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A FAST DIRECT METHOD FOR THE CALCULATION OF HOPF BIFURCATION POINT OF DYNAMIC VOLTAGE STABILITY IN POWER SYSTEM

LI HONG-ZHONG, CHENG HAO-ZHONG, WANG CHENG-MIN
Department of Electrical Engineering,
Shanghai Jiaotong University,
Huashan Road No.1954, Shanghai
P.R.CHINA

Abstract: - In classic direct method to calculate the Hopf bifurcation points, a (2n+2)-dimensional augmented was required for an n-dimensional power system. The computation was expensive to solve the augmented system. A fast method to calculate the Hopf bifurcation points in power system dynamic voltage stability is presented. A (n+2)-dimensional augmented system is founded which includes the differential-algebraic equations set to describe the dynamic characteristics of the power system and 2 scalar equations. The Hopf bifurcation points can be ascertained by solve the new augmented system. The proposed method has been applied to a classic power system dynamic model to illustrate its effectiveness.

Key-Words: - Power system; Dynamic voltage stability; Hopf bifurcation point; Direct method; Augmented system

1 Introduction

Bifurcation theory has been a powerful tool for the analysis of voltage stability in power system. Hopf bifurcation is a kind of representative dynamic bifurcation. In 1980, Van Ness J.E. firstly combined the oscillation of power system with the bifurcation theory and investigated the oscillatory occurrence associated with Hopf bifurcation [1]. Abed and Varaiya demonstrated the existence of subcritical Hopf bifurcation in a simple power system model [2]. Alexander discovered the supercritical and subcritical Hopf bifurcations by analyzing a small power system with 2 generators [3]. Henceforth a lot of bifurcation phenomenons, even chaos, were found in power system [4,5]. Now, Hopf has been regarded as one of the three kinds of bifurcations which can cause voltage instability [6,7]. Therefore it is very important for the dynamic voltage stability research to calculate the Hopf bifurcation points of power system [8~11].

There are two kinds of methods to compute Hopf bifurcation points:

(1) Continuous method, by tracing the equilibrium solution manifold and computing eigenvalues of Jacobi matrix during the continuation process. The Hopf bifurcation was obtained if a pair of complex conjugate eigenvalues crossing the imaginary axis. The numerical computation was complicated and time-consuming.

(2) Direct methods, using an augmented system of time independent equations for which the Hopf bifurcation point is an isolated solution.

The directed method can determine the Hopf bifurcation point on the solution manifold directly. However, in the classic direct method a (2n+2)-dimensional augmented was required for an n-dimensional power system [12~14]. The computation was expensive to solve the augmented system. A fast method to calculate the Hopf bifurcation points in power system dynamic voltage stability is presented. A (n+2)-dimensional augmented system is founded which includes 2 scalar equations and the differential-algebraic equations set to describe the dynamic characteristics of the power system.

2 Direct method

2.1 Hopf bifurcation

Consider the nonlinear dynamic system as show below:

$$\dot{x} = f(x, \lambda), x \in R^n, \lambda \in R^p \quad (1)$$

Assume that $(x(\lambda),\lambda)$ is one-parameter solution of a family of (1) and $A(\lambda)=f_x(x(\lambda),\lambda)$ is the one-parameter matrix family. Then the numerical computation of Hopf bifurcation becomes the problem to determine λ so that $A(\lambda)$ has a pair of pure imaginary eigenvalue. For the given λ_0,ω_0, if there exists sufficient smooth functions $\eta(\lambda),\omega(\lambda)$ which defined in a neighborhood of λ_0 and satisfy the following Hopf conditions H1-H4, then $(x_0,\lambda_0,\omega_0)\in R^n\times R^2$ will be a Hopf bifurcation point of (1).

H1 $\mu(\lambda)=\eta(\lambda)+i\omega(\lambda)$ is an eigenvalue of $A(\lambda)$.

H2 $\eta(\lambda_0)=0$, $\omega(\lambda_0)$£½$\omega_0>0$.

H3 $\eta'(\lambda_0)\neq 0$.

H4 $i\omega(\lambda_0)$ is an eigenvalue of $A(\lambda_0)$ and there is no eigenvalue of $A(\lambda_0)$ in the form of $ki\omega^0, k\neq 1$.

Let $\phi_0=\phi_1^0+i\phi_2^0$ be the corresponding eigenvector of the simple eigenvalue $i\omega(\lambda_0)$ of f_x^0.

