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STOCHASTIC ANALYSIS OF OPERATIONS DECOUPLING IN FLEXIBLE MANUFACTURING SYSTEMS

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Abstract:- In serial production systems, storage may be provided between processes to avoid interference due to lack of synchronization. In order to manufacture a product, a job is divided into individual tasks, typically manufacturing or assembly processes. These tasks are interdependent and should be coordinated. To reduce the interdependence between downstream and upstream operations and to maintain the output of the production line, it is common to introduce buffers between the operations. These buffers decouple operations and eliminate the interdependency unless the buffer is emptied when a shutdown occurs upstream. In this paper we study the buffered flows of matter in a flexible manufacturing system considering only two machines. We develop procedures to compute some steady-state performance measures, including the interference loss and some limiting distributions. We use the Markov processes theory to obtain our results.

Key Words:- Flexible manufacturing systems, Markov processes, Performance evaluation, Dynamical systems, Operations decoupling.

1. Introduction

The dynamics of continuous systems are often modeled by a set of differential equations that can express the relationships between rates of changes in the values of system state variables. Given an initial state and boundary conditions, these equations completely specify a model of the system's dynamic behavior.
When this system of differential equations is particularly simple or has some special properties, it can be solved analytically to find the system's path of motion (trajectory). However, many interesting models are too complex to solve analytically and must be simulated by numerically integrating the set of differential equations (Scruben[1]). If the system is modeled using random processes, then the simulations can be used to generate sample paths for statistical analysis.
Flexible manufacturing systems (FMS) are an important class of discrete event dynamic sys-

tems.
Flexibility means to produce reasonably priced customized products of high quality that can be quickly delivered to customers. Flexible manufacturing systems are computer-enhanced batch of repetitive processes that facilitate the production of high volumes of customized products on highly automated equipment that is responsive to software instructions.

A FMS is a queueing network system where different classes of products are processed contemporaneously. Each product has to perform its own orderly sequence of operations, different for each class, in order to be completed. The same machine can perform operations on different product classes, eventually with different service times: the same operation can be performed on alternative machines. In this sense, flexibility is the capability of the FMS to cope in time with changing product class blend and production inconveniences such as buffer blockages and machine breakdowns, maintaining an optimum production target, machine load balance and, if required, an assigned production mix (Balduzzi and Menga[2]).

Although numerous benefits are associated with automated flexible manufacturing processes, such as reduced labor cost, faster throughput times and faster responses to demand volume changes and to product design changes, the optimization of such a process is a difficult process. In practice, there is major uncertainty about implementation costs, date of on-line availability, and performance characteristics once on line. Many of the benefits typically associated with flexibility, such as improved quality control, reduced work-in-process inventories, and reduced lead times, are not yet fully substantiated and may be difficult to measure.

In this paper, we develop a set of performance measures to evaluate the dynamic behavior of a FMS considering the simplest form of the system: two machines and a buffer for operations decoupling.

2. Background

Scheduling problems encountered in an FMS can be separated into several distinct types which encompass a wide range of resources including parts, robots, machines, and AGVs. Stecke and Solberg[3] categorize different scheduling problems and apply sets of dispatching rules to each problem in an effort to evaluate the impact of various rules on the system performance. Several researchers have since evaluated different problems under different sets of rules. Dar-El et al[4], evaluated the impact of a "good" schedule for a particular problem and the effect of any dispatching rule has been found to vary with several factors such as system layout, system state, and the desired performance measure.
Other researchers (Cho and Wysk[5]; Jones et al[6].) have suggested using neural networks to identify candidate rules for multi-pass simulation analysis.
Jones et al[6] take into account multi-criteria performance measures. When a new schedule is desired, a neural net generates good rules for each performance measure and then simulation is used to predict how each rule does against all performance measures simultaneously. In both cases, the neural network is trained off-line by the simulation under a variety of input conditions. They also examine the application of discrete-event simulation for shop floor control of a flexible manufacturing system.
Cho[7] defines five types of scheduling problems in the context of an automated workstation. At each decision point, the neural network generates candidate rules for each problem type and these rules are then evaluated through simulation. The analysis of a single lines that involves Markov models has been suggested by Hongler[8], and Bharucha-Reid[9].

