

Electromagnetic Scattering Analysis and Radar Cross Section (RCS) Computation of Circular-like Impedance Objects Using the Method of Auxiliary Sources (MAS) in Conjunction with the Standard Impedance Boundary Condition (SIBC)

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Abstract: - A combination of the Method of Auxiliary Sources (MAS) and the Standard Impedance Boundary Condition (SIBC) is employed in the analysis of transverse magnetic (TM) plane wave scattering from various infinite, impedance cylindrical objects. Various cross sections, which can be considered as perturbed duplicates of the circle, are investigated. The computational error related to the numerical solution of the MAS linear system, as well as the condition number of the pertinent impedance matrix, are plotted and the results are discussed. A parametric analysis is conducted in order to probe the effect that, the degree of perturbation from the circle and the number of the auxiliary sources used, have upon the numerical error and the stability of the ensuing system. Furthermore, the monostatic RCS for the location of the auxiliary surface with optimum trade-off between error and condition number is presented. The aim of this study is the extraction of useful conclusions on the optimum location of the auxiliary sources for arbitrarily shaped impedance scatterers.

Key-Words: - Electromagnetic Scattering, Radar Cross Section, Method of Auxiliary Sources, Impedance Scatterers

1 Introduction

The Method of Auxiliary Sources (MAS) [1]-[2] (also known as the Multifilament Current Model [3]) is an alternative approach to more widespread algorithms such as the Moment Method (MoM), for the solution of the forward scattering problem. Extensive applications of MAS [4] have clearly highlighted the merits and advantages of the technique. Specifically, MAS is characterized by algorithmic simplicity and a less time-consuming procedure for the evaluation of the matrix elements compared to MoM, due to a much smaller constant multiplier in the cost estimate, thus rendering the method potentially favourable for electrically large geometries [5].

Nevertheless, the MAS merits cannot be fully exploited, unless the location of the auxiliary sources (AS) is optimized. Otherwise, the technique's performance may range from merely inefficient to completely inaccurate. Recently, rigorous optimization was carried out for infinite cylindrical scatterers with circular cross section, satisfying various boundary conditions, including

perfectly conducting (PEC) [6], impedance [7] and dielectric surfaces [8]. The optimization was based on the theory of circulant matrices, which inherently characterizes all MAS implementations associated with circular geometries. Although the analytical results cannot be straightforwardly extended to other shapes, the physical insight gained can serve as a guideline towards the feasibility of such a generalization.

In this paper scatterers which can be described utilising the SIBC boundary condition are considered. It has been shown [9] that SIBC can accurately model a coated perfect electric conductor unless the relative dielectric constant has a very small magnitude. Coated structures occur in many engineering applications and real-life phenomena (e.g., paint on an aircraft fuselage illuminated by a RADAR). The purpose of this paper is an attempt to optimize MAS for generic impedance scatterers, using mostly numerical schemes, based however on the observations made in [6]-[8]. Specifically, it is useful to examine the behavior of the numerical MAS solution, as the cross section gradually

deviates from the circle. Similar work for PEC and dielectric scatterers is presented in [10] and [11] respectively.

In Section 2, a brief description of the main results presented in [6]-[8] for circular cross sections is given, whereas a number of novel geometries, serving as perturbations of the circle, are introduced. In Section 3, the numerical error of the linear system solution, along with the matrix condition number is presented for these geometries. Also, the results of a parametric analysis are presented and discussed and the Radar Cross Section (RCS) is evaluated for the case of optimum location of auxiliary surface. Finally, in Section 4, several conclusions and possible future work are discussed.

