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Online Detection of Out-of-Step Operation Based on Prony Analysis-Impedance Relaying

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Abstract:- This paper presents an online technique for detecting out-of-step operation of interconnected power systems based on instantaneous apparent impedance relaying. The instantaneous apparent impedance seen by the relay as well as the peak values of the voltage and current signals at the relay location are highly influenced by the actual system frequency. Prony analysis is used to track the actual system frequencies by decomposing the measured voltage and current signals to its modal components: amplitude, frequency, damping factor and phase angle. Once the actual system modes are predicted, the instantaneous apparent impedance seen by the relay is calculated. The simulation results show that loci of the instantaneous apparent impedance are effective and reliable in predicting and describing the behavior of the system during out-of-step operation.

Key-Words:- Out-of-step relaying, power system protection, Prony analysis, digital relay, impedance relay

1 Introduction

Power systems are subjected to a wide range of abnormal conditions such as faults, generator tripping, line switching, loss of excitation, or load shedding. These severe disturbances may cause large separation of generator rotor angles, large swings of power flows, large fluctuations of voltages and currents at generator or transmission line terminals, and eventually lead to a loss of synchronism or what is called out-of-step (OS) operation between a generator and the rest of the power system, or between interconnected power systems [1-4]. Moreover, OS condition may result in torsional resonance and pulsating torques that are severely harmful to the generator-turbine shaft. If such OS condition occurs, it is imperative that the asynchronous generator(s) are isolated to avoid widespread outages, flashovers and equipment damage of circuit breakers [5]. This type of protection is achieved using OS relaying at preselected locations at generator or transmission line terminals [1-4].

Several techniques have been developed for OS detection. Conventional impedance techniques were developed based on unified frequency operation, whereas interconnected systems may have different operating frequencies. In these techniques, the apparent impedance measured at the relay location is compared with a proper impedance characteristic to detect OS condition [1,2,6,7]. These techniques are mostly not secure and usually fail in detecting power system instability before taking any prevention action. Other technique based on the rate of change of apparent resistance [8] and early tripping based on instability prediction, observation of phase differences between substations were developed [9-13]. An adaptive OS relay based on phasor measurement using GPS technology is proposed in [14]. A technique that applies equal area criterion to assess the stability of the generators and determine when pole slipping might occur is presented in [15]. These techniques, however, are highly computational. Recently, neural networks and fuzzy logic techniques have been used in OS protection as they proved to be efficient in many applications in power system areas [16-18].

During OS operation, the voltage and current waveforms contain modal components with frequencies that depend on the rotor slip. At the beginning of asynchronous operation between the interconnected generators, the differences between the frequencies are small which is hard to detect using Fast Fourier transform (FFT). FFT is computationally efficient, however it suffers from two problems; the frequency resolution and spectral "leakage" due to data windowing. Due to the limitations of FFT, other signal processing techniques should be examined [19].

This paper presents an online technique for detecting OS operation using impedance-based relay in conjunction with Prony analysis (PA) [19]. The instantaneous impedance seen by the relay is calculated using online measurements of the voltage and current signals at the relay location. PA is used to track power system frequencies during OS condition by extracting the modal components of the voltage and current signals that characterize the instantaneous impedance.

The instantaneous apparent impedance loci are then used to qualitatively describe the power system behavior during the OS operation.

2 Conventional OS Impedance Relay

The principle of OS detection using the apparent impedance seen by an OS relay can be analyzed using the two-machine system of Fig.1. The machines are interconnected through a transmission line (TL) of impedance Z_L and are represented by voltage sources with constant magnitudes behind their transient impedances.

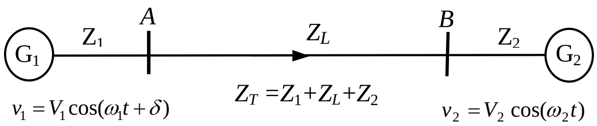

Fig.1: Two-Machine System

Following a large disturbance and during power oscillations, OS operation can be detected by studying the variation (locus) of the apparent impedance as seen by a relay located at A for different power angles δ [1,2]. The apparent impedance Z_A is given by

$$Z_A = \frac{\bar{V}_A}{\bar{I}} = -Z_1 + Z_T \frac{V_1\angle\delta}{V_1\angle\delta - V_2\angle 0} \qquad (1)$$

where V_1 and V_2 are the internal voltages of the machines, Z_1 and Z_2 are the transient impedances, and Z_T is the total connection impedance. $\bar{V}_2$ is assumed to be the reference phasor and δ represent the angle by which $\bar{V}_1$ leads $\bar{V}_2$. Assuming $|V_1|=|V_2|$ as a special case, then,

$$Z_A = \frac{\bar{V}_A}{\bar{I}} = \left(\frac{Z_T}{2} - Z_1\right) - j\left(\frac{Z_T}{2}\cot\frac{\delta}{2}\right) \qquad (2)$$

