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# An Efficient Procedure for Solving Radial Distribution Networks through the Backward/Forward Method

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*Abstract:* - In the paper, after having presented the general backward/forward methodology for radial systems analysis, a new b/f procedure showing some interesting features that improve its performance in terms of convergence speed and calculation effort is presented. The features that fundamentally are responsible for such improvements concern the main steps of the b/f procedure. The starting voltage profile solution is different from the flat profile and is suitably modified. In the backward phase and starting from the second iteration, the branch currents variations due to the loads changes are evaluated; the latter variations are calculated on the basis of the difference of nodal voltages at the beginning and at the end of each iteration. Finally, the adopted convergence criterion is based on the entity of the difference between each load node current in two subsequent iterations. The results of the applications of the proposed methodology to a set of networks taken from the literature on the topic are reported. In this way, the performance of the proposed methodology has been evaluated in terms of computational efficiency. The results of other tests have evidenced the influence of each feature of the modified b/f methodology over its performances.

*Key-words:* - Backward/forward method, Load flow analysis, Power distribution.

## 1 Introduction

For the analysis of distribution systems, characterized by a high R/X ratio and a radial structure, in recent times it has been developed the backward/forward method. Such method proceeds through the following steps:

a) calculation of the currents required by the loads and the lines shunt admittances, on the basis of the calculated or fixed values of nodal voltages;
b) evaluation of the current (or power) flows in the branches composing the electrical system, starting from the terminal branches and going up to the source node (*backward sweep*);
c) nodes voltages calculation, starting from the source node and proceeding to the terminal ones (*forward sweep*);
d) verification of a convergence criterion; if it is satisfied the process stops, otherwise it restarts from step a).

The backward/forward method shows, as compared to the Newton methods, a high probability to converge, a limited required computational effort and an easy implementation; the only inconvenience, especially for heavily loaded systems, is the increased number of iterations to attain a satisfying solution.

The measures to limit such inconvenience mainly are of two kinds:

1. reduction of the number of operations through the adoption of simplified network models;
2. identification of starting solutions that are closer to the final one.

Obviously, in the first case it is necessary to identify the entity of the errors caused by the introduced approximations, in the second case it is required to evaluate the reduction of the number of iterations and of the CPU time as compared to its increase due to the evaluation of the starting solution.

The load flow problem in distribution systems has been widely dealt with in the literature since the eighties, in relation with the development of those problems that are connected to distribution automation. Initially only radial systems have been considered, then also weakly meshed systems have been studied; when there are meshes in the system, it is turned into radial, by means of some cuts and by the introduction, in the cuts sections, of the so-called breakpoint currents calculated using the multi-port compensation method. The basic backward/forward method is presented in [1] where a power flow method is developed for weakly meshed distribution and transmission networks; the radial network is solved by the direct application of Kirchhoff's voltage and current laws. In [2] a simplified method for the load flow analysis of radial distribution systems based on the use of voltage magnitudes and neglecting the voltage phase angle of each bus is presented.

A three-phase power flow, based on the methodology developed in [1], is presented in [3] to solve distribution networks showing meshes, distributed generation, unbalanced loads, voltage regulators and shunt capacitor banks. In [4] the load-flow solution is carried out through the iterative calculation of the bus voltage magnitudes expressed in terms of the real and reactive powers flowing in the branches keeping into account even the losses; the convergence criterion is based on the difference at each branch between the real and reactive power evaluated in two following iterations. The same methodology is again considered in [5] where the convergence criterion is modified as it takes into account the real and reactive powers, flowing from the substation, in two subsequent iterations. For distribution systems having both radial and meshed topology, Haque ([6]) has developed an iterative solution method using the bus voltage magnitudes and phases equations; the meshes are opened adding some dummy buses; the power flows injected at these buses are evaluated by means of impedance matrices of reduced order. The branch by branch computational approach is again considered in [7] to solve both radial and meshed systems in normal and faulty conditions; the load models here considered are with constant admittance, with constant power and with load proportional to the voltage magnitude. In [8] three sets of recursive load flow equations for different load models are compared.

