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An Improved Method for Determining Voltage Collapse Proximity of Radial Distribution Networks

A. AUGUGLIARO, L. DUSONCHET, S. FAVUZZA, S. MANGIONE
Dept. of Electrical, Electronic and Telecommunication Engineering
Università degli Studi di Palermo
Viale delle Scienze, 90128 Palermo
ITALY
http://www.diepa.unipa.it

Abstract: - The two-bus equivalent model is commonly used for voltage stability studies in both distribution and transmission systems. The paper presents a simple method to evaluate, for each bus, the parameters which define the equivalent circuit of a radial distribution network. In particular, a straightforward way for determining the Thévenin equivalent impedance behind a load node is proposed, which allows to better identify the maximum loading point beyond which the voltage collapse takes place in the network. Simulation results show that the proposed method is significantly more accurate than other existing methods on evaluating the critical power at a particular node (i.e. the weak node of the network) starting from any operating point.

Key-words: - Radial distribution network, voltage collapse, voltage-stability index, critical power.

1 Introduction

The voltage stability problem has become one of the major concerns of several researchers and power system planning engineers in the last two decades. This problem and above all the related phenomenon of voltage collapse have been concerning, till now, power transmission networks. The main reason of that is the constant increase in system loading in spite of a limited expansion of these systems for economical and environmental reasons. Furthermore, the deregulated energy market and the trend to a higher exploitation of existing transmission and distribution facilities have worsened even more the problem of voltage instability or collapse, extending the phenomenon also to the distribution networks.

At a given operating point of the system, the determination of the voltage stability features of a network is generally done with the following steps:

1. definition of a proper voltage stability index to assign to each node of the network;
2. identification of the weak node, i.e. the most vulnerable node of the network;
3. evaluation of the maximum loading capability of the weak node or the maximum loadability margin of the entire network, beyond which voltage collapse takes place.

Even though the phenomenon of voltage instability involves dynamic aspects, a static approach is usually adopted for evaluating stability conditions and load capability of either the weak node or the entire network, starting from static models of the system.

Voltage stability problems have been till now widely dealt with in literature by several authors, especially with reference to transmission networks and the heavy consequences of system blackouts due to the voltage collapse [1]. On the contrary, the research on the phenomenon of voltage instability of radial distribution networks is still at an early stage. Jasmon and Lee [2,3] established the mathematical condition for voltage instability of a radial network starting from an single-line/two-bus equivalent system of the entire network. The equivalent system is determined on the basis of the load flow solution of radial distribution networks developed by Baran and Wu [4]. A different equivalent of a radial network, based on voltage phasor measurements, has been proposed by Gubina and Strmčnik [5] in order to asses voltage collapse proximity and active and reactive power margins. A stability index for the load nodes of radial distribution network is proposed by Chakravorty and Das [6], also considering compound loads; the index is defined by solving a single-line equivalent and determining the condition for which the voltage magnitude at the ending node has a real solution. Finally, the loadability limits of real distribution networks has been examined by Prada and Souza [7] with respect to voltage stability as well as to thermal constraints, demonstrating that maximum loading can be limited by voltage stability rather than thermal limit. Thus, it follows the necessity of considering voltage stability as a new constraint in operation and planning of distribution systems, especially when new loads are added to a part of the system because of network reconfigurations.

This paper deals with voltage stability assessment in radial distribution networks. The voltage collapse proximity index proposed by Chebbo et al. [8], for power transmission systems, is assumed. Based on the optimal impedance solution of a two-bus equivalent system, this index indicates how far the load nodes of the actual network are from their voltage collapse points, allowing the weak node and its critical power to be identified, i.e. the maximum load power beyond which voltage collapse takes place.

The proposed method differs from the one used in [8] and the revised method proposed by Haque [9] on the determination of the two-bus equivalent, obtaining much better results in evaluating the critical power at the weak node, with smaller errors between actual and predicted values than others methods.

2 Background Theory

In this section the expressions of the voltage stability index and the critical power for a two-bus system are given and the validity limits for their extension to an actual radial distribution system are defined.

2.1 A two-bus system

Consider a two-bus system in which a load represented by the impedance $Z_L\angle\phi$ is fed by a constant voltage source $\boldsymbol{V_S}$ (Fig. 1).