2 Theoretical Background

2.1 Circular Geometries

In [7], a simple benchmark problem was chosen, i.e., scattering of a transverse magnetic (TM) plane wave from an impedance, infinite, circular PEC cylinder with radius $b - d$, coated with a thin dielectric layer. The dielectric layer is characterized by thickness d , complex permittivity ϵ_r and relative permeability μ_r . The composite structure was modelled in a compact way, by invoking the Standard Impedance Boundary Condition (SIBC) [12]. According to SIBC, in the three-dimensional case, the total electric \mathbf{E} and magnetic field \mathbf{H} on the cylindrical surface with radius $b - d$ are related to each other through

$$\hat{\mathbf{n}} \times \mathbf{E} = \zeta \hat{\mathbf{n}} \times (\hat{\mathbf{n}} \times \mathbf{H}) \quad (1)$$

where $\hat{\mathbf{n}}$ is the outward pointing, normal unit vector to the surface S , ζ is the surface impedance given by

$$\zeta = j \zeta_1 \tan(k_1 d) \quad (2)$$

with $\zeta_1 \equiv \sqrt{(\mu_0 \mu_r) / (\epsilon_0 \epsilon_r)}$ (coating intrinsic impedance) and $k_1 \equiv \omega \sqrt{\mu_0 \epsilon_0 \mu_r \epsilon_r}$ (wavenumber inside the coating). The MAS linear system was formulated by constructing one interior fictitious auxiliary surface conformal to the actual surface and invoking SIBC. The impedance matrix was inverted via eigenvalue evaluation and subsequent diagonalization. The derivation of exact expressions was thus facilitated for the error, associated with the boundary condition satisfaction. Furthermore, an exact expression for the condition number of the

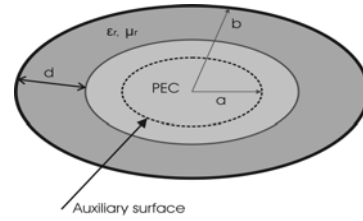


Fig. 1: Scatterer's geometry

system was also derived. Similar work has been completed for circular, PEC [6] and dielectric cylinders [8]. For the impedance case [7], it was shown that all the conclusions drawn for the PEC case are also applicable to the SIBC cylinder. For example, the eigenvalues for the PEC case [6] can be derived from the eigenvalues of SIBC case [7] by setting $\zeta = 0$.

The main conclusion in these papers was that, irrespective of the scatterer's material, the analytical error generated by the discrete nature of MAS generally decreases as the auxiliary surface approaches the center of the cylinder. As an exception to this rule poses a denumerable set of auxiliary surface locations, in the vicinity of which the error bursts abruptly, due to resonances, mathematically associated with the zeroes of Bessel functions. On the other hand, the numerical error, computed by LU decomposition and inversion, agrees with the analytical one only above a certain threshold of the auxiliary surface radius. Below this threshold, it grows erratically, due to severe matrix ill - conditioning. Finally, the condition number was shown to grow exponentially with the number of unknowns.

2.2 Circular-like Geometries

Unfortunately, the analytical tools invoked in [6]-[8] for the circular case are apparently not applicable to generic shapes. The reason is that for any other cross-section, the resulting MAS matrix fails to be circulant, and therefore it is not necessarily amenable to analytical inversion. However, it is expected that small perturbations to the circular geometry should not affect significantly the qualitative behavior of either the error, or of the condition number. It would be very interesting to study both these parameters quantitatively, for geometries close enough to a circle, in order to make observations potentially helpful in a generic MAS optimization. Thus, in this paper, MAS is applied to impedance scatterers, their cross section being geometrically described by a regular ellipse, a "daisy" and a super-ellipse. The computational

error and the matrix condition number (in the 1-norm) are calculated via a direct numerical method, specifically LU decomposition with partial pivoting. For each geometry, the normal unit vector is computed separately and we cast the linear systems in each case using trivial algebra.

