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Surface reconstruction from sparse data by a multiscale volumetric approach

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Abstract: - This paper describes a method for surface reconstruction from sparse three-dimensional (3D) data that performs the reconstruction by building a sequence of surfaces approximating the data at increasing level of details (LOD). The method is simple, fast and suitable for a progressive 3D data/model representation, archiving, transmission. The surface reconstruction is obtained by a volumetric method that differs from other volumetric methods because it does not require implicitly or explicitly information on surface normals. This aspect is important in the case of noisy data sets, such as those coming from image based methods, because normals are often estimated unreliably from 3D data. The method is based on a hierarchical partitioning of the volume data set. The working volume is split and classified at different scales of spatial resolution into surface, internal and external voxels and this hierarchy is described by an octree structure in a multiscale framework. The octree structure is used to build a multiresolution description of the surface by means of compact support Radial Basis Functions (RBFs). A hierarchy of surface approximations at different LOD is built by representing the voxels at the same octree level as RBFs having the same spatial support. At each scale, information related to the reconstruction error drives the reconstruction process at the following finer scale. Preliminary results on real data are presented and discussed.

Key-Words: - Surface reconstruction, sparse data, volumetric methods, radial basis functions, multiscale, octrees

1 Introduction

The reconstruction of surfaces from its sparse three-dimensional (3D) samples is a challenging problem both in computer vision and computer graphics. There is a wide variety of approaches to surface reconstruction from sparse 3D data (see [4] for a survey), among which region growing [9], tensor voting [11], distance fields [10], algebraic and computational geometry methods [16], volumetric regularization and implicit surface methods [13,15].

Implicit surface methods [3] are an active research area which includes among others Moving Least Squares (MLS) [1,2] and Radial Basis Functions (RBFs) [15]. RBF based approaches address the question of the choice between local or global RBFs. Local RBFs lead to fast and simpler computation but have the drawback of being sensitive to the density of scattered data. Global RBFs on the other hand are able to deal better with non-uniform density data but are computationally heavier or even impractical for large data sets.

Some authors [8,12,18] have tried to combine the advantage of both types of RBFs by using locally supported basis functions in a hierarchical framework. A coarse-to-fine approximation/interpolation of the data is built by sets of self-similar local basis functions having their support scaled according to the scale of the details considered. The estimated reconstruction error at a coarse scale drives the reconstruction process at the following finer scale. This approach is very promising respect to the size of the data set that can be managed, the reconstruction time and the control on the level of details (LOD), but it requires a hierarchical partitioning of the data set.

Data set partitioning is commonly realized by the hierarchical subdivision of the data set volume in voxels of different sizes according to a fixed subdivision scheme or an adaptive scheme, based on either the approximation error, or the evaluation cost [1,8]. The hierarchical partition of the data set is commonly coded in an octree structure [14] or in a K-D tree [12].

An important aspect common to several of these approaches, especially those coming from the computer graphics area is that they relay directly or indirectly on information about the surface normals to produce a meaningful output. Generally

this information is assumed available a priori or is estimated from the data.
On the other side, some computer vision approaches to surface reconstruction from images such as Generalized Voxel Coloring (GVC) [7] and space carving [17] do not use information on surface normal nor try to infer it. They produce estimates of 3D surfaces as a partition of the surface volume in empty and surface voxels, whose centres can be assumed as surface points.
Depending on the technique used to measure or to estimate 3D samples of the surface (e.g. laser scanner, stereo, etc.) information obtained on surface normals can be more or less reliable.
This paper presents a volumetric approach to surface reconstruction which does not use this information and hence copes easily with several sources of 3D data. The method tries to blend some of the ideas concerning multiscale partitioning of 3D data, local construction of volumetric functions, successive approximations to the surface driven by LOD, and GVC techniques. It requires a smaller amount of information with respect to more sophisticated methods as [12], still allowing an acceptable quality of the surface reconstruction.
The method classifies at different spatial scales the volume occupied by data into inner, outer and surface voxels. The constraints on the surface reconstruction are then defined in term of volume instead of values of the surface normals and of the volumetric function at inner and outer points as in[15].
The algorithm builds a description of the volume around the data at different spatial scales, by classifying the volume occupied by data into voxels of an octree hierarchical structure. The octree structure is then used to build a multiresolution volumetric description of the surface by means of RBFs of compact support.
A sequence of volumetric functions of different smoothness and LOD is generated from the octree structure. Information related to the partial reconstruction errors at one scale is used to drive the reconstruction process at the following finer scale. The estimated surfaces at different LOD are obtained as the zero level set of the corresponding volumetric functions. Preliminary results produced by the algorithm on real data are presented and discussed.

