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Transient and Static Modeling of Tubular Linear Induction Motors

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Abstract: - This paper is concerned with the transient modeling of tubular linear induction motors with blocked rotor in the assignable respective positions of rotor and stator. The motor is a traveling-wave generator- driven type. This work is based on calculation of self inductances of a link and a coil, mutual inductance and mutual inductance gradient of coaxial coils with current filament method, assumption of the rotor constituted of some rings, and current calculation of stator coils and rotor rings with respect to time. We have written some computer codes for calculating inductances and analyzing the performance. Finally the instantaneously force applied on rotor is calculated.

Keywords: - Air Core Tubular Linear Induction Motor, Coilgun, Induction Launcher

1 Introduction

Tubular Linear Induction Motors (TLIMs) can have unique applications as electromagnetic launchers with very high speed and acceleration abilities, and controllability [1]. As all motors, a TLIM consists of two main parts: stationary (stator) and moving (armature here) members. The motor supplying is by stator which consists of several coaxial coils and number of them is determined with respect to number of phases, poles, and sections. Usually it is necessary that we design the coils in such a way that could be changeable individually [2].

When analyzing the motor by current filament method, the rotor which is usually a conductive and thin circular layer, depended to required accuracy, length and thickness of rotor is divided to several current sections, called as rings. Links of the stator coils and rings of the rotor are assumed as circular and parallel links with uniform current densities, called as current filaments. Mutual inductance equation between two current filaments is [3]:

$$M = 2\mu_0\sqrt{\frac{ab}{m}}\left[\left(1-\frac{m}{2}\right)K(m)-E(m)\right] \qquad (1)$$

Where

a = radius of the first filament
b = radius of the second filament
d = axial distance between two filaments

$$m = \frac{4ab}{(a+b)^2+d^2}$$

K (m) and E (m) are first and second types of elliptical integral.

2 Inductance Calculations

2.1 Self Inductance of a Link

For calculation of a link self inductance, we assume the link is constituted of n filaments with the same current, and use the method of current filament mutual inductance. For carrying out this calculation, we assume a link is constituted of three filaments (Fig. 1), and give the self- inductance of a link as below:

Voltages of all three filaments are equal; the voltage of the first filament is equal to:

$$v = L_{11}\frac{di_1}{dt} + L_{12}\frac{di_2}{dt} + L_{13}\frac{di_3}{dt} \qquad (2)$$

With regard to the assumption:

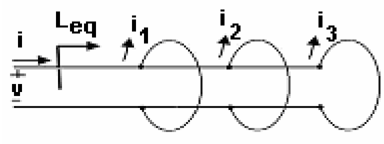

Fig. 1 : a Link Constituted of Three Current Filaments

$$i_1 = i_2 = i_3 = i' = \frac{1}{3} i \qquad (3)$$

Then

$$v = \frac{1}{3}(L_{11} + L_{12} + L_{13})\frac{di}{dt} = L_{eq1}\frac{di}{dt} \qquad (4)$$

So

$$L_{eq1} = \frac{1}{3}\sum_{j=1}^{3} L_{1j} \qquad (5a)$$

And writing the voltages of filaments 2 and 3,

$$L_{eq2} = \frac{1}{3}\sum_{j=1}^{3} L_{2j} \qquad (5b)$$

$$L_{eq3} = \frac{1}{3}\sum_{j=1}^{3} L_{3j} \qquad (5c)$$

Then the equivalent inductance would be

$$L_{eq} = \frac{1}{3}\left(L_{eq1} + L_{eq2} + L_{eq3}\right) = \frac{1}{3^2}\sum_{i=1}^{3}\sum_{j=1}^{3} L_{ij} \qquad (6)$$

Assuming a link consisting of n filaments, one can

$$L_{eq} = \frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n} L_{ij} \qquad (7)$$