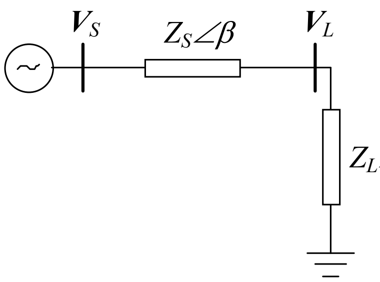

Fig. 1 – Two-bus system.

If $Z_S\angle\beta$ represents the impedance of the line between the source and the load node and the shunt admittances are neglected, the active power transmitted at the load node can be expressed as [8]:

$$P_L = \frac{\dfrac{V_S^2}{Z_S}}{1+\left(\dfrac{Z_L}{Z_S}\right)^2+2\left(\dfrac{Z_L}{Z_S}\right)\cos(\beta-\phi)}\left(\frac{Z_L}{Z_S}\right)\cos\phi \qquad (1)$$

Starting from an operating point, if the load at the receiving end of Fig. 1 is increased but maintaining the power factor constant (i.e. the modulus of load impedance Z_L decreases while ϕ remains constant) the load current I flowing through the feeding line increases causing an increase on voltage drop along the line, so that load voltage V_L decreases; consequently P_L, calculated by (1), at first increases (when $Z_L>Z_S$) and after reaching a maximum it decreases ($Z_L<Z_S$), which make the system become instable when the load is of constant power type or of composite type with a portion of such voltage-independent load within the total load.

2.2 Critical power and voltage stability index

On the basis of equation (1) the condition of the maximum power transferred to the load ($\partial P_L/\partial Z_L=0$) is achieved when the modulus of the load impedance equals that of the line impedance:

$$Z_L = Z_S \qquad (2)$$

In this case the transmitted active power reaches the critical value given by the following expression [8]:

$$P_{crit} = \frac{\left(\dfrac{V_S^2}{Z_S}\right)\cos\phi}{2[1+\cos(\beta-\phi)]} \qquad (3)$$

which represents, for a given load power factor ($\cos\phi$), the maximum loading point beyond which the voltage collapse occurs when constant power loads are considered.

Note that the above condition takes place only when the load impedance and the line impedance are equal in modulus ($Z_S/Z_L=1$). At the normal operation the load impedance is much greater than the line impedance ($Z_S/Z_L<<1$); hence the value assumed by the impedance ratio Z_S/Z_L when load changes can be considered as the node distance indicator from the voltage collapse point.

Thus the following voltage stability index (*SI*) of the load node can be assumed:

$$SI = \frac{Z_S}{Z_L} \qquad (4)$$

The closer this ratio to 1, the nearer to the voltage collapse point the load is.

In conclusion, it is important to point out some properties of relationship (3) when constant power loads are considered. The critical load expressed by (3) really represents the maximum loading capability beyond which voltage collapse occurs only if:

- a two-bus system is considered (Fig. 1);

- the source voltage magnitude V_s, and the impedance Z_s between the source and the load node are kept constant when the load (Z_L) changes.

2.3 Extension of P_{crit} and *SI* to an actual radial distribution system

On the basis of the above conclusion it is evident that is not possible to apply the expressions (3) and (4) directly to an actual radial distribution system constituted by a main feeder, laterals and a great number of load nodes; in order to apply the above expressions for a given load node it is necessary firstly to reduce the radial network to a two-bus equivalent system. Moreover, when loads in the network change, in order to evaluate *SI* and P_{crit} with the equivalent system, the voltage at the feeding bus and the equivalent impedance of the network should remain constant.

A given radial distribution network can be reduced to a two-bus equivalent by applying the Thévenin's theorem at a particular node when the system loads and generators are linearised around the operating point.

Starting from an operating point the voltage stability index can be then computed for every node of the network by applying relationship (4). The node with the maximum value of *SI* specifies the weak node of the system (generally the one where voltage takes on the lowest value); since it represents the most vulnerable node of the network, the critical active power can be evaluated by equation (3), to asses the node power margin before reaching the voltage stability limit. This procedure can be repeated at any operating point and should be implemented as a tool for real-time applications in automated distribution systems.