2.2.1 Regular Ellipse

An ellipse can be considered as a perturbation of a circle with radius a , by defining one of its semi-axes to be equal to a , and the second one equal to $b=\gamma a$, where the positive parameter $\gamma < 1$ is the ellipticity. The equation of the ellipse is, of course,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1. \quad (3)$$

For the two-dimensional (2D) TM case, the SIBC (1) becomes

$$\mathbf{E} = \zeta (\hat{\mathbf{n}} \times \mathbf{H}). \quad (4)$$

As an intermediate step, the total fields are expressed as sums of incident ($\mathbf{E}_i, \mathbf{H}_i$) and scattered ($\mathbf{E}_s, \mathbf{H}_s$) terms and (4) yields

$$\mathbf{E}_s - \zeta \hat{\mathbf{n}} \times \mathbf{H}_s = -\mathbf{E}_i + \zeta \hat{\mathbf{n}} \times \mathbf{H}_i. \quad (5)$$

The incident field at an arbitrary point (x, y) is equal to

$$\mathbf{E}_i(x, y) = E_{0i} \hat{\mathbf{z}} \exp\{jk_0(x \cos \phi_i + y \sin \phi_i)\} \quad (6a)$$

$$\mathbf{H}_i(x, y) = -\frac{E_{0i}}{\zeta_0} (\hat{\mathbf{x}} \sin \phi_i - \hat{\mathbf{y}} \cos \phi_i) \quad (6b)$$

$$\times \exp\{jk_0(x \cos \phi_i + y \sin \phi_i)\}$$

where E_{0i} is the associated electric field amplitude.

The normal unit vector for the regular ellipse case is

$$\hat{\mathbf{n}} = \frac{\nabla f}{|\nabla f|} = \frac{2x}{a^2} \hat{\mathbf{x}} + \frac{2y}{b^2} \hat{\mathbf{y}} \quad (7)$$

$$\text{where } f(x, y) = \frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 = 0. \quad (8)$$

The electric field at the position (x, y) due to the n th auxiliary source at the point (x_n, y_n) is

$$\mathbf{E}_{sn}(x, y) = \hat{\mathbf{z}} E_n H_0^{(2)} \left[k_0 \sqrt{(x-x_n)^2 + (y-y_n)^2} \right] \quad (9a)$$

where E_n is the corresponding unknown weight, $H_0^{(2)}$ is the Hankel function of zero order and second kind. The corresponding magnetic field of the auxiliary source is

$$\mathbf{H}_{sn}(x, y) = -jE_n H_1^{(2)} \left[k_0 \sqrt{(x-x_n)^2 + (y-y_n)^2} \right] \times \frac{(y-y_n)\hat{\mathbf{x}} - (x-x_n)\hat{\mathbf{y}}}{\left[k_0 \sqrt{(x-x_n)^2 + (y-y_n)^2} \right]} \quad (9b)$$

where $H_1^{(2)}$ is the Hankel function of zero order and second kind.

The total scattered field is expressed as

$$\mathbf{E}_s(x, y) = \sum_{n=1}^N \mathbf{E}_{sn}(x, y) \quad (10a)$$

$$\mathbf{H}_s(x, y) = \sum_{n=1}^N \mathbf{H}_{sn}(x, y) \quad (10b)$$

Substitution of (6), (7) and (10) to (4) yields the appropriate matrix equation system.

2.2.2 “Daisy”

To obtain a circular-like geometry with a periodically perturbed circumference, the “daisy” is defined by

$$r(\theta) = r_{\min} + r_0 \sin(\omega\theta) \quad (13)$$

where r_{\min} is the minimum radius, r_0 is the maximum perturbation and ω is the angular frequency (see Fig. 2a). When ω increases, a wedge scheme as in [13] has to be taken into consideration in order to account for edge effects.

2.2.3 Super-ellipse

A super-ellipse is a generalization of an ellipse, its equation given by

$$\frac{x^\nu}{a^\nu} + \frac{y^\nu}{b^\nu} = 1, \nu \in \mathbf{N}. \quad (14)$$

For $\nu \rightarrow \infty$, the super-ellipse approaches a rectangle (see Fig. 2b).