2.2 Numerical method

To compute Hopf bifurcation points, Roose and Hlavacek proposed the following augmented system:

$$F(y)=F(x,p,\lambda,\omega)=\begin{vmatrix} f(x,\lambda) \\ ((f_x(x,\lambda))^2+\omega^2 I)p \\ <p,p>-1 \\ <q,p> \end{vmatrix}=\vec{0} \quad (2)$$

where q is a constant vector without null-projection in the space spanned by ϕ_1^0,ϕ_2^0. The mark $<,>$ denotes computing the scalar product of the vector.

Then there exists a unique vector $p_0\in span\left|\phi_1^0,\phi_2^0\right|$ such that $y_0=(x_0,p_0,\lambda_0,\omega_0)$ is an isolated solution of $F(y)=\vec{0}$ [14]. It is obvious that (2) is (2n+2)-dimensional equations set and to work out it will cost much time and storage. Therefore, it is disadvantageous for the application on the bulk power system.

In [15] a fast direct method was presented in which the augmented was reduced to an (n+2)-dimensional one. Now a sketch is shown as below.

Since $p_0\in span\left|\phi_1^0,\phi_2^0\right|$, there exists constants d_1, d_2 such that:

$$p_0=d_1\phi_1^0+d_2\phi_2^0 \quad (3)$$

Let $[f_x^0-i\omega_0 I]^*$ be the adjoint operator of $[f_x^0-i\omega_0 I]$, then:

$$Ker([f_x^0-i\omega_0 I]^*)=span\left|\psi 0=\psi_1^0+i\psi_2^0\right| \quad (4)$$

ψ_0 satisfies:

$$\begin{cases} <\psi_0,\phi_0> \;=2 \\ <\psi_1^0,\phi_1^0> \;=1,\quad <\psi_2^0,\phi_2^0> \;=1 \\ <\psi_1^0,\phi_2^0> \;=0,\quad <\psi_2^0,\phi_1^0> \;=0 \end{cases}$$

It is obvious that:

$$\begin{cases} Ker([(f_x^0)^2+\omega_0^2 I])=span\left|\phi_1^0,\phi_2^0\right| \\ Ker([(f_x^0)^2+\omega_0^2 I]^*)=span\left|\psi_1^0,\psi_2^0\right| \end{cases}$$

Define:

$$\begin{cases} \psi_3^0=\dfrac{1}{d_1^2+d_2^2}(d_1\psi_1^0+d_2\psi_2^0) \\ \psi_4^0=\dfrac{1}{d_1^2+d_2^2}(d_2\psi_1^0-d_1\psi_2^0) \end{cases} \quad (5)$$

then

$$<\psi_3^0,p_0>=1,\quad <\psi_4^0,p_0>=0 \quad (6)$$

Denote $k=\omega^2, k_0=\omega_0^2, A=\begin{vmatrix} f_x^2+kI & B \\ C^T & 0\end{vmatrix}$. Since $[f_x^0(x_0,\lambda_0)+k_0 I]$ has a dual eigenvalue, we could choose $B=(b_1,b_2)$, $C=(c_1,c_2)$ such that matrix A is nonsingular. Then Lemma 1 is obtained.

Lemma 1 The following systems are unique solvable:

$$A\bullet\begin{vmatrix} v \\ g_1 \\ g_2\end{vmatrix}=\begin{vmatrix}\vec{0} \\ 0 \\ 1\end{vmatrix} \quad (7)$$

$$\begin{vmatrix} u_1^T & k_1 & g_1 \\ u_2^T & k_2 & g_2\end{vmatrix}\bullet A=\begin{vmatrix}\vec{0} & 1 & 0 \\ \vec{0} & 0 & 1\end{vmatrix} \quad (8)$$

where $v,u_i\in R^n, k_i,g_i\in R(i=1,2)$ are functions of (x,λ,k), and

(1) $g_i=-<u_i,(f_x^2+kI)v>$

$$(2)\begin{cases} \dfrac{\partial g_i}{\partial x}=-<u_i,\dfrac{\partial(f_x^2)}{\partial x}v> \\ \dfrac{\partial g_i}{\partial \lambda}=-<u_i,\dfrac{\partial(f_x^2)}{\partial \lambda}v> \\ \dfrac{\partial g_i}{\partial k}=-<u_i,v> \end{cases}$$

The proof of Lemma 1 can be seen in [15].