Calculating mutual inductance of a filament with itself, using current filament method, one obtains an Indefinite value [4]. So for calculating self inductance of each link, we calculate the mutual inductance of each filament only with adjacent filaments. In this case, self inductance of a link would be calculated from the below equation:

$$L = \frac{1}{n(n-1)}\sum_{i=1}^{n}\sum_{j=1}^{n} L_{ij} \qquad j \neq i \qquad (8)$$

Table 1 : Results of the Code, Ref [5] and [6]

Coil No.	Links	L (mm)	Dout (mm)	Din (mm)	inductance (mH)		
					[5]	[6]	code
1	103	60	70	59	0.462	0.450	0.459
2	178	60	98	76	2.07	2.087	2.085

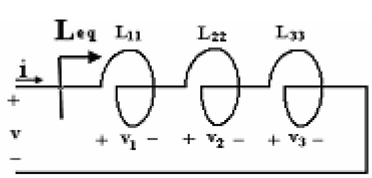

Fig. 2: a Coil with Three Series Links

2.2 Self Inductance of a Coil

Assuming a coil consisted of three links (Fig. 2), the voltage and current relation of coil is

$$\begin{aligned} v &= v_1 + v_2 + v_3 \\ &= \left(L_{11}\frac{di}{dt} + L_{12}\frac{di}{dt} + L_{13}\frac{di}{dt}\right) \\ &+ \left(L_{21}\frac{di}{dt} + L_{22}\frac{di}{dt} + L_{23}\frac{di}{dt}\right) \\ &+ \left(L_{31}\frac{di}{dt} + L_{32}\frac{di}{dt} + L_{33}\frac{di}{dt}\right) \\ &= \left(\sum_{i=1}^{3}\sum_{j=1}^{3} L_{ij}\right)\frac{di}{dt} = L_{eq}\frac{di}{dt} \end{aligned} \qquad (9)$$

Assuming a coil with n links, one obtains

$$L_{eq} = \sum_{i=1}^{n}\sum_{j=1}^{n} L_{ij} \qquad (10)$$

Obviously, inductance calculation of a coil requires calculations of self inductance of links as well as mutual inductance between each two links. Comprising the results of the written computer code in our work, results of the reference [5] and results of the reference [6], for self inductance calculation of a coil with table 1 characteristic, our code was verified (table 1).

With regard to the code format, coil 1 arrangement was assumed to be 4 layers and 26 links in each layer $(4 \times 26 = 104)$, coil 2 arrangement was assumed to be 4 layers and 44 links in each layer $(4 \times 44 = 176)$, and actual number of links was taken into consideration by applying a suitable coefficient.

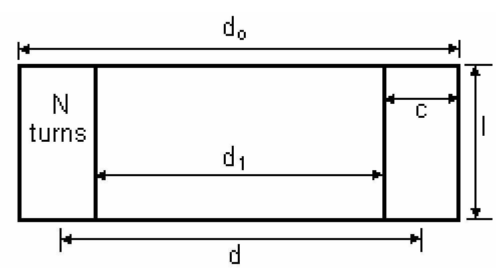

Fig. 3: an Air-Core Cylindrical Coil Dimensions for Calculation of its Inductance [6]

The reference [6] calculates the self inductance of a tubular air-core coil(fig. 3) using number of links, N ,mean diameter, d ,length, L ,thickness, C , as

$$L = \Phi d N^2 (\mu H) \tag{11}$$

Where d is in meters and

$$\Phi = \frac{0.1\pi^2}{0.45 + \alpha + \rho + (\frac{2}{3})\alpha\rho[(\alpha+1)/(\alpha+2)]} \qquad \alpha = \frac{l}{d} \quad and \quad \rho = \frac{c}{d} \tag{12}$$

2.3 Mutual Inductance of Two Coils

We begin investigation of mutual inductance of two coils by a coil of two links and other coil of three links (Fig. 4).