3. Mathematical analysis

Our general model is defined following the ideas developed by Hongler[8]. We suppose that the dynamics of the system is given by the simple production line schematically represented in Figure 1, and it is based on the following assumptions:
1) There are two machines in the system, and they are similar with respect to the average number of breakdowns which each experiences in its unit working or running time.
2) The mean time to repair in both machines is similar an it is exponentially distributed.
3) All random variables are independently distributed.
4) The queueing system (machines and repairmen) is in a state of statistical equilibrium.

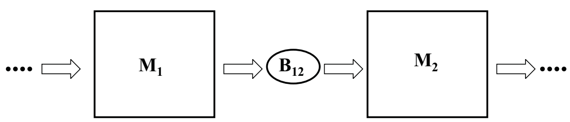

Fig1: Two machine production line

Two failure prone machines M_1 and M_2 are partly decoupled by the introduction of a buffer B_{12} which has a maximum capacity equals to φ [parts]. By assumption (1), the mean time to failure and the mean time to repair will be denoted respectively by λ^{-1} and μ^{-1} for both machines. In this model, $\rho = \lambda\mu^{-1}$ represents the indisposability factor of machine $M_j, j = 1, 2$. The production rate of M_j is ϱ_j [parts/unit time]. The time-dependent content of the buffer B_{12}, $\mathcal{M}(t)$, $t \geq 0$ can be considered as a random variable in the interval $(-\varphi/2, \varphi/2)$. We define the derivative stochastic process:

$$\mathcal{M}(t) = \varrho_1\pi_1(t) - \varrho_2\pi_2(t), \tag{1}$$

where for $j = 1, 2$,

$$\pi_j(t) = \begin{cases} 1, & \text{if } M_j \text{ produces in } t \\ 0, & \text{in other case} \end{cases} \tag{2}$$

The waiting time intervals between transitions from states $\{0\}$ to $\{1\}$ and vice verse are characterized respectively by probability distributions $\psi_j(z)$ and $\phi_j(z)$ on positive random variables. Thus we have

$$\int_0^\infty z d\phi(z) = \lambda^{-1}, \text{ and } \int_0^\infty z d\psi(z) = \mu^{-1},$$

In this model we are interested in the case where $\psi(z)$ and $\phi(z)$ are exponentially functions distributed.

Theorem 1: Let $\{\pi(t), t \geq 0\}$ be the stochastic process defined in (2). Since $\phi(z)$ and $\psi(z)$ have finite means, and $\phi(z) + \psi(z)$ has a continuous distribution, then

$$\lim_{t\to\infty} \mathbf{P}[\pi(t) = 1] = \frac{\mu}{\mu+\lambda}, \tag{3}$$

$$\lim_{t\to\infty} \mathbf{P}[\pi(t) = 0] = \frac{\lambda}{\mu+\lambda}, \tag{4}$$

Proof: See Pérez-Lechuga et al.[11], see also Parzen [12] for a more widespread proof. □

Theorem 2: Let the stochastic process defined in (2), then for any $s, t \geq 0$, the transition probability functions, $p_{jk}(t) = \mathbf{P}\{\pi(t+s) = k \mid \pi(s) = j\}$ are given by

$$\left.\begin{array}{lcl} p_{00}(t) & = & \frac{\lambda}{\mu+\lambda} + \frac{\mu}{\mu+\lambda}\, e^{-(\mu+\lambda)t} \\ p_{10}(t) & = & \frac{\lambda}{\mu+\lambda}\left[1 - e^{-(\mu+\lambda)t}\right] \\ p_{01}(t) & = & \frac{\mu}{\mu+\lambda}\left[1 - e^{-(\mu+\lambda)t}\right] \\ p_{11}(t) & = & \frac{\mu}{\mu+\lambda} + \frac{\lambda}{\mu+\lambda}\, e^{-(\mu+\lambda)t} \end{array}\right\} \tag{5}$$

Proof: The transition probabilities of process (2) can be obtained using the forward Kolmogorov differential equations

$$\frac{\partial}{\partial t} p_{jk}(t) = -q_k\, p_{jk}(t) + \sum_{i\neq k} p_{ji}(t)\, q_{ik}, \tag{6}$$

where $q_j(t)$ and $q_{jk}(t)$ are the homogeneous intensity of passage and the homogeneous intensity of transition respectively, and $p_{jk}(t)$ is the transition probability function

$$p_{jk}(t) = \mathbf{P}\{\pi(t+s) = k \mid \pi(s) = j\}.$$

Let the intensities of passage from 0 to 1 be given respectively by $q_0 = \mu$ and $q_1 = \lambda$. It follows that the transition intensities are given by $q_{01} = \mu$ and $q_{10} = \lambda$.

