

Multi-Feature Edge Detection with the Feature of Local Image Complexity

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Abstract: - In this paper, the local fuzzy fractal dimension (LFFD) is proposed to represent the feature of local image complexity. The definition of LFFD is an extension of the box-counting dimension of discrete sets by incorporating the fuzzy set. The relationship between LFFD and local intensity changing is investigated by experiments, which proves that LFFD is an important feature of edges and is insensitive to noises. Multi-feature edge detection is implemented with the LFFD and the Sobel operator. The experimental results show that the incorporation of LFFD in edge detection improves the quality of edge detection results.

Key-Words: - Edge detection, fractal, local fuzzy fractal dimension, local image complexity, multi-feature image processing

1 Introduction

Edge detection is an important basis of further image processing. It provides essential information for image analysis, pattern recognition, image compression, etc. [1,2,3]. Traditional edge detectors focus on the intensity discontinuity in images, such as the Roberts operator, the Laplacian operator, the Sobel operator, etc. [4]. Because noise in images usually has sudden change of intensity, traditional gradient-based operators are sensitive to noises. Therefore, it is far from sufficiency to identify edge points only by the gradient feature.

Edge detection can be regarded as a problem of classification, which differentiates edge points from the other image points. The performance of the classification is related to the dimension of the feature space. If more features are available, more accurate the classification might be. Images have multiple features besides intensity change, such as texture, complexity, etc. Multi-feature methods have been applied in image processing applications and got promising results, such as corner detection, straight line extraction, boundary finding, etc. [5,6,7]. Therefore, multi-feature edge detection becomes a promising way that considers multiple image features to get better performance of edge detection.

Fractal dimension can reflect the complexity of fractal sets, which has been used in texture analysis, object modeling and image classification, etc. [8,9].

The concept of fractal dimension in fractal geometry is defined for fractal geometric figures. In many applications, the data sets do not strictly follow the definition of fractal, but only follow the strict definition within a certain range of scales [10]. Moreover, the definition of the fractal dimension is for conventional sets. It is not applicable to fuzzy set that is widely used in the domain of artificial intelligence. In this paper, the computation of fractal dimensions for n-dimensional discrete sets is proposed based on the box-counting dimension and the pixel-covering method. The fuzzy concept is introduced and the fractal dimension of fuzzy sets with discrete elements is proposed for the measurement of their complexity. Then the local fuzzy fractal dimension (LFFD) of gray-scale images is proposed to extract the local feature of image complexity.

The fractal dimension is useful to quantify the complexity of details. In this paper, the relationship between the LFFD and local intensity changing in gray-scale images is investigated by experiments, which shows LFFD is an important feature of edges. The LFFD shows good quality in edge detection with noise and between different textures. A method of multi-feature edge detection is proposed based on the LFFD and intensity discontinuities. In the experiments, it is shown that the LFFD greatly improves the performance of edge detection.

2 The Local Fuzzy Fractal Dimension (LFFD)

The fractal geometry was established by Benoit B. Mandelbrot. It is useful in profound description of irregular and random phenomenon in nature [10,11]. The fractal dimension (FD) is the basic concept in the fractal geometry and serves as an important feature of images. Hausdorff dimension is the fundamental definition of the fractal dimension in the fractal geometry theory. However, it is hard to calculate Hausdorff dimension in most cases [11]. Box-counting dimension is easy to compute and widely used. It can be estimated for 2D monochrome images by the pixel-covering method [11]. In this paper, the pixel-covering method is extended to an n-dimensional discrete set, based on which the fuzzy set is incorporated and the LFFD is proposed.

2.1 The box-counting dimension and the pixel-covering method

The definition of the box-counting dimension is as follows [11]:

$$\dim_B F = \lim_{\delta \rightarrow 0^+} \frac{\log N_\delta(F)}{-\log \delta} \quad (1)$$

where F is a non-empty bounded set in R^n . $N_\delta(F)$ is the minimum number of the sets covering F with their radii no larger than δ . To estimate the box-counting dimension, a group of data $(-\log \delta_i, \log N_{\delta_i}(F))$ is obtained by changing the value of δ . The slope of the line derived from the data is estimated as the box-counting dimension by the least-squares linear regression. Fig. 1 shows the estimation of $\dim_B F$.