The voltage of the two – links coil is :

$$\begin{aligned} v = v_1 + v_2 &= \left(L_{11}\frac{di}{dt} + L_{12}\frac{di}{dt} + L_{13}\frac{di'}{dt} + L_{14}\frac{di'}{dt} + L_{15}\frac{di'}{dt} \right) \\ &+ \left(L_{21}\frac{di}{dt} + L_{22}\frac{di}{dt} + L_{23}\frac{di'}{dt} + L_{24}\frac{di'}{dt} + L_{25}\frac{di'}{dt} \right) \\ &= (L_{11} + L_{12} + L_{21} + L_{22})\frac{di}{dt} \\ &+ (L_{13} + L_{14} + L_{15} + L_{23} + L_{24} + L_{25})\frac{di'}{dt} \\ &= L\frac{di}{dt} + M\frac{di'}{dt} \end{aligned} \tag{13}$$

Consequently, mutual inductance of a coil of n links and a coil of n' links is obtained by the equation of

$$M = \sum_{i=1}^{n} \sum_{j=n+1}^{n+n'} L_{ij} \tag{14}$$

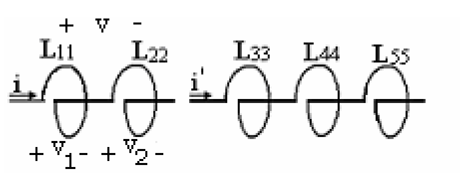

Fig. 4 : a Coil of Two Links and Another Coil of Three Links

3 Currents of Coils and Rings

For an initial design of the required computer code, a motor with 6 coils in stator and 12 rings in rotor was assumed (table 2 and Fig. 5). Mutual inductance matrix of coils and rings is :

$$L = \left[\begin{array}{c|c} L_d & L_{dp} \\ \hline L_{pd} & L_p \end{array} \right] = \left[\begin{array}{c|c} L_d & L_{dp} \\ \hline L'_{dp} & L_p \end{array} \right] \tag{15}$$

Where

L_d = inductance matrix of drive coils
L_p = inductance matrix of rotor rings
L_{dp} = mutual inductance matrix of coils and rings

Table 2: The Simulated Motor Characteristics

Stator

Output radius of coils (mm)	45.5
Radial thickness of coils (mm)	14.9
Longitudinal length of coils (mm)	71.1
Layers number of coils	8
Links number of each layer	40
Distance between each two coils (mm)	3.9
Diameter of cupper wire	1.7
Number of coils	6

Rotor

Output radius (mm)	29.5
Radial thickness (mm)	2
Longitudinal length (mm)	45
material	Aluminum

Supply

Voltage (RMS) (v)	220
Frequency (Hz)	50

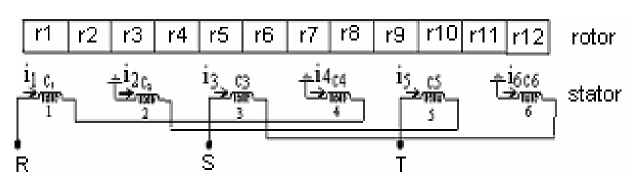

Fig. 5 :a Motor with 6 Stator Coils and 12 Rotor Rings

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And voltage relation of coils and rings is

$$\begin{bmatrix} v_d \\ \hline 0_{N_p\times 1} \end{bmatrix} = \left[\begin{array}{c|c} r_d & 0_{N_d\times N_p} \\ \hline 0_{Np\times Nd} & r_p \end{array}\right]\begin{bmatrix} I_d \\ \hline I_p \end{bmatrix} + \left[\begin{array}{c|c} L_d & L_{dp} \\ \hline L'_{dp} & L_p \end{array}\right]\begin{bmatrix} \dot{I}_d \\ \hline \dot{I}_p \end{bmatrix} \tag{16}$$

Where

- v_d = voltage vector of drive coils
- I_d = current vector of drive coils
- I_p = current vector of rotor rings
- r_d = resistance matrix of drive coils
- r_p = resistance matrix of rotor rings