Equation (2) shows that when δ changes from 0^o to 360^o during an OS operation, the apparent impedance trajectory of Z_A, on the R-X diagram, follows a straight line that offsets from the origin by $Z_A = \left(\frac{Z_T}{2} - Z_1\right)$ and is perpendicular to the total impedance Z_T as shown in Fig. 2. In case $V_1 \neq V_2$, the trajectory of Z_A follows a group of circles as shown in Fig. 2.

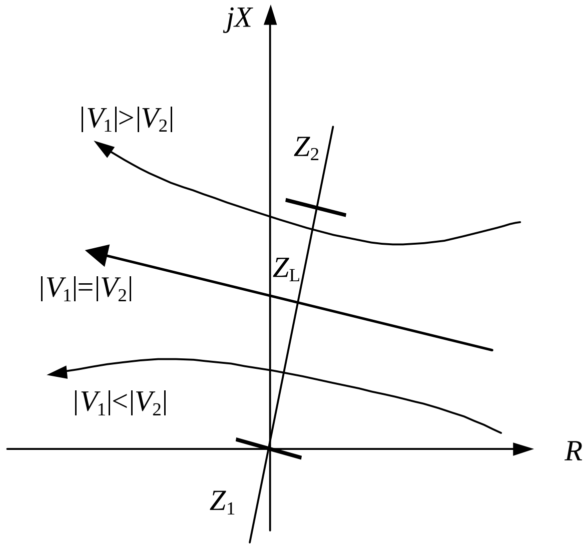

Fig.2: Locus of Z_A at Different $|V_1|$ and $|V_2|$ Levels

In this paper, instantaneous apparent impedance loci as function of time are developed after tracing the actual system frequencies.

3 Proposed Instantaneous Impedance OS Relay

Figure 3 shows the study system. It consists of two machines G_1 and G_2 interconnected via transmission line. The data of the power system is given in the Appendix. The proposed OS impedance relay is assumed to be located at the middle of the TL, though it can be connected any where on the TL as well as at the generator terminals. The proposed relay requires two transducers; a voltage transformer (VT) for measuring the voltage signal at the relay location at point P, and a current transformer (CT) for measuring the current signal. The OS relay is also equipped with A/D converter as shown in Fig.3. The measured samples of the voltage and current signals are processed using Prony method to extract the modal components that are necessary for the calculation of the instantaneous impedance seen by the relay at point P.

Following a major disturbance, an OS operation could take place. This is, usually, a queasy-stationary process characterized by an existence of double-frequency voltage and current signals at the measurement point P. Considering double frequency signals, the voltage at generator G_1 can be represented by $v_1 = V_1\cos(\omega_1 t + \theta_1)$ with corresponding frequency f_1, amplitude V_1 and phase angle θ_1. Likewise, the voltage at the terminals of generator

G_2 can be represented by $v_2 = V_2 \cos(\omega_2 t + \theta_2)$ with corresponding frequency f_2, amplitude V_2 and phase angle θ_2. The power angle δ between the voltage phasors $\overline{V}_1$ and $\overline{V}_2$ is the phase difference $\delta = \left((\omega_1 - \omega_2)t + (\theta_1 - \theta_2)\right)$.