In [9] the distribution network is solved iteratively considering as state variables the bus voltages. The method developed by Haque in [6] is again considered and studied by the same Author ([10]) in order to keep into account shunt elements and more than one supply node.

In [11] three load flow solution methodologies for radial systems are described and compared.

An efficient method for radial networks solution is proposed in [12]. It is based on an iterative algorithm with some special measures to increase the convergence speed; the bus voltages are considered as state variables according to approaches that are common in literature. It uses a simple matrix representation for the network topology and branch current flows management. The solution method developed in [13] is based on the iterative backward/forward process, which is applied to a set of partial networks obtained by the original radial system. As compared to classical backward/forward sweep methods, in the proposed method the state variables calculation is performed starting from the terminal nodes and moving to the source node; this allows to verify the convergence of the process performing a test on the source node voltage only. In [14] the methodology developed by Baran and Wu ([15]) is simplified using the real and reactive powers and the bus voltage magnitudes.

Some measures and procedures to improve the efficiency of the classical backward/forward method are presented in [16] and [17].

In this paper, after the presentation of the state of the art on the solution of radial distribution networks with the b/f technique, the same methodology is described. In particular it is examined considering four main steps: initialization of the state variables; backward sweep; forward sweep; identification of a convergence criterion. For each of these, except for the forward sweep, a new procedure is defined. In particular, a methodology for the identification of a starting solution that is as close as possible to the desired one is proposed; the backward phase is modified for the evaluation of the branch currents and a new convergence criterion is proposed. Finally, the results of the application of the proposed methodology to a set of systems already studied by other Authors are reported.

## 2 The B/F Method for Radial Systems Analysis

The main features of the system are the following. The distribution system has radial topology and a single feeding point with constant voltage; the branches are three-phase; the loads are three-phase and symmetrical and can be modelled as constant power sinks; the capacitive admittances of the branches are negligible. The radial topology allows to know the direction of the power flows in the branches and therefore to identify, for each branch, the sending node through which the power is injected in the branch and the ending bus through which the power is transferred to the buses downstream the branch.

The b/f method can be divided into the following steps:

1. initialization of the bus voltages;
2. backward sweep: evaluation of the branch currents, starting from the terminal branches and going up to the source node;
3. forward sweep: calculation of the bus voltages starting from those immediately downstream the source node and arriving at the terminal nodes;
4. verification of a convergence criterion.

### 2.1 Initialization of the bus voltages

The flat profile of the voltages, i. e. all bus voltages equal to the source node voltage, is the starting profile adopted

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in most of the paper about load flow in distribution systems. Such assumption has the main advantage of a limited computational effort and does not prevent from converging to a final solution, because of the robustness of the b/f analysis method. It is clear that, anyway, supplying a starting solution closer to the final one can reduce the number of iterations and the computational time. Consider the single branch system, supplied at one end by a constant voltage Vo source, that supplies, at the other end, a constant power load ($A$=$P$+$jQ$), fig. 1.

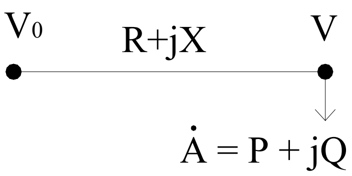


Fig. 1. Single branch system.

It is known that such system can be directly solved without using any iterative method; indeed the complex voltage $\boldsymbol{V}$ at the load bus is given by:

$$\boldsymbol{V} = \boldsymbol{V}_o - \frac{(R + jX)(P + jQ)}{\boldsymbol{V}^*} \qquad (1)$$

where R and X are the resistance and the reactance of the branch and $\boldsymbol{V}^*$ is the complex conjugate of the voltage $\boldsymbol{V}$; expressing the voltages $\boldsymbol{V}_0$ and $\boldsymbol{V}$ as summation of real and imaginary parts:

$$\boldsymbol{V}_o = V_o \qquad (2)$$

$$\boldsymbol{V} = V_r + jV_i \qquad (3)$$

from (1) it can be obtained:

$$V_i = \frac{(-RQ - XP)}{V_o} \qquad (4)$$

$$V_r = \frac{V_o + \sqrt{V_o^2 - 4\left[V_i^2 + (RP - XQ)\right]}}{2} \qquad (5)$$