Due to the strongly non-linearity of the original unreduced system, the parameters of the Thévenin equivalent circuit vary when system loading changes, so that the predicted value of the critical power at a given operating point will differ from that evaluated at a different point and will not match the corresponding actual value exactly. In other words, according to the conclusions drawn at the end of 2.2, there is no way to analytically evaluate the maximum loading capability of an actual radial network working at a given operating point. Nevertheless, the closer to the maximum loading point the considered operating point is, the more the predicted critical power evaluated by (3) will match the corresponding actual value.

The proposed method allows much better results than those of other methods [8,9] in evaluating the critical power at any operating point of the network, with smaller errors between actual and predicted values. Moreover, on determining the two-bus equivalent, the proposed method differs from other methods in that it does not need to evaluate the impedance matrix of the network, but requires only two load flow solutions of the original system, with less time and storage consumption.

3 Two-bus Equivalent of a Radial Network

This section describes in detail the procedures for finding the equivalent of the radial network with existing methods, which utilize the bus impedance matrix $\boldsymbol{Z}$ [8,9], and with the proposed method.

3.1 Problem formulation

Consider a radial network fed by a generator and containing a number of load nodes working at a given operating point (Fig. 2). Assume the loads as constant power with constant power factor.

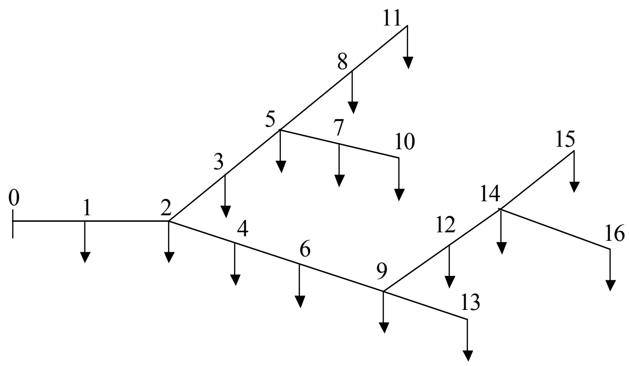

Fig. 2 – Radial distribution feeder with laterals.

For a generic node k the Thévenin's theorem allows the equivalent of the radial network to be obtained as shown in Fig. 3, representing the whole system behind the considered node, where $\boldsymbol{E}_{Th}$ is the no-load voltage at node k and $\boldsymbol{Z}_{Th}$ is the impedance of the network as seen from the same node.

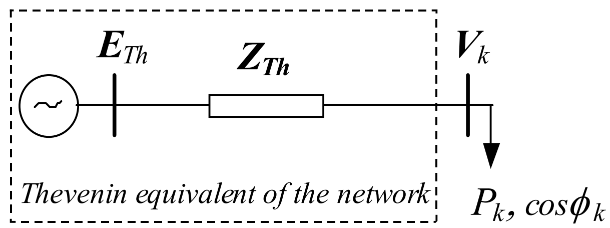

Fig. 3 – Two-bus equivalent of a radial network.

In particular, if the weak node of the network is chosen as the candidate node, the two-bus equivalent of Fig. 3 represents the whole weak-node/network at the given operational point and equation (3) can be applied to establish the node power margin before reaching the voltage collapse point.

3.2 The *Z*-matrix based method [8,9]

As known, the Thévenin equivalent impedance can be calculated by means of the system **Z**-matrix, whose diagonal elements represent the impedance of the network as seen from different nodes.

To determine the two-bus equivalent, taking into account the non-linearity of the system, due to the loads, and assuming k as the candidate node (i.e. the weak node), the following steps should be completed [8]:

1. the load flow solution of the network at the operating point is calculated, to obtain the node voltages profile;
2. all loads are linearised, replacing them by constant admittances:

$$Y_i \angle -\phi_i = \frac{P_i}{V_i^2} \cos\phi_i \angle \phi_i \tag{5}$$

3. the admittance matrix $\boldsymbol{Y}$, not including node k, is evaluated;
4. the impedance matrix $\boldsymbol{Z}$ is obtained by inversion of the $\boldsymbol{Y}$ matrix;
5. $\boldsymbol{Z}_{Th}=\boldsymbol{Z}_{kk}$ (k-th diagonal element of $\boldsymbol{Z}$) is assumed;
6. the load flow solution of the linearised model, ignoring the load at the node k, is calculated, in order to evaluate the no-load voltage $\boldsymbol{E}_{Th}$.