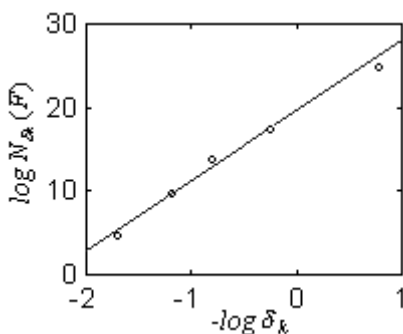


Fig. 1. The estimation of $\dim_B F$ by the least-squares linear regression

In signal processing with computers, data is usually discretized. The pixel-covering method is proposed based on the box-counting dimension to

estimate the FD of fractal geometric figures, which are binarized images where the points of the fractal objects are represented by 1 and the background 0 [11]. The image is divided into squares with the width of δ . $N_\delta(F)$ is the number of the squares containing at least one object point. The FD is estimated as the same way of the box-counting dimension by the least-squares linear regression.

2.2 The estimation of box-counting dimension for n-dimensional discrete sets

The pixel-covering method can only estimate the FD of discrete binarized images. Because in many applications the data to be processed is not limited to 2D images, it is necessary to generalize the pixel-covering method for discrete sets in R^n . The box-counting dimension of a bounded discrete set C in R^n is proposed as follows:

- (1) For a discrete scale δ , divide the region that just contains C into n-dimensional boxes of width δ .
- (2) For any n-dimensional box C_i^δ , its characteristic value $F(C_i^\delta)$ is defined as follows:

$$F(C_i^\delta) = \bigcup_{p \in C_i^\delta} f(p) \quad (2)$$

In Equation (2), $f(p)$ indicates whether the element p belongs to C . $f(p)$ is defined as follows:

$$f(p) = \begin{cases} 1 & \text{if } p \in C \\ 0 & \text{if } p \notin C \end{cases} \quad (3)$$

In Equation (2), the operator \bigcup is defined as follows:

$$a \bigcup b = \begin{cases} 1 & \text{if } a = 1 \text{ or } b = 1 \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

- (3) For the discrete scale δ , the number of boxes that C occupies, i.e. the characteristic value of C on scale δ , is defined as:

$$N_\delta(C) = \sum_{i=1}^M F(C_i^\delta) \quad (5)$$

where M is the number of boxes in the region that just contains C .

According to the above three steps, a series of data $(-\log \delta, \log N_\delta(C))$ can be obtained by changing the value of δ . The FD of C can be estimated by the least-squares linear regression according to the data. It is obvious that the pixel-covering method is a special case for the FD of n-dimensional sets when $n=2$.

2.3 The local fuzzy fractal dimension

The estimation of FD for n-dimensional discrete set is only for binarized sets. In binarized sets, the characteristic values of elements can only be 1 or 0, where the value 1 indicates the element belongs to the set and 0 the opposite. However, in many applications the characteristic values of the elements are not limited as only 0 and 1. Such sets can be regarded as the sets with multiple characteristic values, which can be described by fuzzy sets. For instance, the pixels in the images of 256 gray-scale levels have more values than just black and white. For the pixel-covering method, gray-scales images must be binarized before the FD can be estimated. The binarization will cause much loss of detailed information, which is not preferable to image feature extraction.

In this paper, the n-dimensional set C is generalized to a fuzzy set A with finite elements. By defining the element's characteristic value as the membership function, the fuzzy fractal dimension (FFD) is proposed for n-dimensional sets with multiple characteristic values. In the estimation of FFD, the characteristic value of an element p is the value of its membership function, which indicates to what degree p belongs to A . Therefore, $f'(p)$ is the membership function of A and $f'(p) \in [0, 1]$. The estimation of FFD is as follows:

- (1) For a discrete scale δ , divide the region that just contains the elements of A into n-dimensional boxes of width δ ;
- (2) For any n-dimensional box C_i^δ , its characteristic value is defined as follows:

$$F'(C_i^\delta) = \vee_{p \in C_i^\delta} f'(p) \quad (6)$$

where $f'(p)$ is the membership function. The operator \vee is defined as follows:

$$a \vee b = \max\{a, b\} \quad (7)$$

- (3) For the discrete scale δ , the characteristic value of A on scale δ is defined as:

$$N'_\delta(A) = \sum_{i=1}^M F'(C_i^\delta) \quad (8)$$

where M is the number of the boxes. Therefore, a series of data $(-\log \delta, \log N'_\delta(A))$ can be obtained by changing the value of δ . The FFD of A is estimated by the least-squares linear regression.