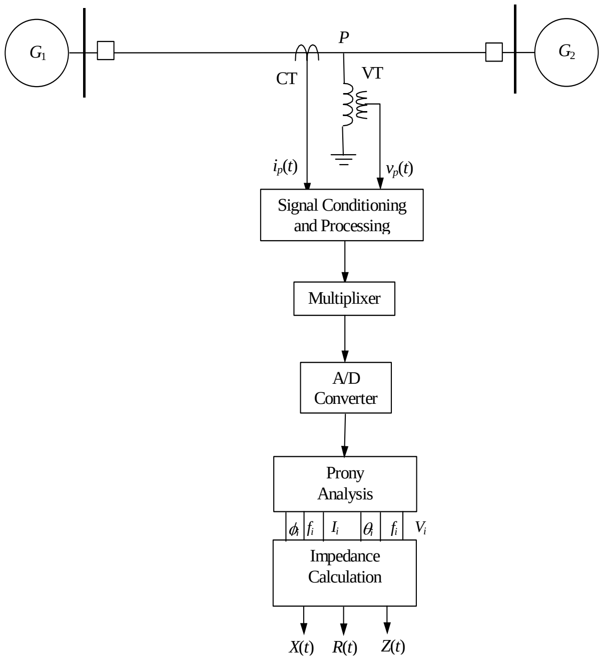

Fig.3: Out-of-Step Relaying Scheme of Two Interconnected Power Systems

Assuming that the proposed OS impedance relay is located at point P of the TL, the voltage and current signals v_P and i_P measured at P can be assumed consisting of two distinct sinusoidal components with two different frequencies ω_1 and ω_2 as given by

$$v_P = A_1 \cos(\omega_1 t + \theta_1) + A_2 \cos(\omega_2 t + \theta_2) \qquad (3)$$

$$i_P = B_1 \cos(\omega_1 t + \phi_1) + B_2 \cos(\omega_2 t + \phi_2) \qquad (4)$$

The modal components A_1, A_2, ω_1, ω_2, θ_1, θ_2, $\phi_1$and ϕ_2 are estimated using PA from samples of measurements of $v_P(t)$ and $i_P(t)$ over a specified time window T_w.

To calculate the instantaneous impedance, the voltage and current, v_P and i_P of (3) and (4), are expressed in complex forms

$$\begin{aligned}\overline{V}_P &= \left[A_1 \cos(\omega_1 t + \theta_1) + A_2 \cos(\omega_2 t + \theta_2)\right] + \\ &\quad j\left[A_1 \sin(\omega_1 t + \theta_1) + A_2 \sin(\omega_2 t + \theta_2)\right] \\ &= v_P(t) + j\tilde{v}_P(t)\end{aligned} \qquad (5)$$

$$\begin{aligned}\overline{I}_P &= \left[B_1 \cos(\omega_1 t + \phi_1) + B_2 \cos(\omega_2 t + \phi_2)\right] + \\ &\quad j\left[B_1 \sin(\omega_1 t + \phi_1) + B_2 \sin(\omega_2 t + \phi_2)\right] \\ &= i_P(t) + j\tilde{i}_P(t)\end{aligned} \qquad (6)$$

Using Euler identity the voltage and current phasors $\overline{V}_P$ and $\overline{I}_P$ of equations (5) and (6) are transformed to the exponential forms

$$\overline{V}_P(t) = V_P(t) e^{j(\psi_v(t))} \qquad (7)$$

$$\overline{I}_P(t) = I_P(t) e^{j(\psi_i(t))} \qquad (8)$$

where $V_P(t)$ and $I_P(t)$ are time-dependent rms values of the measured voltage and current at the relay location at point P. $V_P(t)$ and $I_P(t)$ are given by

$$V_P(t) = \sqrt{v_P^2 + \tilde{v}_P^{\,2}} \qquad (9)$$

$$I_P(t) = \sqrt{i_P^2 + \tilde{i}_P^{\,2}} \qquad (10)$$

The phase angles $\psi_v(t)$ and $\psi_i(t)$ can be calculated using (5) and (6) where

$$\psi_v(t) = \tan^{-1}\left(\tilde{v}_P / v_P\right) \qquad (11)$$

$$\psi_i(t) = \tan^{-1}\left(\tilde{i}_P / i_P\right) \qquad (12)$$

The instantaneous impedance at point P can be calculated as:

$$Z_P(t) = \left(\frac{\overline{V}_P(t)}{\overline{I}_P(t)}\right) = R_P(t) + jX_P(t) \qquad (13)$$

where the instantaneous resistance $R_P(t)$ and the reactance $X_P(t)$ of the instantaneous impedance $Z_P(t)$ are given by

$$R_P(t) = \frac{V_P(t)}{I_P(t)} \cos(\psi_v(t) - \psi_i(t)) \qquad (14)$$

$$X_P(t) = \frac{V_P(t)}{I_P(t)} \sin(\psi_v(t) - \psi_i(t)) \qquad (15)$$

Equations (13)-(15) indicate that the instantaneous impedance and its real and imaginary components vary with time due the difference in frequency between G_1 and G_2. The instantaneous resistance and the reactance, $R_P(t)$ and $X_P(t)$, are used to plot the R-X impedance loci which can be used to explain the behavior of the system during OS operation.