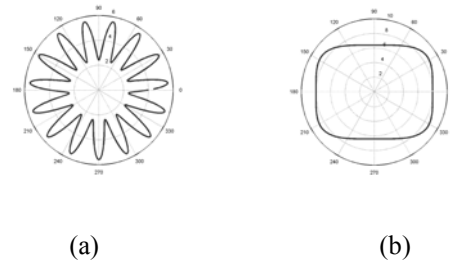


Fig. 2: Geometries

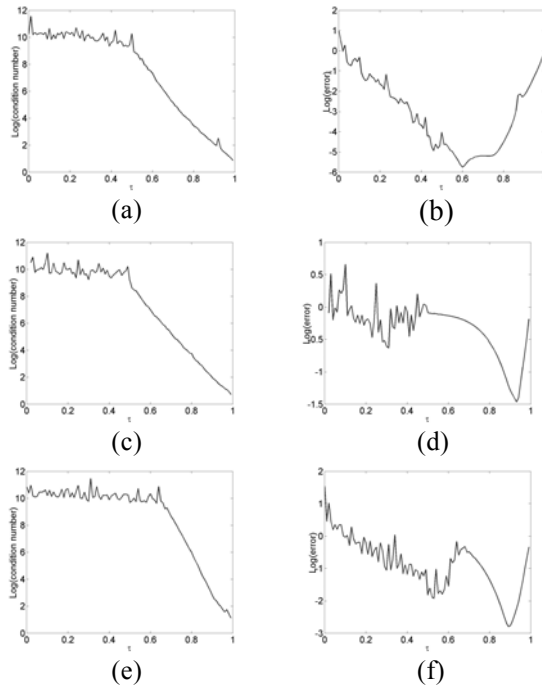


Fig. 3: Condition number and numerical error plots for various geometries. The first column depicts the condition number and the second the numerical error. Plots (a), (b) show the results for the ellipse, while (c), (d) for the "daisy" and (e), (f) for the super-ellipse.

3 Numerical Results and Discussion

In all cases examined, the auxiliary surfaces were defined to be conformal (similar) to the scatterer's boundary, with a similarity ratio $0 < \tau < 1$ for the interior surface. The error was defined exactly as in [6]-[8]. The relative electrical permittivity of the dielectric coating was defined as $\epsilon_r = 3 - j10$ in all cases.

As a first example, we consider an elliptical scatterer with ellipticity $\gamma=0.8$, large semi-axis $a=\lambda$ and dielectric thickness 0.1λ . A number of $N=50$ auxiliary sources was used. The plots of the condition number and the numerical error are presented in Figures 3(a) and 3(b) respectively, as a function of τ . The condition number is large and unstable for $\tau < 0.5$ and is decreasing for increasing τ when τ belongs to the interval $[0.5, 1]$. For $\tau < 0.5$ the numerical error is oscillating while it assumes a smooth form for $\tau > 0.5$. The numerical error is not monotonous, and is minimized approximately for $\tau=0.6$.

Next we consider a "daisy" of minimum radius 0.5λ , maximum perturbation 0.05λ , angular frequency $\omega=10 \text{ rad}^{-1}$ and dielectric thickness of 0.05λ . The number of auxiliary sources used was $N=70$. Figure 3(c) show that again the condition

number becomes large and unstable for values of τ less than 0.5, whereas the numerical error remains stable for the remaining range of values of the parameter τ , as is depicted in Fig. 3(d).

As a third example we considered a super-ellipse with $\nu=4$, $a=\lambda$, $b=0.8\lambda$ and dielectric thickness 0.1λ . Again it is easily noticed that after a critical value for the parameter τ which in this case is approximately 0.65, the condition number is starting to monotonically decrease for increasing τ and the numerical error assumes a smooth variation.

To optimize the AS location, and hence the MAS performance, the computational error should evidently be minimized. Unlike the circular cylinder case [6]-[8], the plots in Fig. 3 show that the error does not necessarily decrease for decreasing τ . However, matrix conditioning invariably deteriorates for values of τ less than 0.5, adding numerical noise to the solution, and therefore concealing the actual error behaviour for very small auxiliary surfaces. Like in [6]-[8], it is clear from the plots that the computational error demonstrates an irregular behaviour when the condition number is larger than 10^7 (a threshold associated with FORTRAN's double precision).