The FD indicates the global feature of complexity for a set. For images, besides global features there are also local features that are important, such as edges, local textures, etc. To represent local image complexity, the local fuzzy fractal dimension (LFFD) at an image point is

defined as the FFD of a small neighboring area around the point. Since the FD indicates the complexity of the geometric structure, the LFFD indicates structure features of the local areas in images.

3 The LFFD as an Important Local Feature of Images

3.1 The LFFD of local image areas

For images of 256 gray-scale levels, the membership function of image points is defined as the quotient of dividing their gray-scale values by 255. In another word, the membership function value of an image point indicates to what degree the point belongs to a white one.

For a local area D in the image, the LFFD is estimated by the method proposed in Section 2.3. The logarithm of $N'_\delta(D)$ is calculated as follows:

$$\begin{aligned} \log N'_\delta(D) &= \log \left[\sum_{i=1}^M F'(D_i^\delta) \right] \\ &= \log \sum_{i=1}^M \left[\vee_{p \in D_i^\delta} f'(p) \right] = \log \sum_{i=1}^M \left\{ \vee_{p \in D_i^\delta} \left[\frac{G(p)}{255} \right] \right\} \\ &= \log \sum_{i=1}^M \left[\frac{\vee_{p \in D_i^\delta} G(p)}{255} \right] = \log \left\{ \frac{1}{255} \sum_{i=1}^M \left[\vee_{p \in D_i^\delta} G(p) \right] \right\} \\ &= \log \sum_{i=1}^M \left[\vee_{p \in D_i^\delta} G(p) \right] - \log 255 \end{aligned} \quad (9)$$

where $G(p)$ is the gray-scale of p . D_i^δ is the i -th box of width δ and M is the number of such boxes in D . Therefore,

$$\begin{aligned} \log N'_\delta(D) &= \log \sum_{i=1}^M \left[\vee_{p \in D_i^\delta} f'(p) \right] \\ &= \log \sum_{i=1}^M \left[\vee_{p \in D_i^\delta} G(p) \right] - \log 255 \end{aligned} \quad (10)$$

The value $\log 255$ is a constant and it does not affect the slope of the line derived from the data. Therefore, when estimating the LFFD of a local area in the image, $G(p)$ can substitute for $f'(p)$ in calculating the characteristic value $F'(D_i^\delta)$. The flowchart of the LFFD calculation is shown in Fig. 2.