In summary

$$[V] = [R][I] + [L][\dot{I}] \tag{17}$$

Coils currents are related to phases currents as:

$$[I_d] = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}\begin{bmatrix} i_R \\ i_S \\ i_T \end{bmatrix} = T_d I_{ph} \tag{18}$$

Where I_{ph} is the vector of phase currents and T_d is called transformation matrix. Generally , vector of coils and rings currents, I, is related to vector of phases and rings currents as :

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \\ \hline i_7 \\ \vdots \\ i_{18} \end{bmatrix} = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & & & \\ 0 & 0 & -1 & & & \\ 0 & 1 & 0 & & 0 & \\ -1 & 0 & 0 & & & \\ 0 & 0 & 1 & & & \\ 0 & -1 & 0 & & & \\ \hline & & & 1 & \cdots & 0 \\ & 0 & & \vdots & \ddots & \vdots \\ & & & 0 & \cdots & 1 \end{array}\right]\begin{bmatrix} i_R \\ i_S \\ i_T \\ \hline i_7 \\ \vdots \\ i_{18} \end{bmatrix} = [T][i]' \tag{19}$$

One can write from equations of (17) and (19)

$$[L][\dot{I}] = [V] - [R][I] \tag{20}$$

$$[L]\left([T][\dot{i}]'\right) = [V] - [R][T][I]' \tag{21}$$

Multiplying left sides of equation (21) by T′

$$\underbrace{[T]^t[L][T]}_{[L]'}[\dot{I}]' = \underbrace{[T]^t[V]}_{[V]'} - \underbrace{[T]^t[R][T]}_{[R]'}[I]' \tag{22}$$

$$[L]'[\dot{I}]' = [V]' - [R]'[I]' \tag{23}$$

Therefore state equations governing on the motor are :

$$[\dot{I}]' = [L]'^{-1}\left\{[V]' - [R]'[I]'\right\} \tag{24}$$

Where [V]' is voltage vector of phases and rings :

$$[\mathrm{V}]' = \begin{bmatrix} \mathrm{V_R} \\ \mathrm{V_S} \\ \mathrm{V_T} \\ \cdots \\ \mathrm{V_7} \\ \vdots \\ \mathrm{V_{18}} \end{bmatrix} \tag{25}$$

By solving equation (24) , one can calculate phases and rings currents. This solution requires performing [R], [L] and then [R]′ and [L]′ matrices. These equations have been solved by Rung-Kutta method and the results for phases and rings currents are represented in Fig. 6. As you seen. Phases S, T currents have transient states, and rings currents are distributed unequally as a result of end effects.

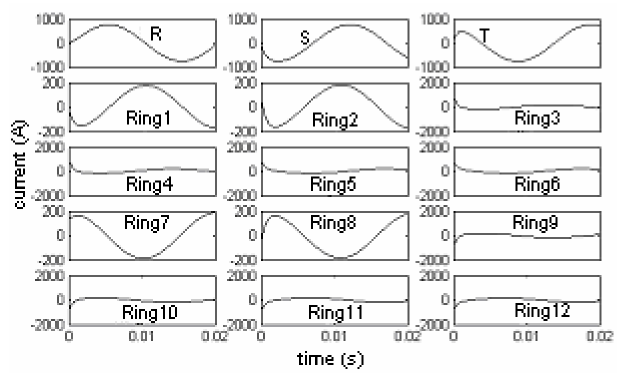

Fig. 6: Phases and Rings Currents

4 Force Calculations

Calculation of force applied to the rotor is based on mutual inductance changing with respect to longitudinal displacement between stator coils and rotor rings. For more calculation speed, one can first calculate sum of mutual inductances between filaments of two coils, and then obtain derivation of the sum with respect to longitudinal displacement.