Finally, given the problems caused by matrix ill-conditioning, it would be beneficial to construct a suitable pre-conditioner applicable to the MAS linear system. Since the optimum pre-conditioner for the circular cylinder is analytically known (it is actually the inverse of the MAS matrix), the pre-conditioner for a geometrical perturbation of the circle may be designed as the inverse of some MAS matrix, associated with a closely related circular configuration. Intense research on this topic is currently in progress.

3.1 Parametric Analysis

In this sub-section, the effect of two parameters on the stability and the accuracy of the numerical solution of the ensuing numerical system is examined. The first set of simulations aims to investigate how the numerical properties of the linear system are affected when the scatterer's cross section departs from the circular geometry. For example, a decrease in the value of the ellipticity (b/a) for the standard ellipse or the super-ellipse, signifies an increased perturbation from the circular shape as does an increase in the value of ω for the "daisy". As can be easily seen from Fig.4 when the cross section resembles the circular shape to a smaller degree, the condition number increases while the numerical error deteriorates.

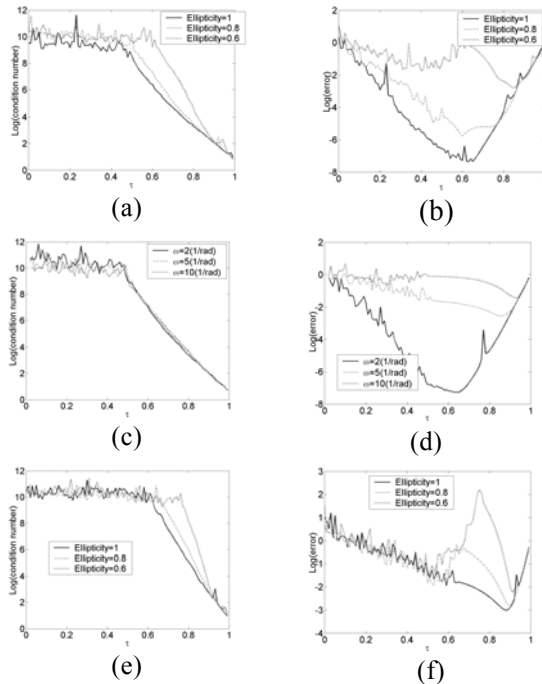


Fig. 4: Comparative condition number and numerical error plots for various geometries. The first column depicts the condition number and the second the numerical error. Plots (a), (b) show the results for the ellipse, (c), (d) for the “daisy” and (e), (f) for the super-ellipse.

In Fig.5 the effect of the number of auxiliary sources on the condition number and the numerical error is presented. The conclusion which can be readily obtained from these results is that an increase on the number of auxiliary sources improves the accuracy of the numerical solution at the cost of an increased condition number.

3.2 Radar Cross Section (RCS)

As it has been shown in circular cases [6]-[8], the analytical error decreases as the auxiliary surface approaches the center of cylinder while the numerical error, computed by LU decomposition and inversion, agrees with the analytical one only above a certain threshold of the auxiliary surface radius. Based on this rule, for circular-like geometries, we compute RCS for the optimum location of the auxiliary surface which corresponds to the value where error and condition number are best balanced. The RCS represents a convenient way to describe the strength of scattered fields observed in the far region. By definition [14]-[15], for 2-D objects, the monostatic RCS (where angles of observation and incidence coincide) becomes the scattering width (SW) determined by the equation

$$SW = \lim_{r \rightarrow \infty} 2\pi r \frac{|\mathbf{E}_{sc}|^2}{|\mathbf{E}_{inc}|^2} \quad (15)$$

The scattered electric field in the far region is written as $\mathbf{E}_{sc} \sim f(\phi, z) \frac{\exp(-jk_0 r)}{\sqrt{r}}$ (16)

where $f(\phi, z)$ is a function of the transverse polar coordinates (ϕ, z) and k_0 is the wave-number.