In this way, under the hypothesis of constant voltage at the sending bus of the branch, and once the real and reactive powers are known at the ending bus of the branch, using (4) and (5) the complex voltage components can be calculated for the ending node. In the particular case of the network in fig. 1, the voltage calculated in this way is the exact one; it is anyway possible, for a generic radial system, to identify, for each of its nodes, an equivalent at a single branch that allows an approximate evaluation of the voltage at that node.

The approximation is due to the fact that losses and voltage drops (for load current calculation) are neglected.

Consider the system in fig. 2 a), made of two branches connected in series, supplied by node 0 at constant voltage $V_o$; the loads at the two nodes are: $\boldsymbol{A}_1$=$P_1$ + $jQ_1$ and $\boldsymbol{A}_2$=$P_2$ + $jQ_2$.

In the aim of evaluating the voltage $\boldsymbol{V}_1$ the single branch equivalent is depicted in fig. 2 b), where the load at bus 1 is the summation of the loads $\boldsymbol{A}_1$ and $\boldsymbol{A}_2$, and the impedance of branch 1 of the equivalent network remains the same as that of the original system.

For the evaluation of the voltage at node 2, the equivalent network, fig. 2 c), shows in node 2, in the same way as before, the summation of the loads $\boldsymbol{A}_1$ and $\boldsymbol{A}_2$, whereas the single branch impedance between the source node 0 and node 2 is given by the weighted sum of the impedances of the branches that, in the original network, connect node 0 and node 2; for each of these branches the weight is given by the ratio between the power flowing in the branch (which, neglecting the losses, is the total power required by the loads downstream the branch) and the power flowing in the branch connected to the source node (which, neglecting the losses, is the total power required by the networks loads).

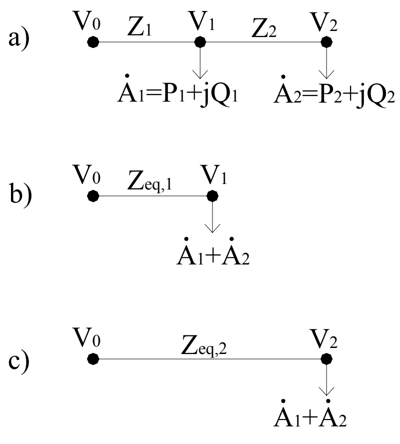


Fig. 2. Radial network with two branches connected in series, a); network equivalent for the approximate evaluation of the voltage at node 1, b), and at node 2, c).

As a result, for the evaluation of the voltage at the two ends, the equivalent impedances to be considered in a single branch system are:

$V_1$)
$$\boldsymbol{Z}_{eq,1} = \boldsymbol{Z}_1 \tag{6}$$

$V_2$)
$$\boldsymbol{Z}_{eq,2} = \boldsymbol{Z}_1 + \frac{\boldsymbol{Z}_2 \boldsymbol{A}_2}{\boldsymbol{A}_1 + \boldsymbol{A}_2} \tag{7}$$

The evaluation of the voltages $\boldsymbol{V}_1$ and $\boldsymbol{V}_2$ through the equivalent network gives rise to errors; indeed the losses in the branches downstream the branch connected to the source node are not considered in the evaluation of the power sink at the end of the single branch equivalent.