The Kolmogorov differential equations (6) then become

$$\begin{aligned}
\tfrac{\partial}{\partial t} p_{00}(t) &= -\mu\, p_{00}(t) + \lambda\, p_{01}(t) \\
\tfrac{\partial}{\partial t} p_{01}(t) &= -\lambda\, p_{01}(t) + \mu\, p_{00}(t) \\
\tfrac{\partial}{\partial t} p_{11}(t) &= -\lambda\, p_{11}(t) + \mu\, p_{10}(t) \\
\tfrac{\partial}{\partial t} p_{10}(t) &= -\mu\, p_{10}(t) + \lambda\, p_{11}(t)
\end{aligned}$$

Since $p_{01}(t) = 1 - p_{00}(t)$ then, the first of these equations can be rewritten (Parzen[12])

$$\frac{\partial}{\partial t} p_{00}(t) = -(\mu+\lambda)\; p_{00}(t) + \lambda, \qquad (7)$$

Equation (7) is an ordinary differential equation of the form (with $g(t) = p_{00}(t), v = \mu + \lambda$ $h(t) = \lambda$)

$$g^{'}(t) = -v\, g(t) + h(t), \quad a \le t < \infty$$

whose general solution is

$$g(t) = \int_a^t e^{-v(t+s)}\, h(s)\, ds + e^{-v(t-a)}\, g(a).$$

Then using the boundary condition $p_{00}(0) = 1$ we obtain

$$p_{00}(t) = \lambda \int_0^t e^{-(\mu+\lambda)(t-s)}\, ds + e^{-(\mu+\lambda)t}. \quad (8)$$

Using equation (8) it follows the proposed results. □

Corollary 1: Let $p_0 = \mathbf{P}[\pi(0) = 0]$ then

$$\mathbf{E}\,[\pi(t)] = \frac{\mu}{\mu+\lambda} - \left(p_0 - \frac{\lambda}{\mu+\lambda}\right) e^{-(\mu+\lambda)t} \qquad (9)$$

To know the behavior of the zero-one process $\{\pi(t), t \ge 0\}$ after it has been operating for a long time, we evaluate the stochastic integral

$$\mathcal{R}(t) = \frac{1}{t} \int_0^t \pi(t^{'})dt^{'} \qquad (10)$$

Equation (10) represents the fraction of time during the interval $[0,t]$ that the stochastic process takes the value 1. Then, from (9) we follows that, in the limit as $t \to \infty$

$$\mathbf{E}\,[\mathcal{R}(t)] = \frac{1}{t}\int_0^t \mathbf{E}\left[\pi(t^{'})\right] dt^{'} \to \frac{\mu}{\mu+\lambda}$$

Corollary 2: For the Markov process defined in (1) we have:

$$\lim_{t\to\infty} \mathbf{E}\,[\mathcal{M}(t)] = \frac{\varrho_1\, \mu_1}{\mu_1+\lambda_1} - \frac{\varrho_2\, \mu_2}{\mu_2+\lambda_2} \qquad (11)$$

Let us define the lost of production due to the period the machines have to wait for service as the *machine interference* (Palm[13]).