4 Prony Analysis

Prony analysis is a technique for modeling sampled data of an exponentially damped signal as a linear combination of damped sinusoids [19]. It gives an optimal fit to the measured signal in the sense of the least-squared error technique (LSE). If N samples of the response $y(t)$ is recorded as $y(k\Delta t) = y(k)$, k=0, 2, …, N-1, then $y(k)$ can be expressed as linear summation of n distinct modes

$$y(k) \cong \sum_{i=1}^{n} B_i e^{\lambda_i k \Delta t} = \sum_{i=1}^{n} B_i z_i^k \qquad (16)$$

where $z_i = e^{\lambda_i \Delta t}$, and Δt is the sampling time. The n distinct eigenvalues λ_i's and amplitudes B_i's can be identified using the three-step Prony analysis as follows [20-22]:

1. Construct a linear prediction model (LPM)

$$y(k) = a_1 y(k-1) + a_2 y(k-2) + ... + a_n y(k-n) \quad (17)$$

Repeating (17) (N-n) times to form the LPM

$$\begin{bmatrix} y(n) \\ y(n+1) \\ : \\ y(N-1) \end{bmatrix} = \begin{bmatrix} y(n-1) & y(n-2) & ... & y(0) \\ y(n) & y(n-1) & ... & y(1) \\ : & : & ... & : \\ y(N-2) & y(N-3) & ... & y(N-n-1) \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ : \\ a_n \end{bmatrix}$$

or

$$Y = \Phi A \quad (18)$$

The least-square estimate of A can be obtained using the psuedo inverse of matrix Φ

$$A = \Phi^{\dagger} Y = (\Phi^T \Phi)^{-1} \Phi^T Y \quad (19)$$

2. Find the roots (eigenvalues) of the characteristic polynomial associated with the LPM of step 1

$$z^n - a_1 z^{n-1} - a_2 z^{n-2} - ... - a_n = 0 \quad (20)$$

where $\lambda_i = \log(z_i/\Delta t) = \sigma_i \pm j\omega_i$

3. Estimate the amplitude and phase angle of each mode obtained in step 2.

$$\begin{bmatrix} y(0) \\ y(1) \\ : \\ y(N-1) \end{bmatrix} = \begin{bmatrix} z_1^0 & z_2^0 & ... & z_n^0 \\ z_1^1 & z_2^1 & ... & z_n^1 \\ : & : & ... & : \\ z_1^{N-1} & z_2^{N-1} & ... & z_n^{N-1} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ : \\ B_n \end{bmatrix} \quad (21)$$

or

$$Y = \Lambda B$$

The least-square estimate of B can be obtained as

$$B = \Lambda^{\dagger} Y = (\Lambda^T \Lambda)^{-1} \Lambda^T Y \quad (22)$$

The degree of fitness of the Prony LPM (16) to the simulated (measured) signal can be evaluated in terms of the signal-to-noise-ratio (SNR) defined as [20]

$$SNR = 20 \log\left(\|y(k)\| / \|y(k) - \hat{y}(k)\| \right) \quad (23)$$

where $\|y(k)\|$ is the second norm of the measured signal $\|y(k) - \hat{y}(k)\|$ is the norm of the error signal between the measured and estimated signals. For perfect fitting the numbers of samples N and modes n are varied until a $SNR \geq 40$ dB is verified.

5 Simulation Results

Figures 4 and 5 depicts the simulated (measured) voltage and current signals, $v_P(t)$ and $i_P(t)$ at P, respectively. In this study, the two signals are generated by simulating an OS operation with the voltages v_1 and v_2 of G_1 and G_2 are assumed to be

$$v_1 = 34.5\cos(2\pi(51)t + 0°)kV$$

$$v_2 = 34.5\cos(2\pi(50)t + 0°)kV$$

The superposition is then used to calculate $v_P(t)$ and $i_P(t)$ for simulation purposes.