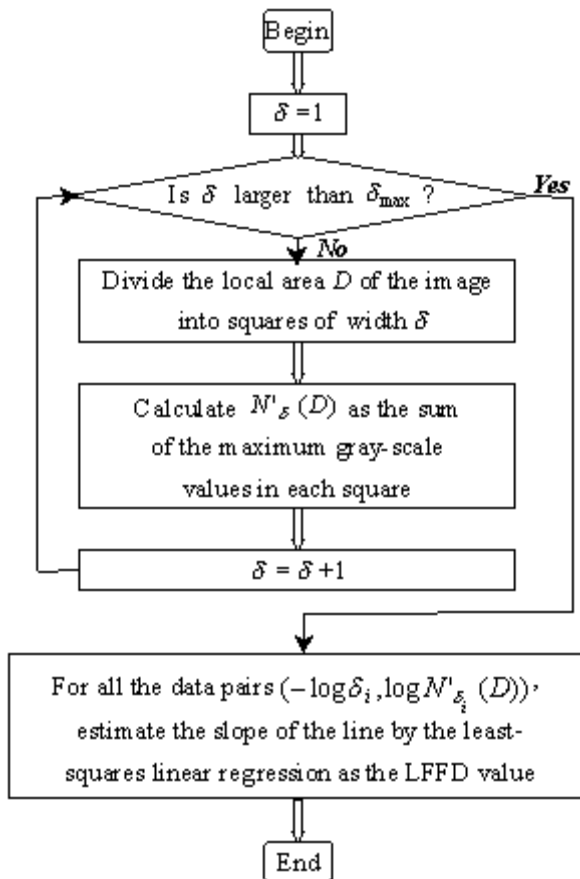


Fig. 2. The flowchart of LFFD estimation for local area D in the image

3.2 The LFFD as a feature of intensity discontinuity

Edges are usually defined as the borders where the intensity changes violently. Since neighboring areas of edge points usually have larger image entropy values than the other areas [12,13], the edge areas have higher degree of complexity than the other areas in a sense. On the other hand, fractal dimension is a measurement of complexity and the LFFD values reflect the complexity of local areas in images. Therefore, the LFFD and edges are interrelated.

In this paper, LFFD is proved to be an important feature of edges. The relationship between LFFD and local intensity changing is investigated by calculating the LFFD value of a test image. The test image is a 6×6 gray-scale image. In the test image, the pixels of the same column have the same gray-scale value, while the gray-scale values increase from left to right evenly. The degree of intensity changing is represented by Δ , which is the gray-scale difference between the adjacent columns.

In the experiment, the value of Δ is increased from 0 to 50. For different values of Δ , the corresponding LFFD values are calculated. For the 6×6 test image, δ_{\max} is 3. Therefore, δ is assigned the value 1, 2 and 3 successively in LFFD estimation of the test image. The average gray-scale value of the image is kept as 128 for all the different values of Δ . The test image series with increasing values of Δ is shown in Fig. 3.

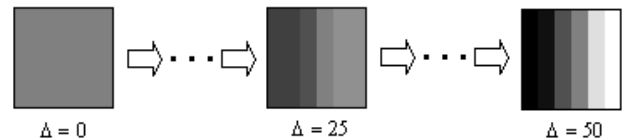


Fig. 3. The image series with increasing values of Δ (The images are 10 times of original size for a clear view)

According to the data obtained in the experiment, the relationship between the LFFD values and different Δ values is shown as Fig. 4, where the x coordinate represents the values of Δ and the y coordinate represents the corresponding LFFD values. Fig. 4 indicates that with a constant average gray-scale of 128, the LFFD value has a linear relationship with the degree of intensity changing. Since edge areas have steep change of intensity, the experiment proves that LFFD is an important feature of edges.

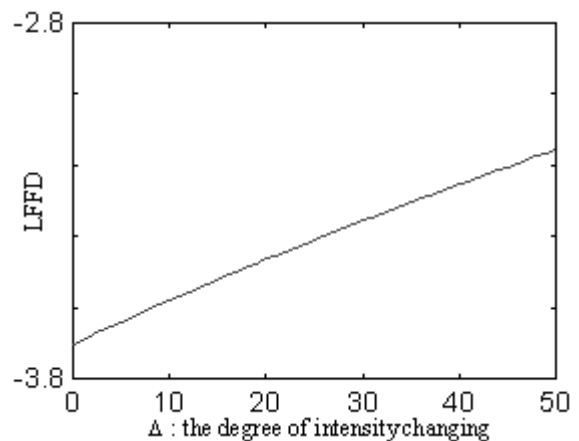


Fig. 4. The relationship between LFFD and the degree of intensity changing

Fig. 5 and Fig. 6 show the result of edge area detection for a test image. At each point, the LFFD value is estimated within a 6×6 neighboring area surrounding it. Fig. 5 is the test image. In Fig. 6 the black areas are the areas where the LFFD values are larger than zero. Fig. 6 shows that the black areas are the edge areas of the test image.



Fig. 5. The original image

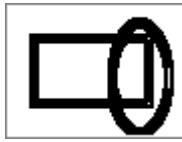


Fig. 6. The edge areas segmented with LFFD

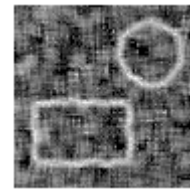


Fig. 9. The LFFD distribution of Fig. 8



Fig. 10. The edge area detected by the LFFD distribution



Fig. 11. The edges detected by the Gaussian filter and the Sobel operator

3.3 The LFFD as a noise-insensitive feature of edge areas

Images acquired in the real world applications usually contain noise due to on-chip electronic noise of image capture devices, quantization noise, etc [14,15]. The noise usually has discrete and random properties. The gradient-based edge detectors are sensitive to such noise [16]. The LFFD values are calculated based on the gray-scale of all the image points within a local area. Therefore, it has a filter-like effect, which makes it insensitive to noise.