4.1 Mutual Inductance Gradient

Longitudinal force between two current filaments is [7] :

$$F_Z = I_d I_P \frac{dM}{dZ} \qquad (26)$$

A code was written for calculation of mutual inductance and gradient of mutual inductance between a coil and a ring for a range of longitudinal positions which results of it were obtained for the coil and ring with reference [5] specifications (Fig. 7)and it is seen a good agreement between this figures and figures of reference[5] (Fig 8).

Simulation results also demonstrated that assuming a constant thickness and length for coil, the arrangement of links does not affect the results for coils self and mutual inductances. Table 3 shows the mutual inductances for several arrangements of table 1 coils. N1 refers to layers number and N2 refers to links number in each layer. The mutual inductance calculated by [5] for axial distance of zero is 685 uH.

Table 3: Mutual Inductances of several arrangements of table 1 coils

	N1	N2	Mutual inductance
Coil 1	5	21	664 uH
Coil 2	3	60	
Coil 1	4	26	685 uH
Coil 2	4	44	
Coil 1	3	34	683 uH
Coil 2	5	36	

4.2 Instantaneous Force

With regard to Fig. 5, instantaneous force applied on rotor with 12 rings is

$$\begin{aligned} f &= f_{17} + f_{18} + \cdots + f_{1,18} \\ &\quad + f_{27} + f_{28} + \cdots + f_{2,18} \\ &\quad \vdots \\ &\quad + f_{67} + f_{68} + \cdots + f_{6,18} \\ &= i_1 i_7 \frac{dM_{17}}{dZ} + i_1 i_8 \frac{dM_{18}}{dZ} + \cdots + i_1 i_{18} \frac{dM_{1,18}}{dZ} \\ &\quad + i_2 i_7 \frac{dM_{27}}{dZ} + i_2 i_8 \frac{dM_{28}}{dZ} + \cdots + i_2 i_{18} \frac{dM_{2,18}}{dZ} \end{aligned} \qquad (27)$$

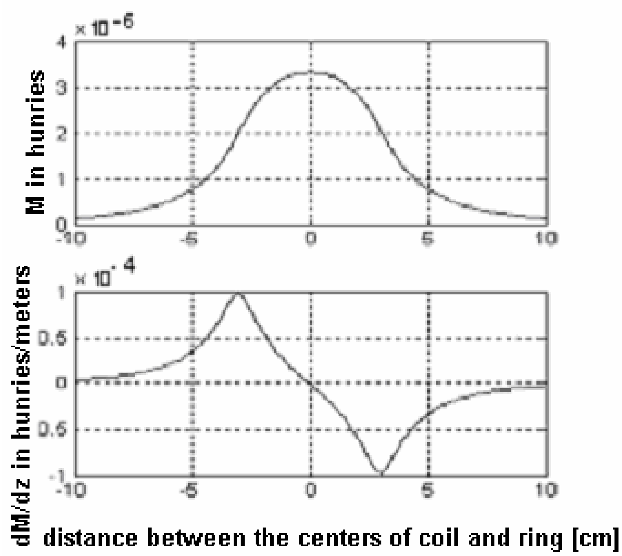

Fig. 7: Mutual Inductance and Mutual Inductance Gradient Between Coil and Ring (code results)

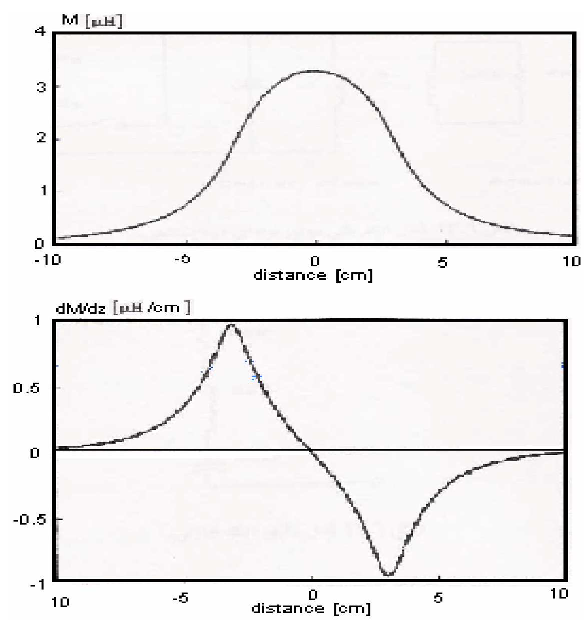

Fig. 8: Mutual Inductance and Mutual Inductance Gradient Between Coil and Ring[5]

