sea level and $300m$.

VII. Conclusion

A computationally efficient method has been presented to model tropospheric radio wave propagation in the presence of height-dependent nonstandard environmental conditions, using Finite Element approach. Finite Element Method formulation can easily process complex refractivity profiles of any kind. Moreover, the refractive index being independent between consecutive range steps, giving the ability to include inhomogeneous tropospheric profiles. In these cases, the method response can be directly adjusted to the refractivity variations, by properly modifying the size of the finite elements and the range step.

Acknowledgment

The authors acknowledge the support of school of computing science, Middlesex University, London.

References

[1] H. V. Hitney, *"Refractive effects from VHF to EHF, Part A: Propagation mechanism,"*, AGARD-LS-196, 4A-1-4A-13, 1994.

[2] J. R. Wait, *"Coupled mode analysis for a non-uniform troposphere wave guide,"*, Radio Sri., vol. 15, no. 3, pp. 667-673, 1980.

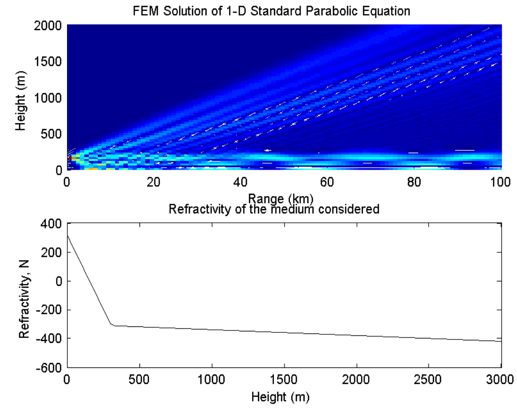

Fig. 7. Coverage diagrams at different waveguide intensity profiles

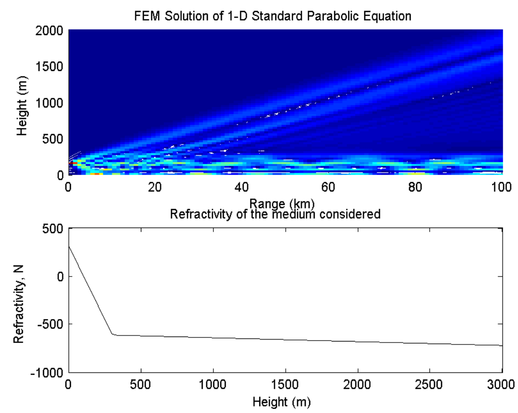

Fig. 8. Coverage diagrams at different waveguide intensity profiles

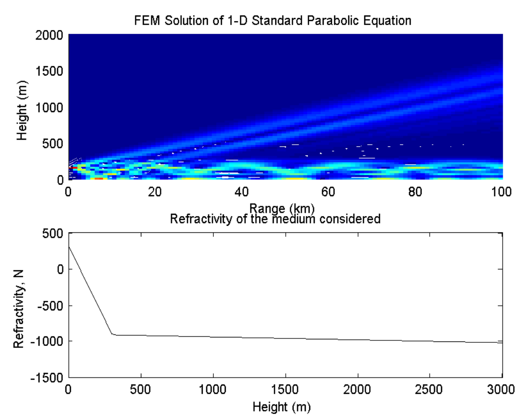

Fig. 9. Coverage diagrams at different waveguide intensity profiles

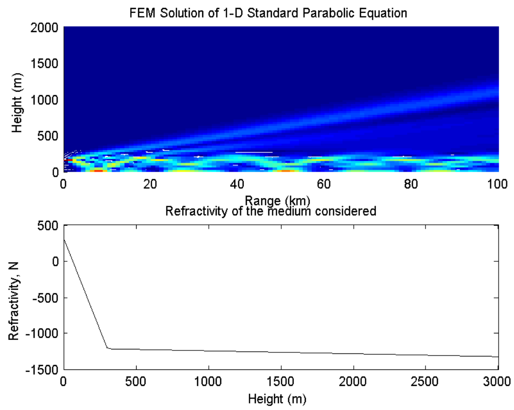

Fig. 10. Coverage diagrams at different waveguide intensity profiles

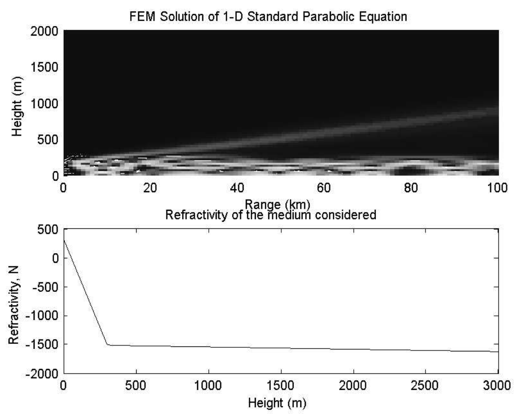

Fig. 11. Coverage diagrams at different waveguide intensity profiles

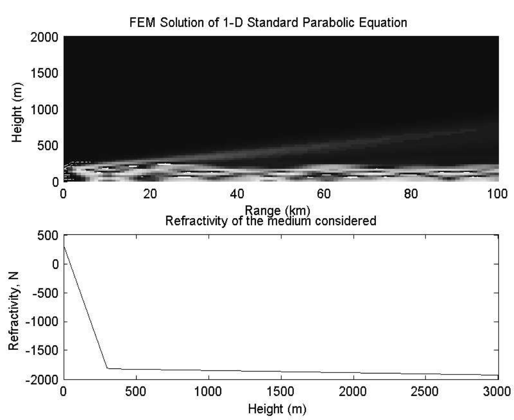

Fig. 12. Coverage diagrams at different waveguide intensity profiles

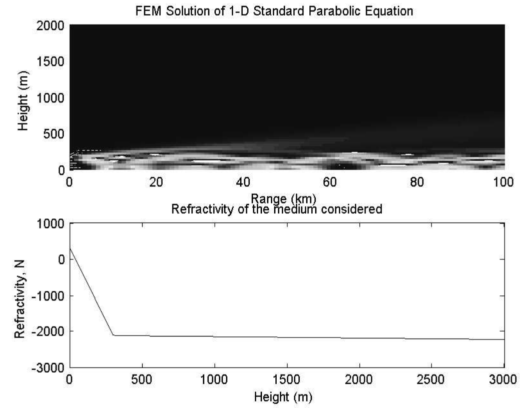

Fig. 13. Coverage diagrams at different waveguide intensity profiles

[3] S. H. Cho and J. R. Wait, *"Analysis of microwave ducting in troposphere,"*, Pure Appl. Geophys., vol. 116, pp. 1118-1142, 1978.

[4] L. B. Felsen, *"Hybrid ray-mode fields in inhomogeneous waveguides and ducts,"*, J. Acoust. Soc. Am., vol. 69, no. 2, pp. 352-361, 1981.

[5] P . L. Slingsby, *"Modeling tropospheric ducting effects on VHF/UHF propagation,"*, IEEE Transactions on Broadcasting, Vol. 37, Issue 2, 2534, 1991.

[6] G. B. Baumgartner, H. V. Hitney and R. A. Pappert, *"Duct propagation modeling for the integrated-refractive-effects prediction program (IREPS),"*, IEE Proc., Pt. F. Vol. 130, No. 7, 630642, 1983.

[7] G. B. Baumgartner, *"XWV G: A waveguide program for trilinear tropospheric ducts,"*, Tech. Doc. 610, Naval Ocean Systems Center, ADA133667, 1983.

[8] C. H. Shellman, *"A new version of MODESRCH using interpolated values of the magnetoionic reflection coefficients,"*, Interim technical report NOSC/TR-1143, Naval Ocean Systems Center,AD A179094, 1986.