2 Hierarchical representation of 3D data
The algorithm makes a hierarchical partitioning of the data volume containing the surface into voxels of different sizes. These voxels are classified into surface, inner, and outer voxels respectively if they contain data, or they are empty and inside or outside the surface. This description of the surface and its surrounding volume is represented by an octree structure in a multiscale framework where different levels correspond to different scales and different voxel sizes (see Fig. 1). The tree from the root to a specific level contains a description of the surface occupancy in the working volume up to that spatial resolution or scale.

2.1 Building the octree structure
The volume partitioning and the construction of the octree structure starts at the coarsest scale where the working volume is split into a cube made of 3x3x3 voxels of the same size. These voxels consist of a central surface voxel where are contained all the 3D data, surrounded by 26 empty voxels classified as outer voxels. Other initial configurations are possible depending on initial hypothesis on the surface and its volume occupancy.
The initial surface voxel, root of the octree, is then split into 8 voxels of the same size halving the voxel size along every dimension [6,14] (see Fig. 1). The son voxels of the root voxel are then classified into surface or empty voxels, according to the occupancy of the voxels by the 3D data. Empty voxels are the classified into inner and outer voxels depending on their adjacency to voxels of the same type. At the first step no inner voxels are generated; these voxels are usually generated in the following steps when are created empty voxels not adjacent to outer ones.
Only the surface voxels are then split again and classified proceeding toward smaller scales of spatial resolution. Empty voxels (inner and outer) are not split anymore, but can be reclassified at the following steps of the octree building from inner to outer voxels. The volume subdivision stops at a specified scale or when every surface voxel contains a 3D point only.
Concerning the decomposition algorithm complexity and memory requirements, it should be noted that the maximum number of voxels produced is lower bounded by $N\log_8(N)$; in practical cases this bound is too conservative. Further, the octree structure is efficiently managed by pointer structures.

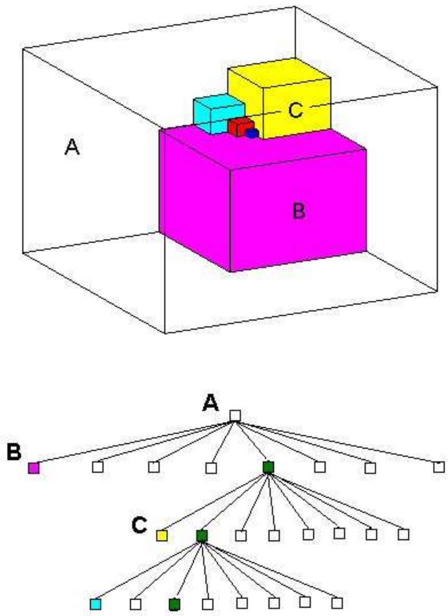

Fig. 1 Volume subdivision in voxels and corresponding octree structure

3 Volumetric regularization by using RBF

3.1 Data approximation and interpolation

The recovering of a function f defined on $\mathbf{R^d}$ from the set $g = \{(\mathbf{x}_i, y_i) \in \mathbf{R^d} \times R\}_{i=1}^{N}$ of N function values sampled at random locations is *ill-posed* [5]. The problem can be constrained and solved assuming some *a priori* knowledge on f. A commonly adopted constraint consists in assuming that the function is *smooth*. A variational approach [5,13,15] is then adopted and the original problem is recast into that of finding the function f which minimize a functional G which take into account both the smoothness and the data values:

$$F[f] = \sum_{i=1}^{N} (f(\mathbf{x}_i) - y_i)^2 + \lambda \phi[f] \qquad (1)$$

The first term of F takes into account the closeness to the data and the functional ϕ at the second term the smoothness of the solution f. The trade-off between the two terms is controlled by the so called *regularizing parameter* λ. In the case $\lambda = 0$ the solution leads to pure interpolation of the data. For a wide class of possible functionals ϕ the solution $f(\mathbf{x})$ has the form [13]:

$$f(\mathbf{x}) = \sum_{i=1}^{N} w_i H(\mathbf{x} - \mathbf{x}_i) + \sum_{j=1}^{k} v_j P(\mathbf{x}) \qquad (2)$$

where the first term consists of a sum of *basis functions* H having radial symmetry which are centred on the $\mathbf{x}_i$ values of the 3D data and are weighted by w_i. The second term in (2) consists of linear combination of polynomial terms $P(\mathbf{x})$.