The entire procedure has to be repeated when the system loading changes or when the Thévenin equivalent circuit behind a different node is needed because the weak node has changed.

In the above procedure at step 3 the admittance matrix $\boldsymbol{Y}$ should be evaluated including also the candidate node k; in this case the Thévenin impedance and voltage of the equivalent circuit can be obtained as [9]:

$$\boldsymbol{Z}_{Th} = \frac{(\boldsymbol{Z}_k - \boldsymbol{Z}_{kk})}{\boldsymbol{Z}_k \boldsymbol{Z}_{kk}} \tag{6}$$

$$\boldsymbol{E}_{Th} = \left(1 + \frac{\boldsymbol{Z}_k}{\boldsymbol{Z}_{kk}}\right) \boldsymbol{V}_k \tag{7}$$

where $\boldsymbol{Z}_k$ is the equivalent impedance of the load at node k and $\boldsymbol{V}_k$ is the voltage at node k obtained at step 1.

Steps 5 and 6 of the above procedure are then replaced by relations (6) and (7).

In this manner it is possible to evaluate the Thévenin equivalent circuit behind all nodes without the need of repeating the process of the **Z**-matrix determination for every candidate node.

3.3 The proposed method

A conceptually similar but methodically less sophisticated algorithm for determining the two-bus equivalent system is the base of the proposed method. Two load flow solutions of the network are required; the first with all the loads, the second without the load at the weak node. For this purpose a backward/forward method, suitable for radial networks and which requires small time and storage consumptions, can be employed.

Starting from a given operating point of the original network and referring to the circuit of Fig. 3 and to the meaning of its parameters, the following steps are performed:

1. run the load flow program for the network with all the actual loads (considered as constant power), obtaining the load voltage $\boldsymbol{V}_k$ at the candidate node k and the load currents $\boldsymbol{I}_k$, with:

$$\boldsymbol{I}_k = \frac{(P_k - jQ_k)}{\boldsymbol{V}_k^*} \tag{8}$$

   where P_k and Q_k are the active and reactive power at node k and $\boldsymbol{V}_k^*$ is the complex conjugate of the voltage;
2. run the load flow program without the load at the node k ($P_k = 0;\ Q_k = 0$), obtaining the no-load voltage $\boldsymbol{E}_{Th}$ of the Thévenin equivalent circuit;
3. evaluate the equivalent impedance of the two-bus system $\boldsymbol{Z}_{Th}$ as:

$$\boldsymbol{Z}_{Th} = \frac{(\boldsymbol{E}_{Th} - \boldsymbol{V}_k)}{\boldsymbol{I}_k} \tag{9}$$

The applications demonstrated that, on computing the critical power defined by (3) at the weak node, the proposed methodology produces a smaller error than the methods using the bus impedance matrix **Z**. Moreover, as the error is inversely proportional to the load at the weak node, when load increases the predicted value of the critical power will become more and more close to the actual value. This favorable circumstance makes the method particularly suitable for on-line monitoring and control of a distribution network, to evaluate how much the critical node can be loaded before reaching the point of voltage collapse.

The process described above has to be repeated when the system loading changes. Moreover, if loads vary non conformally, the weak node on the original unreduced network should change; in this case the new candidate node k has to be located by evaluating the stability index of all the nodes as defined by (4), being:

$$SI_k = \max_{i \in N} \left\{ \frac{Z_{Th,i}}{Z_i} \right\} \tag{10}$$

where N is the number of the nodes in the network.
To this end, and in the perspective of real-time applications, the evaluation of the $\boldsymbol{Z}$-matrix which includes all of the nodes of the network can be performed at step 1, to easily check by (10) if any change occurs on the weak node location when the system loading changes.