For a generic system, with laterals and sub-laterals, the approximated evaluation of the bus voltages can be executed by means of the following procedure:

1. evaluation of the apparent complex powers, $\boldsymbol{A}_{b,i}$, circulating on each branch i and due to the loads;
2. for the generic node j, identification of the set $\{B_j\}$ of the branches connecting it to the source node;
3. calculation of the equivalent impedance:

$$\boldsymbol{Z}_{eq,i} = \boldsymbol{Z}_1 + \frac{\sum \boldsymbol{Z}_i \boldsymbol{A}_{b,j}}{\boldsymbol{A}_{b,1}} \tag{8}$$

where the summation is extended to the branches that belong to the set $\{B_j\}$;

4. calculation of the real and imaginary components of the voltage $\boldsymbol{V}_j$ through (4) and (5) where:

$$R = \text{Real}\,[\boldsymbol{Z}_{eq,j}] \tag{9}$$

$$X = \text{Imag}\,[\boldsymbol{Z}_{eq,j}] \tag{10}$$

$$P = \text{Real}\,[\boldsymbol{A}_{b,1}] \tag{11}$$

$$Q = \text{Imag}\,[\boldsymbol{A}_{b,1}] \tag{12}$$

where $\boldsymbol{A}_{b,1}$ is the complex apparent power required by all the network loads and flowing, not considering the losses, on branch 1 connected to the source node 0. The bus voltages calculated in this way can be used as starting solution of the iterative backward/forward process.

## 2.2 Backward sweep

In this phase, starting from the terminal branches and going to the source node, the current flows in each branch are evaluated starting from the load demand. The bus voltages, calculated as starting solution or attained at the end of the preceding iteration, are useful to evaluate the currents required at the bus loads.

At the beginning of the first iteration the load current, $\boldsymbol{I}$, at node is given by:

$$\boldsymbol{I}_i^{(1)} = \frac{\boldsymbol{A}_i}{\boldsymbol{V}_i^{(0)*}} \tag{13}$$

where $\boldsymbol{A}_i$ is the complex apparent power at node i and $\boldsymbol{V}_i^{(0)*}$ is the complex conjugate voltage at the same node used as starting solution. At the end of the first iteration, the voltage at node i, $\boldsymbol{V}_i^{(1)}$, differs from the starting voltage value, $\boldsymbol{V}_i^{(0)}$, thus the complex power required by the load, evaluated with the new voltage value and with the initial value of the current (13), is not equal to the fixed value.

Therefore, at the end of the first iteration, and in general, at the end of the k-th iteration, a difference between the calculated and fixed values of power required by the loads comes up.

In the methodology here proposed, for each load, the current due to the difference between the calculated and fixed values of power required by the loads is evaluated at the end of each iteration.

Such current, in what follows, will be indicated as "load current difference". This value is summed to the load current at the bus; in this way, the load current at the beginning of each iteration is the same as the one in the preceding iteration increased, from the second iteration, of the load current difference. In general terms, the load complex power at node i, at the end of iteration k, is given by:

$$\boldsymbol{A}_i^{(k)} = \boldsymbol{I}_i^{(k-1)} \boldsymbol{V}_i^{(k)*} = \frac{\boldsymbol{A}_i}{\boldsymbol{V}_i^{(k-1)*}} \boldsymbol{V}_i^{(k)*} \tag{14}$$

and differs from the required power at node i, $\boldsymbol{A}_i$, of the following quantity:

$$\Delta^{(k)} \boldsymbol{A}_i = \boldsymbol{A}_i - \boldsymbol{A}_i^{(k)} \tag{15}$$

The load current difference associated to this power difference is given by:

$$\Delta^{(k)} \boldsymbol{I}_i = \frac{\Delta^{(k)} \boldsymbol{A}_i}{\boldsymbol{V}_i^{(k)*}} = \frac{(\boldsymbol{A}_i - \boldsymbol{A}_i^{(k)})}{\boldsymbol{V}_i^{(k)*}} = \boldsymbol{A}_i \left(\frac{1}{\boldsymbol{V}_i^{(k)*}} - \frac{1}{\boldsymbol{V}_i^{(k-1)*}}\right) \tag{16}$$

Thus, the load current at node i, at the beginning of the subsequent iteration, is given by:

$$\boldsymbol{I}_i^{(k+1)} = \boldsymbol{I}_i^{(k)} + \Delta^{(k)} \boldsymbol{I}_i \tag{17}$$

The load current difference in (16) approaches zero, as the bus voltage difference in two subsequent iterations gets smaller , thus when the overall voltage profile converges to the final value.