The regularizing parameter λ can vary from sample to sample, to take into account different local trade-off between reconstruction error and smoothness. The choice of a specific smoothness functional ϕ leads to radial gaussian basis functions H of the type:

$$G(\mathbf{r}) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\mathbf{r}^2/2\sigma^2} \qquad (3)$$

and the polynomial term in (2) reduces to a constant p_0. The unknown weights w_i and p_0 in (2) can be determined from the data [13] and the choice of λ by the linear system of equations:

$$(H + I\,\mathbf{l})\mathbf{w} + p_0 = \mathbf{y} \qquad (4)$$

and the constraint $\sum_{i=1}^{N} w_i = 0$, where I the identity matrix, and

$$(H)_{ij} = H(\mathbf{x}_i - \mathbf{x}_j), \quad (\mathbf{l})_i = \lambda_i, \quad (\mathbf{y})_i = y_i, \quad (\mathbf{w})_i = w_i$$

The isosurface related to the zero level set $\{\mathbf{x} \mid f(\mathbf{x}) = 0\}$ of the volumetric function f gives an estimate of the surface associated to the 3D data.

3.2 Radial basis function of compact support

According to the algorithm description in Sec. 1 the volume occupancy of voxels of different size is cast into a corresponding volume occupancy by gaussian radial basis functions of compact support (See Eq. 3) centred on the voxel centres c_i. In the case of wide support basis functions the system of equations (4) becomes untractable for more than few thousands data. On the other hand a compact support of the radial basis functions means that at a given scale the influence on the volumetric function f of the subset of 3D data contained into a specific voxel is localized around the voxel volume. Hence the compact support choice has the benefit that the system of equations (4) becomes sparse and can be solved very efficiently by standard methods [5]. For the gaussian RBF used in this work the support of the functions depends on the parameter σ ; this parameter changes with the scale.

3.3 Multiresolution volumetric reconstruction by RBF

Multilevel approximation by adaptive domain decomposition is a promising approach to sparse data approximation [5, 8]. Here the data domain decomposition in subsets produces the octree structure of Sec. 2, which is the starting point to build up a multiresolution volumetric description of the surface by means of RBF of compact support.

The global volumetric function S_i which approximates up to the scale i the surface associated to the data $g = \{(\mathbf{x}_j, y_j) \in \mathbf{R^d} \times R\}_{j=1}^{N_i}$ is given by the sum of a sequence of volumetric functions f_k

$$S_i(\mathbf{x}) = \sum_{k=1}^{i} f_k(\mathbf{x}) = f_0(\mathbf{x}) + f_1(\mathbf{x}) + \ldots + f_i(\mathbf{x}) \qquad (5)$$

At a given scale i the function f_i approximates the reconstruction error due to the difference between the actual values of the S_{i-1} at the voxel centres c_{ij} and the prescribed values y_{ij}. Hence the reconstruction error propagates from the scale $i-1$ to the scale i according to:

$$f_i(c_{ij}) = y_{ij} - S_{i-1}(c_{ij}) \qquad (6)$$

where c_{ij} are the j voxel centres at the scale i corresponding to the level i of the octree, and

$$y_{ij} = \begin{cases} K_s \text{ if } j \text{ is a surface voxel}, \\ K_o \text{ if } j \text{ is an outer voxel}, \\ K_i \text{ if } j \text{ is an inner voxel}. \end{cases} \quad (7)$$

The constant values K_s, K_o, K_i are chosen as seen in Sec. 4 while the evaluation of the functions f_i is performed as shown in Subsec. 3.1.

4 Implementation details and choice of the parameters

The choice of the parameter values and of the voxel classification rules are crucial for the quality of reconstruction process. Some parameters have standard values in the literature, and those values have been implicitly assumed here, as the geometry of the working volume partitions (voxels) or the shrinking value (2^{-1}) along every co-ordinate at every scale step.

The constants for the function value assignments at the voxel centres in Eq. 7 are chosen to be $K_s = 0,\ K_o = -1,\ K_i = 1$ as in [15] , but other choices can be exploited. Other parameters as the regularizing parameter λ and the support parameter σ along with the classification rules deserve a discussion.