4 Simulation Results

To demonstrate the effectiveness of the proposed method for determining the voltage stability limit of a general radial system through the two-bus equivalent, the 85-node test system represented in Fig. 4 and whose data are reported in [10] has been used in the simulations. All loads are considered as constant power with constant power factor. At the base case operating point, node 53 is the weakest node and its active critical power has been evaluated for different load conditions and magnitude of the source voltage. For load flow computations a computer program based on the backward/forward method developed by the Authors [11] has been used and the convergence tolerance was set at 10^{-5}.
In the following, the computational results are compared with those obtained by means of repetitive load flow (RLF) calculations and the $\boldsymbol{Z}$-matrix based method in the formulation proposed by Haque [8]. The RLF computation allows the maximum loading capability of the network to be determined by gradually increasing the loading system and computing the load flow solution in the original unreduced network until the method fails to converge. This straightforward method is very time consuming and is not suitable for on-line applications; nevertheless, in spite of some numerical problems in the neighbourhood of the voltage instability point [12], the results obtained are very close to the actual values and usually taken as reference for comparison.
For the network of Fig 4, by setting the magnitude of the source voltage 1 p.u. and the loads power factor 0.9, the load at the node 53 has been gradually increased as a multiple of the nominal value. For each load increase the two-bus equivalent has been determined by the proposed method and by the Haque method, evaluating the predicted critical power at the weak node.
Finally, the maximum loading capability has been calculated on the original unreduced network by the RLF method. Results are shown in Fig. 5, which represents the variation of the critical power of node 53 vs. the load power demand as obtained with the different methods.
For the above reason, if it is considered that the actual value of P_{crit} should be slightly higher than the value obtained by the non-convergence criterion of the RLF method, it can be seen that the values obtained by the proposed method at each operating point are much closer to the actual value than the $\boldsymbol{Z}$-matrix based method; the maximum error occurs with both methods at the base case and is 4.78% with the proposed method and 22.36% with the other method, while at the verge of voltage collapse the errors are 0.36% and 4.70%, respectively.

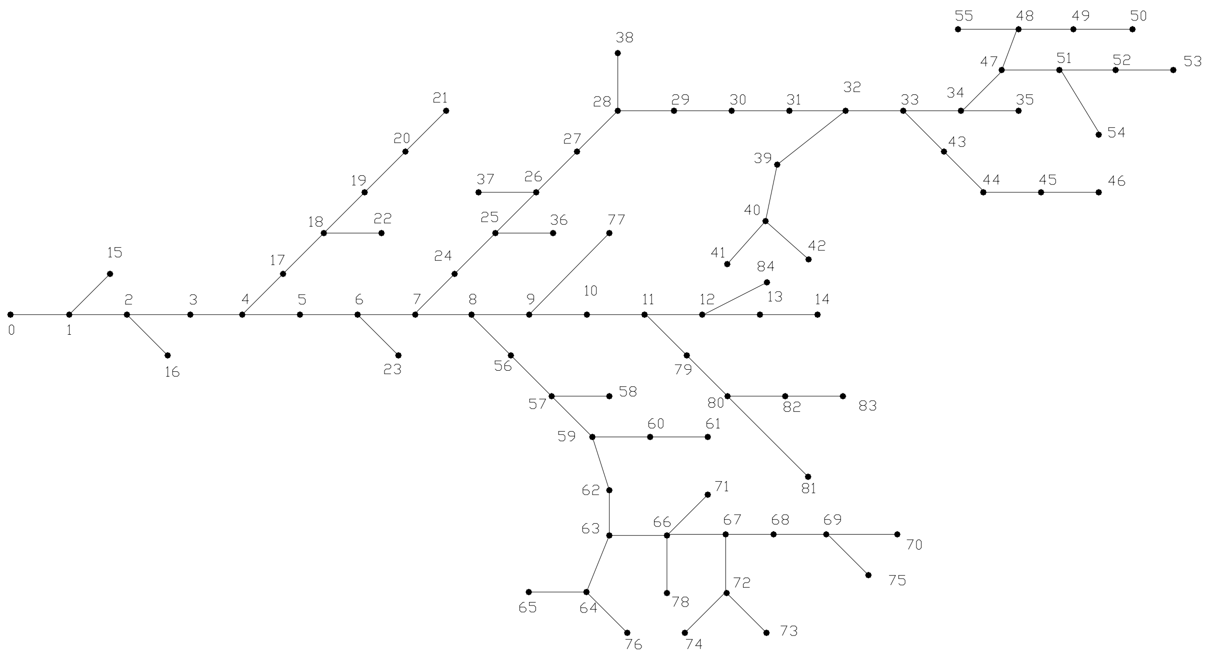

Fig. 4 – The 85-node test system.