## 2.3 Forward sweep

In this phase, for each branch i, the voltage at the ending bus $EB_i$, is calculated on the basis of the known values of the sending node, $SB_i$, voltage and of the current, $\boldsymbol{I}_{b,i}$, flowing in the branch.
The need to know the sending node voltage makes it mandatory to proceed from the source node towards the ending branches.
For each branch then, if in the backward sweep the currents calculation was performed, the following equation can be used:

$$\boldsymbol{V}_{EBi} = \boldsymbol{V}_{SBi} - \boldsymbol{Z}\boldsymbol{I}_{b,1} \qquad (18)$$

## 2.4 Convergence criterion

Each iteration terminates with the calculation of the bus voltages; to decide whether to continue the iterative process or not, the attained results are compared to those obtained in the preceding iteration; this is done in order to evaluate the errors and, on the basis of a prefixed convergence factor, $\varepsilon$, to verify whether the errors are larger or smaller than this factor; if these are greater than this factor, another iteration is performed calculating the currents difference and assuming as bus voltages the last calculated values; otherwise, if they are smaller, the iterative process stops and the required results are printed out.
The easiest convergence criterion consists in the comparison between the voltages in two subsequent iterations; the error is then evaluated as:

$$|\boldsymbol{V}_i^{(k)} - \boldsymbol{V}_i^{(k-1)}| \leq \varepsilon \qquad (19)$$

If, for all the nodes, (19) is verified, the convergence has been reached. It is possible to reduce the overall number of operations to execute, and thus the calculation time, by verifying the criterion not for the set of all nodes, but simply node by node. If for the generic node i, (19) is verified, the voltage at the i-th node is considered constant on the subsequent iterations; in this way, in the backward phase, the calculation of the load current difference at node i can be avoided as well as, in the forward phase, the calculation of the voltage at node i.
Another convergence criterion, similar to the latter that can be adopted, consists in the comparison for each node of the calculated and fixed values of power required by the loads at the end of each iteration.
The load current difference is a feature that is representative of the error in terms of power since it reaching zero indicates the equivalence between the calculated and fixed values of power required by the loads, or also indicates the equivalence at each node of the voltage in two subsequent iterations.
Thus the inequality to be verified is the following:

$$|\Delta^{(k)} I_i| \leq \varepsilon \qquad (20)$$

In the same way as before, the convergence criterion expressed by (20) can be verified, at each iteration, for all the load nodes of the network (global convergence criterion) or, node by node, not considering in the following evaluations the nodes for which the criterion has been already verified (local convergence criterion).
In fig. 3 the flow-chart of the proposed algorithm in shown.