4.1 Empty voxel classification

The general voxel classification rules have been discussed before in Subsec. 2.1. When empty voxels of different type become adjacent, the classification depends on the scale. At scales coarser than or equal to typical scale i_{TYP} , which depends on the mean distance between 3D data points, volume carving is supported. Inner voxels adjacent to outer ones are by reclassified as outer, and this classification propagates along adjacent voxels of the same or coarser scale. The partially build octree structure is analyzed bottom up to propagate the reclassification. At scales finer than i_{TYP} the reclassification propagation is partially inhibited to avoid the creation of holes (outer-inner voxel adjacency) on the surface along which inner voxels can change to outer destroying in this way the surface continuity.

Empty voxels are classified outer only if they are adjacent to at least one bigger outer voxel. Empty voxels are classified inner only if they are adjacent to at least one bigger inner voxel. In all the other cases empty voxels are classified as a surface voxels of particular type because they will never be split further.

4.2 Support and Regularizing parameters

To force the support extension of an RBF at a given scale i to be contained inside the first neighboring voxels the *support parameter* σ is chosen as:

$$\sigma^2 \le a_i^2, \qquad a_i = 2^{-i} \quad (8)$$

where a_i is the length of the voxel side at the scale i.

The *regularizing parameter* λ is crucial to control the reconstruction process. By varying *locally* its value for every voxel it is possible to stress the closeness to the data, such as near edges or sharp surface variations, or the smoothness, such as near planar zones of the reconstructed surface.

A reasonable constraint on the reconstruction process could be the preservation of surface continuity at scales finer than i_{TYP}, if it is supposed no holes are present at that scale and at the finer ones. Hence not only space carving should be inhibited by the classification rules on empty voxels, but also smoothness should increased by increasing λ. In any case the surface should be close to the 3D data points inside the surface voxels, hence a measure of the approximation error has to be defined to deal with the trade-off between *smoothness* and *goodness of fit* to the data.

As a measure of the *local approximation error* of the reconstruction has been chosen the Euclidean distance $\mathbf{e}_i$ between the point $\mathbf{x}_i$ and the estimated surface, or more formally, between the 3D data point $\mathbf{x}_i$ and the point $\mathbf{y}_i$ on the isosurface $f = 0$ intersected by the normal $\hat{n}_i = \nabla f(\mathbf{x})\big|_{\mathbf{x}_i}$ to the isosurface $f_i = f(\mathbf{x}_i)$ passing though $\mathbf{x}_i$, that is :

$$\mathbf{e}_i = \mathbf{x}_i - \mathbf{y}_i \simeq f(\mathbf{x}_i) \Big/ \left(\nabla f(\mathbf{x})\big|_{\mathbf{x}_i} \right) = \widehat{\mathbf{e}_i} \quad (9)$$

For a given surface voxel V, the maximum $e_{\max} = \max_i \left\{ \left\| \widehat{\mathbf{e}_i} \right\| \right\}$ of the moduli of the estimates $\widehat{\mathbf{e}_i}$ of the approximation errors $\mathbf{e}_i$ related to the points $\mathbf{x}_i$ that belong to V defines the local maximum deviation of the surface from the data. Note that the estimates are valid only for a first order expansion of f ; for this reason $e_{\max}$ has been bounded to be at most a_i where a_i is the length of the voxel side at the scale i. To trade smoothness for accuracy, at the next scale step the values of the smoothness parameter λ for all the eigth descendants of the surface voxel V (if it will be split) is locally set according to :

$$\lambda_{surf} = \frac{1}{e_{\max}} \quad (10)$$

Inner and outer voxels are not split hence their local approximation error does not propagates along the scales. For this reason their smoothness parameters are constant at a particular scale; they only increase while proceeding along the scales to compensate for the corresponding reduction of the smoothness part of the functional $\phi[f]$ of Eq.1, according to :

