Proceedings of the 5th WSEAS/IASME Int. Conf. on SYSTEMS THEORY and SCIENTIFIC COMPUTATION, Malta, September 15-17, 2005 (pp213-219)

An Application of Multiobjective Optimization in Electronic Circuit Design

JAN MÍCHAL JOSEF DOBEŠ
Department of Radio Engineering, Faculty of Electrical Engineering
Czech Technical University in Prague
Technická 2, 16627 Praha 6
Czech Republic

http : //www.cvut.cz

Abstract: Optimization is undoubtedly playing an ever more important role in CAD of electronic circuits. As the complexity of practical designs grows, so does the number of objectives to be optimized simultaneously. Even though a large number of multiobjective optimization methods have been developed in other disciplines like Operations Research, they are still generally unknown to electrical engineers and creators of CAD tools. The present paper provides a brief introduction to multiobjective optimization as such, explains two of the most frequently used methods, and discusses their advantages as well as their drawbacks. Several application guidelines and recommendations based on the authors' experience are proposed. The use of both methods is demonstrated on a practical example of a video amplifier design followed by evaluation and discussion of the obtained results.

Key-Words: Multiobjective optimization, Pareto optimality, goal attainment.

1 Introduction

Numerical optimization can be used in the process of electronic circuit design as a means of determining parameter values in order to bring the designed circuit as close as possible to some prescribed behavior or a set of characteristics. Requirements on a designed circuit can be of two different kinds: a minimization or maximization of some quantity (called an objective function) or a constraint condition that needs to be met by the solution. If the latter kind of requirements is present, a method for constrained optimization needs to be used.

1.1 Multiobjective Optimization Problem

In practical designs, there are often multiple mutually contradicting requirements on the designed circuit. In such cases, our aim is to solve the corresponding *multiobjective optimization problem*. This can be formally written as

$$\underset{x\in S}{\text{minimize}} \quad \{f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_k(\mathbf{x})\}, \qquad (1)$$

where we have k objective functions f_i: $\mathbf{R}^n \leftarrow \mathbf{R}$, $k \geq 2$. The decision vectors $\mathbf{x} = (x_1, x_2, \ldots, x_n)^T$, belong to the (nonempty) feasible region S, $S \subseteq \mathbf{R}^n$, which can be defined by a number of equality constraints, inequality constraints, and/or bounds on the decision variables x_i.

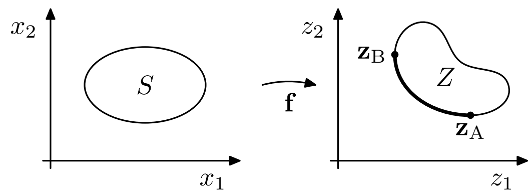

Figure 1: Feasible solutions and Pareto front.

The vector of objective functions will be denoted by $\mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_k(\mathbf{x}))^T$, and the image of the feasible region, also called the *feasible objective region*, will be denoted by $Z = \mathrm{f}(S)$, $Z \subseteq \mathbf{R}^k$. The elements of Z are called *objective vectors* and denoted by $\mathbf{f}(\mathbf{x})$ or $\mathbf{z} = (z_1, z_2, \ldots, z_k)^T$, where $z_i = f_i(\mathbf{x})$ for all $i = 1, 2, \ldots, k$ are *objective values*. The geometrical representation can easily be illustrated on a two-dimensional case, as shown in Figure 1 for $n = 2$ and $k = 2$.

1.2 Pareto Optimality

The word "minimize" in (1) means that we want to minimize all the objective functions simultaneously. However, because of the contradiction between the objective functions, it is not possible to find a single solution that would be optimal for all the objectives simultaneously. The concept of *noninferiority*, also called *Pareto optimality* after the French-Italian economist and sociologist Vilfredo Pareto (1848–1923), who developed this concept in 1896, must be used to characterize the objective vectors. A noninferior solution is one in which an improvement in one objective requires a deterioration of another. The set of all noninferior solutions is also called the *Pareto front*. In Figure 1 it is marked by the thick curve segment between points $\mathbf{z}_\mathrm{A}$ and $\mathbf{z}_\mathrm{B}$.