Let the random variable $\mathcal{W}(t)$ denote the number of machines *not* working at time t and let

$$\Pi_w = \lim_{t\to\infty} \mathbf{P}\,[\mathcal{W}(t) = w], w = 0, 1, \ldots, m$$

where m is the number of machines in the system (2 in this case). Using (3) and (4) as estimators of the limiting distribution, and by the independence hypothesis between machines we have

$$\begin{aligned}
\Pi_0 &= \lim_{t\to\infty} \mathbf{P}\,[(\pi_1(t) = 1) \cap (\pi_2(t) = 1)] \\
&= \frac{\mu^2}{(\mu+\lambda)^2} \qquad (12)
\end{aligned}$$

and

$$\Pi_1 = \lim_{t\to\infty} \mathbf{P}\,[(\pi_1(t) = 0) \cap (\pi_2(t) = 1) +$$
$$(\pi_1(t) = 1) \cap (\pi_2(t) = 0)] = \frac{2\mu\lambda}{(\mu+\lambda)^2} \qquad (13)$$

Let α, β, and γ denote the average number of machines working, being serviced, and waiting to be serviced, respectively. We have the following identities (Barucha-Reid[9]):

$$\alpha + \beta + \gamma = m, \tag{14}$$

$$\frac{\alpha}{\beta} = \frac{\mu}{\lambda}, \tag{15}$$

$$\beta = r - \sum_{w=0}^{r-1} (r - w)\pi_w \tag{16}$$

where r denotes to the number of repairmen assigned to the system.

Equation (16) relates to the equality of the number of engaged repairmen and the number of machines being serviced. From (14) we obtain

$$\gamma = m - \left(\frac{\mu + \lambda}{\lambda}\right)\left[r - \sum_{w=0}^{r-1}(r - w)\Pi_w\right] \tag{17}$$

Thus, for $m = r = 2$, and using (12) and (13), $\gamma = 0$.

Similarly, for the case of one repairman and $m = 2$,

$$\gamma = 2 - \left(\frac{\mu + \lambda}{\lambda}\right)(1 - \pi_0) = 2 - \left(\frac{2\mu + \lambda}{\mu + \lambda}\right).$$

Equation (17) give the average number of machines in the waiting line, i.e., the interference loss.

Other quantities of interest are the following ones (Bharucha-Reid[9]). For $m = 2$ and $r \in [0, 2]$, the average number of idle repairmen given by

$$\mathcal{I} = r - \beta = r\left[1 - \frac{\lambda^2}{(\mu + \lambda)^2}\right] - \frac{2\mu\lambda}{(\mu + \lambda)^2},$$

and the coefficient of loss for repairmen (for $r > 0$)

$$\mathcal{L} = \frac{r - \beta}{r} = \left[1 - \frac{\lambda^2}{(\mu + \lambda)^2}\right] - \frac{2\mu\lambda}{r(\mu + \lambda)^2}.$$

The operative efficiency that is defined as the ratio of the number of machines waiting to be serviced to the number of repairmen (for $r > 0$)

$$\eta = \frac{\beta}{r} = \frac{\lambda^2}{(\mu + \lambda)^2} + \frac{2\mu\lambda}{r(\mu + \lambda)^2}.$$

The coefficient of normal loss due to repairs

$$\mathcal{N}_r = \frac{\beta}{2} = \frac{\lambda(r\lambda + 2\mu)}{2(\mu + \lambda)^2}.$$

The coefficient of normal loss due to machine interferences

$$\mathcal{N}_m = \frac{\gamma}{2} = \frac{\lambda(1 - \frac{r}{2})}{2\,(\mu + \lambda)}.$$

The combined coefficient of loss equals

$$\mathcal{N}_{\mathcal{I}} = \frac{\beta + \gamma}{2} = 1 - \frac{\mu}{2}\left[\frac{r\lambda + 2\mu}{(\mu + \lambda)^2}\right].$$

Finally, the machine efficiency (or machine availability) of the system is given by

$$\mathcal{A} = \frac{\alpha}{2} = \beta\frac{\mu}{2\lambda} = \frac{\mu\,(r\lambda + 2\mu)}{2\,(\mu + \lambda)^2}.$$

Conclusions

The dynamics of the population level of a buffer, when approximated by a continuous variable, can be described by stochastic differential equations and Markov processes. In this document we only have considered a modeling in which the noise source is a continuous Markov Chain. We develop a set of performance measures in a single case of FMS. Some of our results were based on the classic work of Palm[13] about the machine interference problem. In our proposal it is only required to know the intensities λ and μ as well as the number of repairmen in the system.

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