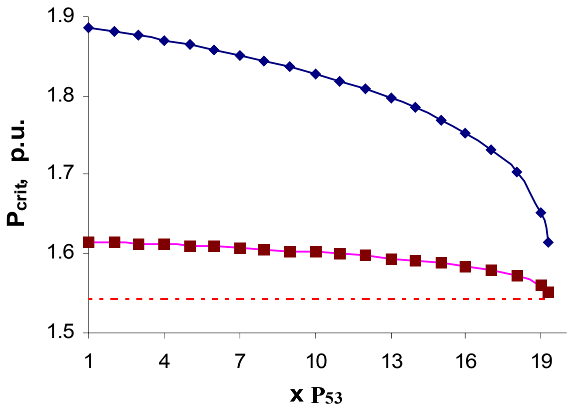

Fig. 5 - Variation of P_{crit} with load power demand at node 53.

RLF method (actual values)
Proposed method
Z-matrix method [6]

In Table 1 the predicted critical power at node 53 obtained at the base case by the two methods is compared with the actual value (RLF method), for various loads power factors and magnitudes of the source voltage (V_0).

Table 1 – Critical power at node 53 for different loads and source voltage conditions (Lf = 1.0)

V_0 [pu]	Cos ϕ	RLF (actual) [kW]	Propo-sed method [kW]	Err %	Z-matrix method [kW]	Err %
1.1	1	2,578	2,656	**3.0**	2,911	**12.9**
	0.9	2,050	2,123	**3.5**	2,396	**16.9**
	0.8	1,743	1,817	**4.3**	2,091	**20.0**
	0.7	1,459	1,536	**5.3**	1,808	**23.9**
1	1	1,988	2,066	**3.9**	2,321	**16.7**
	0.9	1,542	1,616	**4.8**	1,887	**22.4**
	0.8	1,285	1,359	**5.8**	1,631	**26.9**
	0.7	1,045	1,123	**7.5**	1,393	**33.3**
0.9	1	1,450	1,528	**5.5**	1,782	**22.9**
	0.9	1,078	1,152	**6.8**	1,419	**31.7**
	0.8	863	938	**8.7**	1,205	**39.8**
	0.7	661	739	**11.7**	1,004	**51.8**

The same comparison when a load factor (Lf) of 1.25 is applied to all of the loads of the original network is given in Table 2.
Simulation results show that the proposed method is significantly more accurate than the ***Z***-matrix based method proposed in [8,9].

Table 2 – Critical power at node 53 for different loads and source voltage condition (Lf = 1.25).

V_0 [pu]	cosϕ	RLF (actual) [kW]	Propo-sed method [kW]	Err %	Z-matrix method [kW]	Err %
1.1	1	2,373	2,469	**4.1**	2,788	**17.5**
	0.9	1,830	1,923	**5.1**	2,261	**23.5**
	0.8	1,518	1,611	**6.2**	1,950	**28.5**
	0.7	1,226	1,324	**8.0**	1,660	**35.4**
1	1	1,776	1,874	**5.5**	2,192	**23.4**
	0.9	1,316	1,409	**7.0**	1,743	**32.4**
	0.8	1,049	1,144	**9.1**	1,478	**40.9**
	0.7	799	898	**12.4**	1,228	**53.8**
0.9	1	1,230	1,327	**7.9**	1,644	**33.7**
	0.9	838	932	**11.2**	1,259	**50.3**
	0.8	608	704	**15.9**	1,029	**69.4**
	0.7	383	481	**25.8**	800	**109.1**

5 Conclusion

The two-bus equivalent model is commonly used for voltage stability studies in both distribution and transmission systems.
The paper presents a simple method to evaluate the parameters which define the equivalent circuit of a radial distribution network.
In particular, a straightforward way for determining the Thévenin equivalent impedance behind a load node is proposed, which allows to better identify the maximum loading point beyond which the voltage collapse takes place.
Simulation results show that the proposed method is significantly more accurate than the ***Z***-matrix based method, proposed in literature, in evaluating the critical power at a particular node (i.e. the weak node of the network) starting from any operating point.
Nevertheless, in a view of a real-time stability control, the ***Z***-matrix as proposed in [9] can be utilized in order to easily evaluate the nodes stability index and identify the weak node location at any loading change of the system.

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