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The following augmented system is presented to compute the Hopf bifurcation point.

$$G(y)=G(x,\lambda,k)=\begin{vmatrix} f(x,\lambda) \\ -g_1(x,\lambda,k) \\ -g_2(x,\lambda,k) \end{vmatrix} = \begin{vmatrix} f(x,\lambda) \\ <u_1,[(f_x(x,\lambda))^2+kI]>v \\ <u_2,[(f_x(x,\lambda))^2+kI]>v \end{vmatrix} = \vec{0} \quad (9)$$

where $u_i, v, g_i (i=1,2)$ are obtained by Lemma 1 and

$$\begin{cases} u_1(x_0,\lambda_0,k_0)=\psi_1^0 \\ u_2(x_0,\lambda_0,k_0)=\psi_2^0 \\ v(x_0,\lambda_0,k_0)=p_0 \end{cases}$$

Theorem 1 Assume that (x_0,λ_0,k_0) is a Hopf bifurcation of $f(x,\lambda)=\vec{0}$ with $G(y)$ defined as (9), then $y_0=(x_0,\lambda_0,k_0)$ is an isolated point of $G(x,\lambda,k)=\vec{0}$.

Before we prove Theorem 1, the following Lemma 2 is presented.

Lemma 2 If (x_0,λ_0,k_0) is a Hopf bifurcation of $G(x,\lambda,k)=\vec{0}$, then

$$<\psi_4^0,((f_x^0 f_{xx}^0 p_0+f_{xx}^0 f_x^0 p_0)v_0+f_x^0 f_{x\lambda}^0 p_0+f_{x\lambda}^0 f_x^0 p_0)> =2\eta'(\lambda_0)\omega_0$$

The proof of Lemma 2 can be seen in [15].

Proof of Theorem 1:

It is needed to prove the following linear system has only trivial solution.

$$D_y G(y_0)\vartheta=\vec{0}, \quad \vartheta=(x,\lambda,k)\in R^{n+2} \quad (10)$$

Expanding (10) yields:

$$\begin{vmatrix} f_x^0 & f_\lambda^0 & 0 \\ <\psi_1^0, f_x^0 f_{xx}^0 p_0+f_{xx}^0 f_x^0 p_0> & <\psi_1^0, f_x^0 f_{x\lambda}^0 p_0+f_{x\lambda}^0 f_x^0 p_0> & <\psi_1^0,p_0> \\ <\psi_2^0, f_x^0 f_{xx}^0 p_0+f_{xx}^0 f_x^0 p_0> & <\psi_2^0, f_x^0 f_{x\lambda}^0 p_0+f_{x\lambda}^0 f_x^0 p_0> & <\psi_2^0,p_0> \end{vmatrix} \bullet \begin{vmatrix} x \\ \lambda \\ k \end{vmatrix} = \vec{0} \quad (11)$$

Namely

$$\begin{cases} f_x^0 x+f_\lambda^0\lambda=\vec{0} & (12-1) \\ <\psi_1^0, f_x^0 f_{xx}^0 p_0+f_{xx}^0 f_x^0 p_0>\cdot x+<\psi_1^0, f_x^0 f_{x\lambda}^0 p_0+f_{x\lambda}^0 f_x^0 p_0>\cdot\lambda+<\psi_1^0,p_0>\cdot k=0 & (12-2) \\ <\psi_2^0, f_x^0 f_{xx}^0 p_0+f_{xx}^0 f_x^0 p_0>\cdot x+<\psi_2^0, f_x^0 f_{x\lambda}^0 p_0+f_{x\lambda}^0 f_x^0 p_0>\cdot\lambda+<\psi_2^0,p_0>\cdot k=0 & (12-3) \end{cases}$$

Since f_x^0 is nonsingular, it follows from (12-1) that

$$f_x^0 x+\lambda f_\lambda^0=\vec{0}$$
$$x=\lambda v_0$$

where v_0 satisfies

$$f_x^0 v_0+f_\lambda^0=\vec{0} \quad (13)$$

Multiplying (12-2) by $(d_1^2+d_2^2)^{-1}\cdot d_1$, and (12-3) by $(d_1^2+d_2^2)^{-1}\cdot d_2$, then take the sum, yields

$$<\psi_3^0,(f_x^0 f_{xx}^0 p_0+f_{xx}^0 f_x^0 p_0)x> +\lambda<\psi_3^0, f_x^0 f_{x\lambda}^0 p_0+f_{x\lambda}^0 f_x^0 p_0>+k<\psi_3^0,p_0>=0 \quad (14)$$