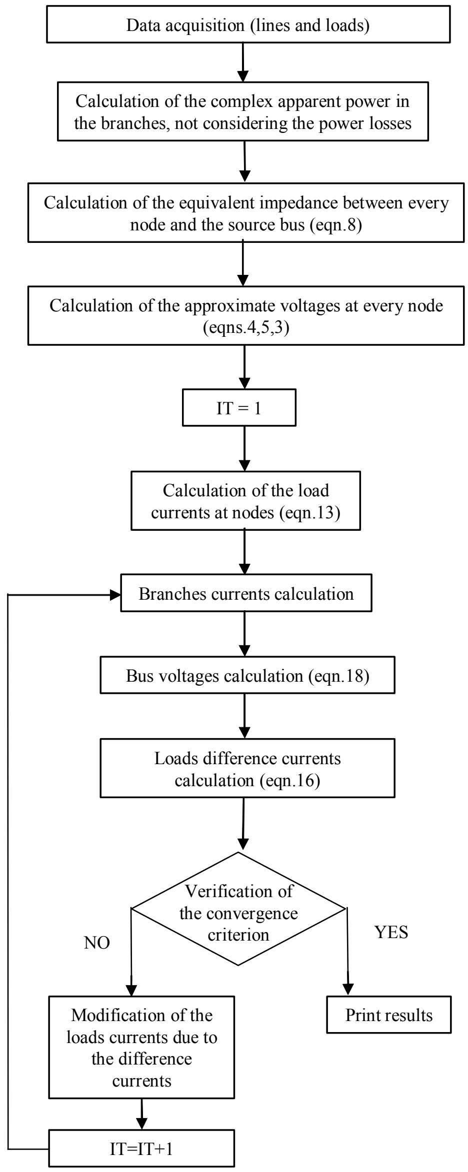


Fig. 3. Flow-chart of the proposed algorithm.

## 3 Applications

In order to evaluate the performance of the proposed methodology, the relevant algorithm has been implemented in FORTRAN 90 and the program has run on a mainframe IBM S/390-2003/225. The applications concerned the solution of some networks with 12, 15, 28, 33, 69 and 85 nodes; the data about the lines and the loads of these networks are respectively reported in [5], [4], [5], [18], [15], [4].

These networks have been used by some of the cited Authors to test different b/f radial networks solution methodologies. In particular the 85 nodes test systems has been studied by Das et al. ([5]) with other several Indian rural distribution networks.

In Table 1 for each of the considered systems are reported:

col. 1 – the number of network buses, NB;

col. 2 – the reference in which the system was analyzed, [..];

col. 3 and 5 – the number of iterations, IT, and the CPU time, in s, attained in the ref. reported in col. 2;

col. 4 and 6 – the number of iterations, IT, and the CPU time, in s, attained with the procedure here presented;

col. 7 – the maximum percent error in the voltage magnitudes evaluated comparing the starting and the final solution, err $x_0$ %(= max{(|$\boldsymbol{V}_{io}$| - |$\boldsymbol{V}_{if}$|) 100/|$\boldsymbol{V}_{if}$|});

col. 8 – the maximum percent error in the voltage magnitudes evaluated, comparing the end of the first iteration and the final solution, err $x_1$ %(= max{(|$\boldsymbol{V}_{i1}$| - |$\boldsymbol{V}_{if}$|) 100/|$\boldsymbol{V}_{if}$|}).

In all cases, the proposed procedure reaches the final solution with a number of iterations that is smaller than that of other methodologies. The convergence factor has been fixed to 0.0001 for all the considered cases; this value is the same as that fixed by other Authors to test the developed b/f methodologies. In order to verify the influence of the modifications concerning the starting solution and the convergence criterion, the same networks have been solved using the proposed procedure, also considering a flat voltage profile and the global convergence criterion. In Table 2 the number of iterations, the CPU time and the maximum errors attained considering the global convergence criterion are shown; in Tables 3 and 4 are instead reported the same features considering, as starting solution, the flat voltage profile and adopting either the local convergence criterion (Table 3) or the global convergence criterion (Table 4). In these Tables err $x_j$ %(= max{(|$\boldsymbol{V}_{ij}$| - |$\boldsymbol{V}_{if}$|) 100/|$\boldsymbol{V}_{if}$|}) is the maximum percent error in the voltage magnitudes evaluated comparing the end of the j-th iteration and the final solution.

From the analysis of the results, the following considerations can be deduced:

- the initialization of the bus voltages through the approximated solution gives rise to a reduction of the number of iterations and of the CPU time;
- the local convergence criterion does not sensibly modify the number of iterations (only in one case a reduction can be observed); it allows a limited reduction of the CPU time when the number of iterations is greater than 2;
- the course of the maximum errors shows that choosing an approximated voltage profile as starting solution increases the efficiency and provokes a reduction of the number of iterations.

## 4 Conclusions

The b/f method specifically developed for radial distribution systems, even if it is conceptually quite easy, shows, in its implementation, a set of alternatives in each of the steps into which it can be divided.