And in matrix form

$$f = \begin{bmatrix} i_1 & \cdots & i_6 \end{bmatrix} \begin{bmatrix} \frac{dM_{17}}{dz} & \cdots & \frac{dM_{1,18}}{dz} \\ \frac{dM_{67}}{dz} & \cdots & \frac{dM_{6,18}}{dz} \end{bmatrix} \begin{bmatrix} i_7 \\ \vdots \\ i_{18} \end{bmatrix} \qquad (28)$$

And in summary

$$f = [i_d]^T \left[\frac{dM}{dZ} \right] [i_P] \tag{29}$$

So, it is necessary to obtain dM/dz. Instantaneous longitudinal force obtained of a written code for a motor with table 2 characteristics is seen in Fig. 9. After a transient period, the force reaches to its steady state value with a sinusoidal form and a constant mean value. As it is seen, the force at initial instants is negative. Also, a code was written for mean value calculation of force applied on the rotor at steady state, and was run for the motor with table 2 characteristic and 24 rotor rings at several longitudinal positions of rotor, and Fig. 10 was obtained. We see that the maximum mean force applied on the rotor is obtained when rotor is completely inside the stator, and the force becomes nearly zero when the rotor lies entirely outside the stator. Simulation results showed that:

1) assuming minimum 24 rings for rotor is necessary for obtaining sufficient accuracy,
2) increasing rotor thickness and changing rotor material of aluminum to cupper result in increased force,
3) In this level of voltage, the temperature effect is ignorable.

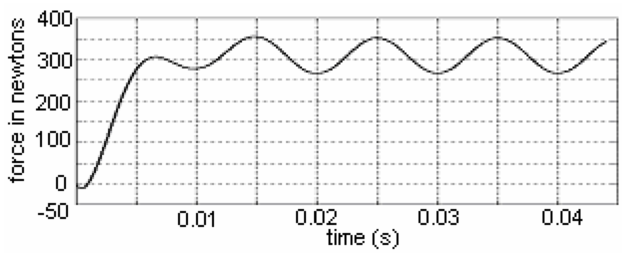

Fig. 9: Instantaneous Longitudinal Force Applied on Rotor

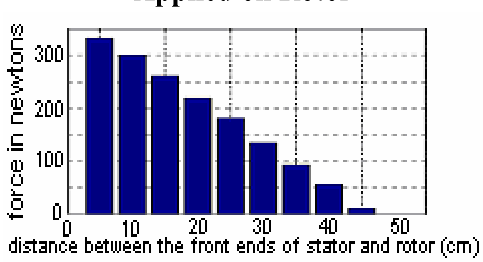

Fig. 10: Mean Value of Force at Given Positions

5 Conclusion

Assuming a constant thickness and length for coil, the arrangement of links for driving the number of coil links does not affect the results for coils self and mutual inductances.

The coupling between stator and rotor is very low, so that three phases currents when rotor is completely inside the stator is approximately as equal as the currents when rotor is completely outside the stator.

Transient state of force is very short, and in the steady state, force has an alternating form. The maximum mean value of force on rotor is obtained when rotor is completely inside the stator, and the force is nearly zero when the rotor is completely outside the rotor.

Considering minimum 24 rings for rotor is necessary for sufficient calculations accuracy. Increasing rotor thickness as well as changing rotor material from aluminum to cupper increases the force. At low voltage levels, the effect of temperature is insignificant.

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