In a similar way, multiplying (12-2) by $(d_1^2+d_2^2)^{-1}\cdot d_2$, and (12-3) by $[-(d_1^2+d_2^2)^{-1}\cdot d_1]$, then take the sum, yields

$$<\psi_4^0,(f_x^0 f_{xx}^0 p_0+f_{xx}^0 f_x^0 p_0)x> +\lambda<\psi_4^0, f_x^0 f_{x\lambda}^0 p_0+f_{x\lambda}^0 f_x^0 p_0>+k<\psi_4^0,p_0>=0 \quad (15)$$

Substitute $x=\lambda v_0$ into (15) and take (6) into account, then

$$\lambda<\psi_4^0,((f_x^0 f_{xx}^0 p_0+f_{xx}^0 f_x^0 p_0)v_0 + f_x^0 f_{x\lambda}^0 p_0+f_{x\lambda}^0 f_x^0 p_0)>=0 \quad (16)$$

It follows from the result of Lemma 2 that (16) becomes:

$$\lambda\cdot 2\eta'(\lambda_0)\omega_0=0 \quad (17)$$

According H2, H3, (17) has one unique solution $\lambda=0$, and consequently $x=\vec{0}$. It follows from (16) that $k<\psi_3^0,p_0>=0$, so $k=0$. So far the proof is finished.

2.3 Arithmetic procedure

The numerical computational process of Hopf bifurcation points is shown as below:

Step 1: Suitably choose *B*, *C* and initial value (x_0,λ_0,k_0) of $y=(x,\lambda,k)$.

Step 2: Solve the linear equations (7), (8) and obtain $u_i, v, g_i (i=1,2)$.

Step 3: Solve the augmented system (9) based on Newton's method and obtain $\Delta y_m (m = 0,1,\ldots)$.

Step 4: Let $y_{m+1} = y_m + \Delta y_m$ and go back to step 2 until the desired accuracy is satisfied.

It can be seen from the above steps that there are 3 linear (n+2)-dimensional equations sets and a nonlinear one need to be solved. Thus it takes less time and storage compared with the algorithm by solving (2) which is of order 2n+2.

3 Numerical example

3.1 mathematic models

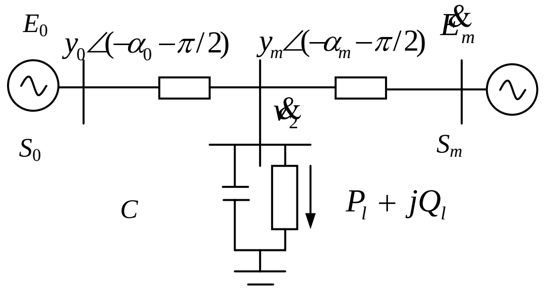

Fig.1 Power system connecting diagram

Consider the power system shown in Fig.1 that consists of a load which is supplied by two generators**.** One generator is a slack bus and the other one has constant voltage magnitude and the dynamics model is given as below.

$$\begin{cases} \dot{\delta}_m = \omega \\ M\dot{\omega} = T_m + E_m v_2 y_m \sin(\theta - \delta_m + \alpha_m) \\ \quad + E_m^2 y_m \sin\alpha_m - D_m\omega \end{cases} \quad (18)$$

where M, D_m, T_m are the generator inertia, damping and input angular force respectively.

The load is represented by a capacitor in parallel with an induction motor. Instead of including the capacitor in the circuit, it is convenient to account for the capacitor by adjusting E_0 and Y_0 to give the Thevenin equivalent of the circuit with capacitor as shown in Fig.2.

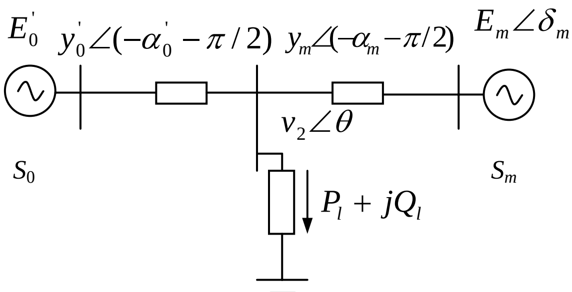

Fig.2 Power system connecting diagram

The adjusted values are shown as below.