In this paper some modifications concerning a suitable initialization of the bus voltages, the backward phase and the convergence criterion are presented.

The results of some applications carried out on some test systems show the efficiency of these modifications in terms of reduction of the number of iterations and of CPU time.

Improving the performance of the b/f methodology implies also an improvement for those methods oriented to the solution of weakly meshed networks and/or with PV nodes (nodes injecting only real power, produced by means of renewable sources, cogeneration plant, etc., and requiring voltage regulation systems).

These networks are usually solved reducing them to radial systems, by means of cuts in real and fictitious meshes, the latter being associated to the PV nodes, and by means of the introduction of compensation currents.

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Table 1 - Performance of the proposed methodology for some networks studied in the literature

| | | **IT** | | **CPU [s]** | | **err x [%]** | |
|---|---|---|---|---|---|---|---|
| **nodes** | **Ref.** | **[ ]** | **proposed** | **[ ]** | **proposed** | **err $x_0$** | **err $x_1$** |
| 12 | 5 | 3 | 2 | - | 0.00013 | -0.082 | -0.0019 |
| 15 | 4 | 3 | 2 | - | 0.00015 | 0.054 | -0.00064 |
| 28 | 5 | 4 | 2 | - | 0.00029 | 0.14 | -0.0038 |
| 33 | 6; 9 | 3 | 2 | 0.17;0.09 | 0.00039 | -0.16 | -0.0039 |
| 69 | 9;10 | 3 | 2 | 0.16;0.27 | 0.00080 | 0.16 | -0.0046 |
| 85 | 5 | 4 | 2 | | 0.0010 | -0.42 | -0.017 |

Table 2 - Performance of the proposed methodology for the networks of Table 1 with global convergence criterion

| | | | err x [%] | |
|---|---|---|---|---|
| **nodes** | **IT** | **CPU [s]** | **err $x_0$** | **err $x_1$** |
| 12 | 2 | 0.000138 | -0.082 | -0.0019 |
| 15 | 2 | 0.000154 | 0.054 | -0.00064 |
| 28 | 2 | 0.000297 | 0.14 | -0.0038 |
| 33 | 2 | 0.000397 | -0.16 | -0.0039 |
| 69 | 2 | 0.00087 | 0.16 | -0.0046 |
| 85 | 2 | 0.0010 | -0.42 | -0.017 |

Table 3 - Performance of the proposed methodology for the networks of Table 1 for flat voltage profile and local convergence criterion

| | | | err x [%] | | |
|---|---|---|---|---|---|
| **Nodes** | **IT** | **CPU [s]** | **err $x_1$** | **err $x_2$** | **err $x_3$** |
| 12 | 3 | 0.00014 | 0.31 | 0.0069 | - |
| 15 | 3 | 0.00016 | 0.28 | 0.0071 | - |
| 28 | 3 | 0.00037 | 0.70 | 0.016 | - |
| 33 | 4 | 0.00050 | 0.69 | 0.043 | 0.00015 |
| 69 | 4 | 0.00088 | 0.79 | 0.0063 | 0.00004 |
| 85 | 3 | 0.0011 | 1.51 | 0.062 | - |

Table 4 - Performance of the proposed methodology for the networks of Table 1 for flat voltage profile and global convergence criterion

| | | | err x [%] | | | |
|---|---|---|---|---|---|---|
| **nodes** | **IT** | **CPU [s]** | **err $x_1$** | **err $x_2$** | **err $x_3$** | **err $x_4$** |
| 12 | 3 | 0.00014 | 0.32 | 0.014 | - | - |
| 15 | 3 | 0.00016 | 0.28 | 0.014 | - | - |
| 28 | 3 | 0.00038 | 0.76 | 0.057 | - | - |
| 33 | 4 | 0.00062 | 0.70 | 0.054 | 0.0038 | - |
| 69 | 5 | 0.0016 | 0.86 | 0.076 | 0.0070 | 0.00057 |
| 85 | 4 | 0.0016 | 1.71 | 0.20 | 0.022 | - |