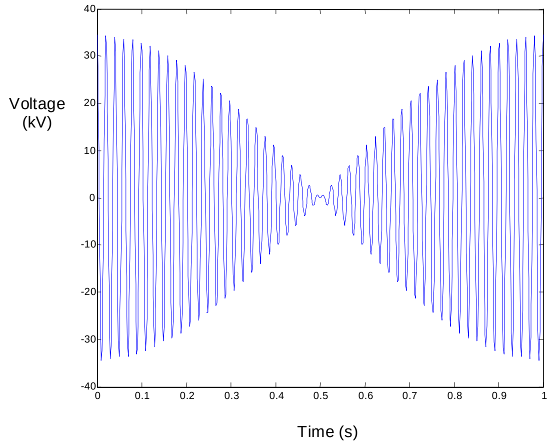

Fig. 4: Measured Voltage Signal at point P

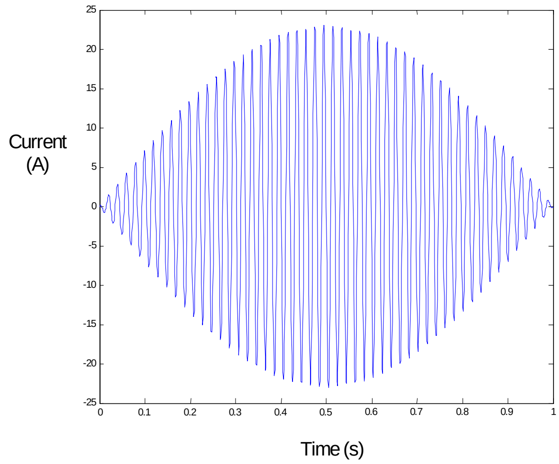

Fig. 5: Measured Current Signal at point P

After simulating (measuring) $v_P(t)$ and $i_P(t)$, the two signals are processed using PA to compute the modal components of $v_P(t)$ and $i_P(t)$. The number of modes n is set to $n = 4$ and the number of samples N and the time window T_w is varied until perfect estimate of the modal components is obtained with $SNR \geq 40$ dB. Tables 1 and 2 shows the modal components of $v_P(t)$ and $i_P(t)$, respectively, with T_w = 10ms and N = 10 with sampling frequency f_s = 1000 Hz.. Once the modal components are obtained, equations (5)-(15) are applied to compute the instantaneous complex apparent impedance $Z_P(t)$ at the relay location.

Table 1: Voltage Modal Components Estimates of the LPM Modes, f_s = 1000Hz, n=4, N=10

Mode Order	σ_i	f_i (Hz)	Amplitude A_i (kV)	θ_i (rad)
1	0.0	50.9019	17.25	0.0
2	0.0	-50.9019	17.25	0.0
3	0.0	50.0000	17.25	0.0
4	0.0	50.0000	17.25	0.0

Table 2: Current Modal Components Estimates of the LPM Modes, f_s = 1000Hz, n=4, N=10

Mode Order	σ_i	f_i (Hz)	Amplitude B_i (A)	ϕ_i (rad)
1	0.0	50.9019	11.50	0.0
2	0.0	-50.9019	11.50	0.0
3	0.0	50.0000	11.50	0.0
4	0.0	50.0000	11.50	0.0

Figure 6 shows the *R*-*X* diagram at the above-mentioned OS conditions. The circle of the instantaneous apparent impedance locus indicates the presence of an OS condition. It can be observed that for a slip frequency of 1 Hz, the locus circle is completed in 1s interval. It can also observed in Fig. 6 that for the first half second, the power flow is from G_2 to G_1 as indicated by the negative resistance locus, while the power flow reverses from G_1 to G_2 during the other half cycle. As far the reactive power flow is concerned, the positive values of the reactance indicate that reactive power flows from G_1 to G_2 during the entire period of oscillation. The *R*-*X* diagram can also be used to predict the instability condition before the power angle δ reaches 180^o. This occurs at t=0.5s when $R(t)$ falls down to zero. Moreover, at this instant the line current i_P(t) reaches its maximum value, while the voltage v_P(t), falls to its minimum value. This situation, makes the OS operation resembles a three-phase fault at *P*. Therefore, this situation has to be detected for example, by determining the rate of change of the resistance and then, a proper control action has to be taken, as quickly as possible, before the distance relays results in false tripping of the TL.