The computed RCS for the aforementioned geometries are shown in Fig. 6. For the ellipse, the large semi-axis a lies on x -axis. It can be observed that the scattered field assumes peak values when the incident field “faces” a wider front of the scatterer’s surface. This is especially pronounced in Fig. 6(c) where the RCS for the “daisy” is depicted.

4 Conclusions and Future work

In this paper, a numerical analysis of the MAS accuracy and stability for impedance scatterers associated with geometrical perturbations of a circular cylinder was presented. For all cases, the computational error and the 1-norm condition number of the MAS linear system were calculated. Plots of the condition number are reminiscent of the circular case, slowly increasing for shrinking auxiliary surfaces, and finally becoming highly irregular. Isolated peaks are also present, corresponding to resonance effects related to roots of Bessel functions in the circular case. On the other hand, plots of the computational error prove that the error does not necessarily decrease for retracting auxiliary surfaces. However, the true, global behavior of the error is not easily discernible, due to severe ill - conditioning of the system for small auxiliary surfaces. Therefore, it is imperative that the system conditioning of a general MAS linear system be reduced, using a suitable pre-conditioner, before an optimization procedure for arbitrary geometries is proposed.

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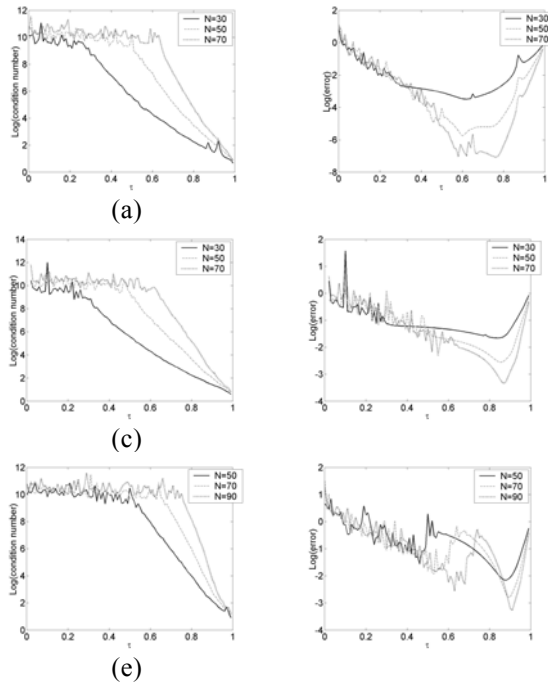


Fig. 5: Comparative condition number and numerical error plots for various numbers of auxiliary sources. The first column depicts the condition number and the second the numerical error. Plots (a), (b) show the results for the ellipse, (c), (d) for the “daisy” and (e), (f) for the super-ellipse.

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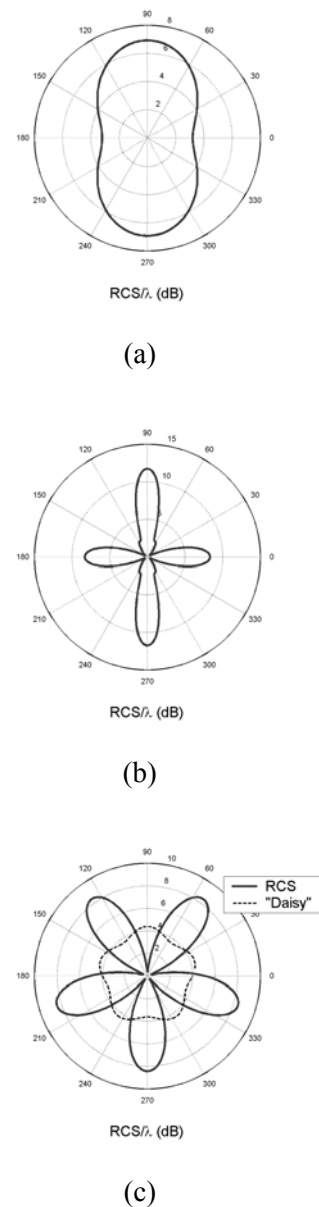


Fig. 6: RCS for the simple ellipse (a), the super ellipse (b) and the “daisy” (c).

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