$$\begin{cases} E_0^{'} = E_0 \big/ (1 + C^2 y_0^{-2} - 2Cy_0^{-1}\cos\alpha_0)^{1/2} \\ y_0^{'} = y_0 (1 + C^2 y_0^{-2} - 2Cy_0^{-1}\cos\alpha_0)^{1/2} \\ \alpha_0^{'} = arctg\dfrac{y_0 \sin\alpha_0}{y_0\cos\alpha_0 - C} \end{cases} \quad (19)$$

The real and reactive powers supplied to the load by the network are

$$\begin{cases} P_l = -E_0^{'} v_2 y_0^{'} \sin(\theta + \alpha_0^{'}) - E_m v_2 y_m \sin(\theta - \delta_m + \alpha_m) \\ \quad + (y_0^{'}\sin\alpha_0^{'} + y_m \sin\alpha_m) v_2^2 \\ Q_l = E_0^{'} v_2 y_0^{'} \cos(\theta + \alpha_0^{'}) + E_m v_2 y_m \sin(\theta - \delta_m + \alpha_m) \\ \quad - (y_0^{'}\cos\alpha_0^{'} + y_m\cos\alpha_m) v_2^2 \end{cases} \quad (20)$$

The combined load model is

$$\begin{cases} P_l = P_0 + P_1 + K_{p\omega}\dot{\theta} + K_{pv}(v_2 + T\dot{v}_2) \\ Q_l = Q_0 + Q_1 + K_{q\omega}\dot{\theta} + K_{qv}v_2 + K_{pv2}v_2 \end{cases} \quad (21)$$

where P_0, Q_0 are the real and reactive powers of the motor and P_1, Q_1 represent the constant PQ load.

Considering (18)~(21), the system differential equations are obtained in the form of equation (22).

$$\begin{cases} \dot{\delta}_m = \omega \\ M\dot{\omega} = T_m + E_m v_2 y_m \sin(\theta - \delta_m + \alpha_m) \\ \quad + E_m^2 y_m \sin\alpha_m - D_m\omega \\ K_{q\omega}\dot{\theta} = -K_{qv}v_2 - K_{qv2}v_2^2 + Q_l - Q_0 - Q_1 \\ TK_{q\omega}K_{pv}\dot{v}_2 = K_{p\omega}K_{qv2}v_2^2 + (K_{p\omega}K_{qv} - K_{q\omega}K_{pv}) \\ \quad + K_{p\omega}(Q_0 - Q_l + Q_1) - K_{q\omega}(P_0 - P_l + P_1) \end{cases} \quad (22)$$

The equations of this system consist of four state variables that correspond to generator angle δ_m, generator angular velocity ω, the angle θ and magnitude v_2 of load voltage. The load reactive power Q_1 is chosen as the system controlling parameter.

Parameters of the power system are shown in Tab.1.

Table 1 Parameters of the power system

E_0	T_m	D_m	M	E_m
1.0	1.0	0.05	0.3	1.0
y_0	α_0	C	y_m	α_m
20.0	-5^0	12.0	5.0	-5^0
$K_{p\omega}$	$K_{q\omega}$	K_{pv}	K_{qv}	K_{qv2}
0.4	-0.03	0.3	-2.8	2.1
T	P_0	Q_0	P_1	
8.5	0.6	1.3	0.0	

3.2 Computing result

One Hopf bifurcation point (H_a) of the example system is obtained if the initial values of state variables and controlling parameter are (0.30, 0.0, 0.10, 1.0) and $Q_1 = 10.8$ respectively by applying the method presented and another one (H_b) is found with the initial values (0.35, 0.0, 0.12, 0.9) and $Q_1 = 11.3$. The result is shown in Tab.2 in detail.

Table 2 Hopf bifurcation points

	δ_m	ω	θ	v_2	Q_1
H_a	0.3059	0.0000	0.1157	1.0986	10.944
H_b	0.3471	0.0000	0.1534	0.9239	11.404

The result is in conformity with that presented in [16] which illustrate the effectiveness of the method put forward in this paper. However this method takes less time and storage compared with the classic ones.

4 Conclusion

A fast method to calculate the Hopf bifurcation points is introduced to the analysis of dynamic voltage stability in power system. This method is a predigestion of the classic ways. A (n+2)-dimensional augmented system is established which includes the differential-algebraic equations set to describe the dynamic characteristics of the power system and 2 scalar equations. The Hopf bifurcation points can be ascertained by solve the new augmented system. The application to a classic power system dynamic model illustrates its effectiveness. The presented method takes less time and storage compared with the classic ones, so it is suitable for the application of dynamic voltage stability analysis in bulk power system.

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