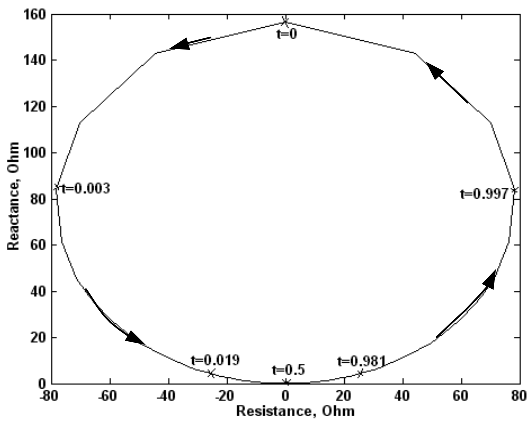

Fig.6: Instantaneous Apparent Impedance Locus for Frequency Slip of 1 Hz

Figure 7 shows the instantaneous impedance locus at a frequency deviation of 0.5 Hz. Examining Fig.7, one can see that the OS operation cycle is longer and takes 2s.

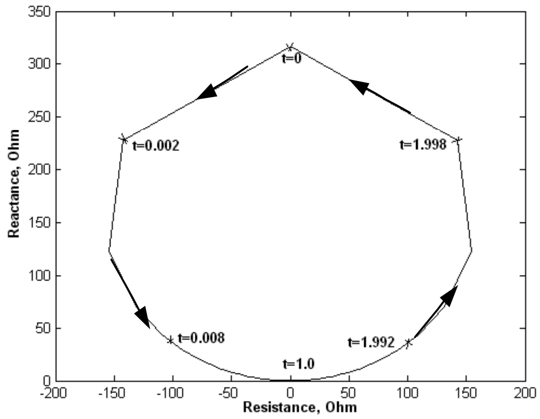

Fig.7: Instantaneous Apparent Impedance Locus for Frequency Slip 0.5 Hz

Figure 8 shows the instantaneous impedance locus for the condition, when the voltage of the generator G_1 falls to 7.5 kV and G_1 runs faster than G_2 as a result of loss of excitation or major disturbance. In this case, the locus starts at t=0s and continues clockwise indicating that G_1 is delivering active power to G_2. Figure 8 indicates that the reactance $X_P(t)$ is negative during the OS condition, which means that the reactive power flows toward G_1. This situation is similar to the generator loss of excitation situation.

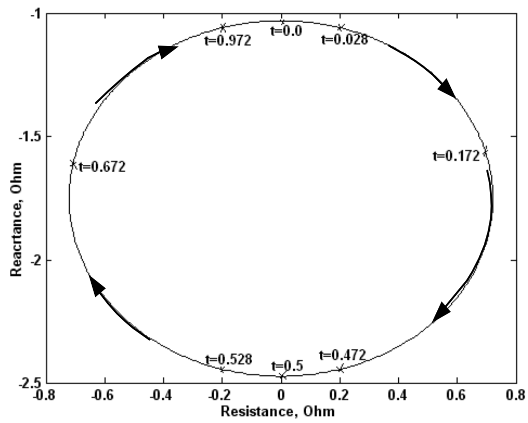

Fig.8: Instantaneous Apparent Impedance Locus for Frequency Slip of 1 Hz and $|V_1|$ = 7.5 kV

5 Conclusion

An impedance relay is developed in this paper for the detection of out-of-step operation based on the instantaneous apparent impedance measured at a relay location. An efficient technique based on Prony analysis was used for extracting the modal components of the voltage and current signals from online measured samples of the voltage and current signals at the relay location. Once the actual frequencies of the interconnected machines are determined, the instant at which the power angle δ reaches 180^o (instant of instability) can be determined. The simulation results show that Prony analysis technique is able to track the system frequency in half a cycle permitting enough time to detect the system instability. Moreover, the results show that the rate of change of instantaneous apparent impedance can be very helpful in distinguishing between out-of-step or fault condition.

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Appendix

The power system data are given below:

$XG_1 = 0.6$ p.u, $XG_2 = 0.2$ p.u, $X_T = 0.1$ p.u

Power Base =1000 MVA, Voltage Base = 34.5 kV.