[9] K. H. Craig, *"Propagation modelling in the troposphere: Parabolic equation method,"*, Electronics Letters, vol. 24, pp. 1136-1139, 1988.

[10] K. H. Craig and M. F. Levy, *"Parabolic equation modelling of the effects of multipath and ducting on radar systems,"*, IEEE Proc. F, Vol. 138, No. 2, 153162, 1991.

[11] S. T. McDaniel, *"Propagation of a normal mode in the parabolic approximation,"*, J. Acoust. Soc. Am., vol. 57, no. 2, pp. 307-311, 1975.

[12] F. D. Tappert, *"The parabolic approximation method,"*, in Wave Propagation and Underwater Acoustics Vol 70 (Lectures Notes in Physics),

[13] S. W. Marcus, *"A hybrid (finite difference-surface Greens function) method for computing transmission losses in an inhomogeneous atmosphere over irregular terrain,"*, IEEE Trans. Antennas Propagat., vol. 40, no. 12, pp. 1451-1458, Dec. 1992.

[14] R. J. McArthur, *"Propagation modelling over irregular terrain using the split-step parabolic equation method,"*, IEEE Intl Conf: Radar 92, no. 365, pp. 54-57, Oct. 12-13, 1992.

[15] F. J. Ryan, *"Analysis of electromagnetic propagation over variable terrain using the parabolic wave equation,"*, Rep. NOSC Tr 1453, Oct. 1991.

[16] A. E. Barrios, *"A terrain parabolic equation model for propagation in the troposphere,"*, IEEE Transactions on Antennas and Propagation,V ol. 42, Issue 1, 9098, 1994.

[17] M. Levy, *"Parabolic Equation Methods for Electromagnetic Wave Propagation"*, 2005 IEE Publisher.

[18] W. L. Flock, *"Propagation effects on satellite systems at frequencies below 10 GHz,"*, A Handbook for Satellite System Design,Second edition, NASA Reference Publication 1108 (02), 1987.

[19] K. Salonen, *"Observation operator for Doppler radar radial winds in HIRLAM 3D-Var,"*, Proceedings of ERAD (2002),405408, Copernicus GmbH, 2002.

[20] F.A. Katsriku, B.M.A. Rahman, & K.T.V. Grattan, *"Numerical Modelling of Second Harmonic Generation in Optical Waveguides Using the Finite Element Method"*, IEEE Journal of Quantum Electronics, Vol. 33, pp. 1727 - 1733. Oct. 1997.

[21] F.A. Katsriku, B.M.A. Rahman, & K.T.V. Grattan, *"Finite Element*

Analysis of Second Harmonic Generation in AlGaAs Waveguides", IEEE Journal of Quantum Electronics, Vol. 36, pp.282 - 289. March 2000.

[22] J. Jin, *"The Finite Element Method in Electromagnetics"*, John Wiley & Sons, New York, 1993.

[23] G. D. Dockery, *"Modeling electromagnetic wave propagation in the troposphere using the parabolic equation,"*, IEEE Transactions on Antennas and Propagation,V ol. 36, Issue 10, 14641470, 1988.

[24] W. Kwon Young, *"The Finite Element Method Using MATLAB"*, Second Edition, CRC Press, 2000.

[25] ITU-R, *"The radio refractive index: its formula and refractivity data,"*, International Telecommunication Union, Recommendation 453459, 2003.