Experiments have been done to investigate the noise-insensitive property of LFFD. Fig. 7 is the original image. Fig. 8 is the result of adding salt and pepper noise to Fig. 7. Fig. 9 shows the LFFD distribution of Fig. 8. In Fig. 9, the image points of larger gray-scale values have higher LFFD values. Fig. 10 shows the edge areas detected by a threshold of the LFFD value. Fig. 11 shows the edge detection result with a Gaussian filter and a Sobel operator. The experimental results show that the LFFD is insensitive to noise compared with the Sobel operator.

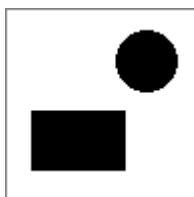


Fig. 7. The original image



Fig. 8. The image with noises

3.4 The LFFD as a feature of textures

The objects in the real world usually have various textures. The difference between textures is the different local spatial structure of gray-scale distribution [17]. The edges between different textures can not be well detected based on the intensity-discontinuity feature. Generally, in order to segment regions of different textures, texture features should first be extracted. Fractal dimension has been used in representing texture features [18,19]. Different textures usually have different complexity. Therefore, the LFFD values can represent local texture features.

In this paper, preliminary experiments have been done to investigate the relationship between the LFFD values and image textures. In Fig. 12 there are two different kinds of textures. Fig. 14 and Fig. 15 show the two different texture elements that make up the two different parts of Fig. 12, which are shown 16 times of original size for a clear view. Fig. 13 shows the LFFD distribution of Fig. 12. In Fig. 13, the image points of larger gray-scale correspond to higher LFFD values. Fig. 13 shows that the edge between the two areas of different textures can be

detected based on the LFFD distribution. Therefore, the LFFD can be taken as a feature of textures.

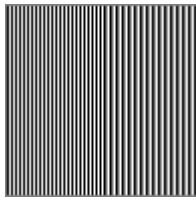


Fig. 12. The texture image

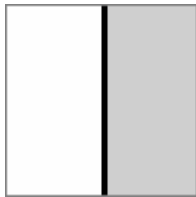


Fig. 13. The LFFD distribution of the texture image



Fig. 14. One texture element

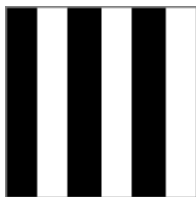


Fig. 15. The other texture element

4 Multi-Feature Edge Detection with LFFD and the Sobel Operator

The discontinuity of intensity is a basic characteristic of the areas where edges exist. Besides the intensity discontinuities, there are other features that identify edges. Multi-feature image analysis can get more information from the image than a single feature. For example, a continuous surface of an object usually has consistent texture, while the texture feature varies between different objects in the image. In this paper, the Sobel operator is combined with the local fuzzy fractal dimension to detect edges. Since the LFFD is insensitive to noise and can reflect edges between different textures, the performance of edge detection is improved. There are three steps in the proposed method:

- Step1** The gradient feature is extracted by the Sobel operator.
- Step2** The local fuzzy fractal dimension is calculated to extract local feature of complexity.
- Step3** For each image point, if the gradient value and the LFFD value are larger than certain thresholds respectively, the point is identified as an edge point.

The experimental results are shown in Fig. 16 to Fig. 19. Fig. 16 is the original images. Fig. 17 shows the LFFD distribution of Fig. 16, where the image points of lower gray-scale values correspond to larger LFFD values. Fig. 18 is the edge detection results of the Sobel operator. Fig. 19 is the result of the multi-feature edge detection. Fig. 19 shows that the multi-feature edge detection method can get more accurate results.