$$\lambda_{inner} = \lambda_{outer} \propto \frac{1}{a_i} \qquad (11)$$

5 Reconstruction results

The reconstruction performance of the algorithm has been tested on two sets of 3D data generated by laser scanners.
The Horse data, made of about 50000 surface points, is available at the Large Geometric Models Archive at Georgia Inst. of Tech., while the Bunny data, made of about 360000 surface points, is available at the at The Stanford 3D Scanning Repository. The typical scale i_{TYP} for the two data set was estimated by the mean inter-point distance of the two data sets.
The two data sets were normalized in a working volume of 2×2×2 units. Starting at scale 0, where the side of the voxel is 2, a first surface voxel with 26 adjacent outer voxels is refined up to scale 7, where the side of the voxel is 0.015. The Horse surface reconstructed at scale 2 is shown in Fig. 2. The surface voxels (32 voxels) are shown on the left and the corresponding reconstructed isosurface on the right. The final result is obtained at scale 7 (33000 surface voxel) and is shown in Fig. 3.
The Bunny surface reconstructed at scale 5 is shown in Fig. 4. The surface voxels (3400) are shown on the left and the corresponding reconstructed isosurface on the right. More details are visible respect to Fig.3, even if a surface "voxelization" is evident. The final result is obtained at scale 7 (56000 surface voxels) and the surface is shown in Fig. 5. The reconstruction times up to scale 7 were 20 s for the Horse data and 90 s for the Bunny data on a PentiumIII 1GHz. The mean reconstruction errors were 0.005 and 0.004 for the Horse data and for the Bunny data respectively. All the surfaces shown were obtained by standard isosurface extraction techniques [3] applied to the volumetric functions estimated by the algorithm from the test data.

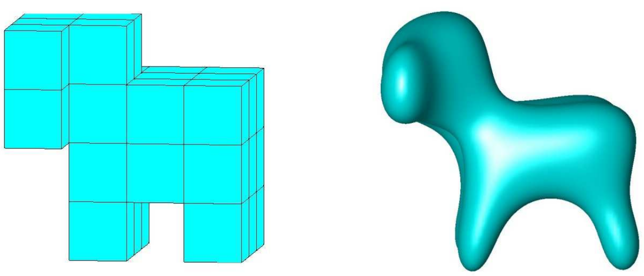

Fig. 2 Horse data reconstruction at scale 2

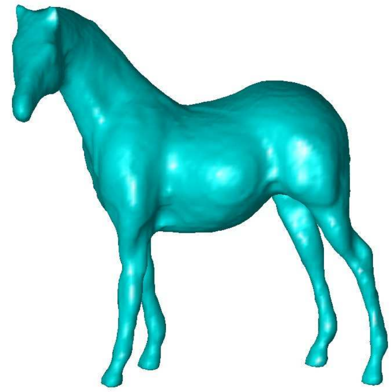

Fig. 3 Horse data reconstruction at scale 7

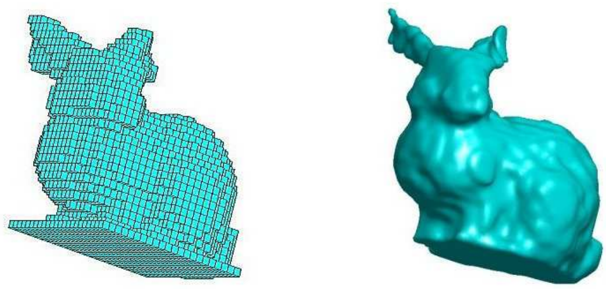

Fig. 4 Bunny data reconstruction at scale 5

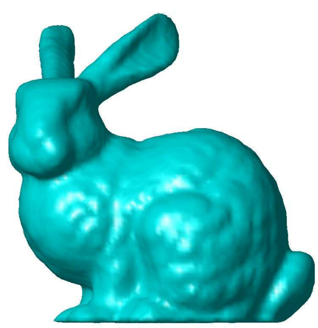

Fig. 5 Bunny data reconstruction at scale 7

6 Conclusions

This paper describes a multiscale volumetric approach to surface reconstruction from non-uniform data which is based on a hierarchical partitioning of the volume data set into an octree structure. The surface is reconstructed from the 3D data as a sequence of surfaces approximating different level of details in the space of spatial scales. Information related to reconstruction error propagates along the scales to drive the reconstruction process. A test on the performance of the method has been made on commonly used data sets available on the web.

The results show the algorithm is effective to produce from the original 3D data a sequence of reconstructions that converge to a final good-quality surface. An important feature of the algorithm is its ability to incrementally refine the reconstruction. When the object represented by 3D data should be archived or transmitted at lower LOD than the finest one, the algorithm output a sequence of information able to upgrade an initial rough representation up to the desired LOD. This is particular attractive for applications to progressive transmission of 3D data [14].

The multiscale approach, the octree data structure and the choice of local support RBFs allow both efficient and fast surface reconstruction.

Further investigations and improvements are necessary to understand in deep how the inter-relations between decomposition rules, local error and smoothing parameters affect the final quality of the reconstruction.

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