Fig. 16. The original image



Fig. 17. The LFFD distribution of Fig. 16



Fig. 18. The edges detected by the Sobel operator



Fig. 19. The edges detected by the multi-feature edge detection method

5 Conclusion

In this paper, the local fuzzy fractal dimension of gray-scale images is proposed as the feature of local image complexity. The estimation of LFFD is an extension of the FD of n-dimensional discrete sets by incorporating the fuzzy set. LFFD is proved to be an important feature of edges. Experimental results have also proved that LFFD is insensitive to noise and can be taken as a feature of textures. The multi-feature edge detection method is proposed with the LFFD and the Sobel operator. Experimental results show that the incorporation of LFFD improves the performance of edge detection. Further research will investigate the application of LFFD in other image processing tasks besides edge detection.

References:

- [1] W. Liu and R. Ge, Research on ambient region of edge model-based compression using JPEG, *Journal of Image and Graphics*, Vol. 5, No. 6, 2000, pp. 501-503.
- [2] D. Marr, E. Hildrith, Theory of Edge Detection. *Proc. Roy. Soc. London*, B207, 1980, pp. 187-217.
- [3] D. Zhao, Y. Chen, W. Gao, An adaptive DPCM quality image compression method based on edge direction, *Journal of Harbin Institute of Technology*, Vol. 32, No. 3, 2000, pp. 118-122.
- [4] H. Yuan, R. Cen, Q. Teng, et. al, The Advances of Edge-Detection in Medical Images, *Journal of Jinan University (natural science)*, Vol. 21, No. 1, 2000, pp. 69-72.
- [5] K. Zhang, J. Wang, Q. Zhang, Corner detection based on multi-feature, *Journal of Image and Graphics*, Vol. 7, No. 4, 2002, pp. 319-324.
- [6] X. Xi, X. Huang, R. Wang, An algorithm of extracting straight lines based on multi-feature fusion, *Computer Engineering and Application*, No. 1, 2002, pp. 51-54.
- [7] A. Chakraborty, M. Worring, J. Duncan, On multi-feature integration for deformable boundary finding. *Proceedings of the Fifth International Conference on Computer Vision*, 1995, pp. 846.
- [8] F. Espinal, T. Huntsberger, B. Jawerth, T. Kubota, Wavelet-Based Fractal Signature Analysis for Automatic Target Recognition, *Optical Engineering*, Vol. 37, No. 1, 1998, pp. 166-174.
- [9] B. Mandelbrot, *The Fractal Geometry of Nature*. W. H. Freeman Company, New York, 1983.
- [10] R. Wang, Characters of Fractal Geometry and Fractal Dimensions, *Journal of Dezhou University*, Vol. 17, No. 2, 2001, pp. 21-24.
- [11] Z. Feng, H. Zhou, Computing Method of Fractal Dimension of Image and Its Application, *Journal of Jiangsu University of Science and Technology*, No. 6, 2001, pp. 92-95.
- [12] A. Shiozaki, Edge extraction using entropy operator, *Computer Vision, Graphics, and Image Processing*, 1986, pp. 1-9.
- [13] S. Kim, D. Kim, J. Kang, J. Song, R. Park, Detection of moving edges based on the concept of entropy and cross-entropy. *Proc. SPIE Capture, Analysis, and Display of Image Sequences III*, San Jose, CA, Vol. 4308, 2001, pp. 59-66.

- [14] Z. Xiang, Elimination of image noise with CCD device characteristics, *Opto-Electronic Engineering*, Vol. 28, No. 6, 2001, pp. 66-68
- [15] G. Holst, *CCD Arrays, Cameras, and Displays*. Bellingham, WA: SPIE Optical Engineering Press, 1996.
- [16] H. Yuan, Q. Teng, Z. Yuan, et. al., Edge-detection and its application in medical image processing, *J. Biomed. Eng.*, Vol. 18, No. 1, 2001, pp. 149-153.
- [17] H. Wan, Morshed U. Chowdhury, H. Hu, et. al., Texture feature and its application in CBIR, *Journal of Computer Aided Design & Computer Graphics*, Vol. 15, No. 2, 2003, pp. 195-199.
- [18] G. Wu, D. Liang, Y. Tian, Texture image segmentation using fractal dimension, *Chinese Journal of Computers*, Vol. 22, No. 10, 1999, pp. 1109-1113.
- [19] Z. He, K. Bao, H. Dong, et. al., Texture image segmentation based on the fractal dimension, *Journal of Data Acquisition & Processing*, Vol. 11, No. 3, 1996, pp. 163-167.