By solving the problem (1) we understand obtaining a sufficient number of noninferior solutions covering parts of the Pareto front that are of interest to the designer. This will allow him or her to fully understand the available trade-offs and to take a qualified decision based on this knowledge.

2 Multiobjective Optimization Methods

There exist a large number of conventional multiobjective methods [1, 2]. They typically take a number of controlling parameters and provide a noninferior solution by converting the problem (1) into an unconstrained or constrained scalar (i.e., single-objective) optimization problem, which is then solved by an adequate scalar optimization method (e.g., the Levenberg-Marquardt method in case of an unconstrained problem or the Penalty Function or Sequential Quadratic Programming methods for a constrained one).

In this paper, we will present two of the most commonly used methods.

2.1 Weighted Sum Strategy

The Weighted Sum Strategy (WSS) converts the multiobjective problem of minimizing the vector $\mathbf{f}(\mathbf{x})$ into a scalar problem by constructing a weighted sum of all the objectives

$$\underset{x \in S}{\text{minimize}} \sum_{i=1}^{k} w_i f_i^p(\mathbf{x}), \quad (2)$$

where $p \in \mathbf{R}$, $p \geq 1$, $w_i \geq 0$ for all $i = 1, \ldots, k$, and $\sum_{i=1}^{k} w_i = 1$. If the exponent $p > 1$, it is also assumed that $f_i(\mathbf{x}) \geq 0$ for all $\mathbf{x} \in S$ and for all $i = 1, \ldots, k$.

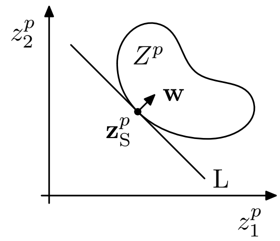

Figure 2: Geometrical representation of Weighted Sum Strategy.

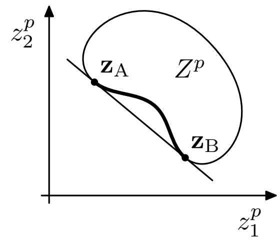

Figure 3: Nonconvex feasible objective region.

By the choice of the weighting vector $\mathbf{w} = (w_1, w_2, \ldots, w_k)^T$ we can control the location on the Pareto front. Only the direction of the vector $\mathbf{w}$ has effect on the found solution. This direction is perpendicular to the image of the Pareto front in the $Z^p = \mathbf{f}^p(S)$ domain. This is illustrated on a two-dimensional case in Figure 2. By minimizing the weighted sum (2) we obtain the objective vector $\mathbf{z}_\mathrm{S}^p$ in which a straight line L perpendicular to vector $\mathbf{w}$ touches the image Z^p of the feasible region S.

However, if the objective feasible region is nonconvex, its image Z^p may also be nonconvex, which may make certain noninferior solutions inaccessible, as shown in Figure 3. This problem can be partially alleviated by convexifying the nonconvex Pareto front. The convexification is performed by choosing a sufficiently large p under certain assumptions [1, p. 79].

Another disadvantage of WSS relates to its different properties depending on the Pareto front curvature in particular areas. In some area a small change in the weighting coefficients may cause big changes in the objective vectors while in other areas dramatically different weighting coefficients may produce nearly similar objective vectors.

2.2 Goal Attainment Method

The Goal Attainment Method (GAM) [2] is defined as a scalar constrained optimization problem of the form

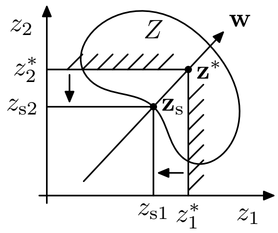

Figure 4: Geometrical representation of Goal Attainment Method.

$$\begin{array}{ll} \underset{\gamma \in \mathbf{R},\, x \in S}{\text{minimize}} & \gamma \\ \text{subject to} & f_i(x) - w_i\gamma \le z_i^*, \\ & \qquad i = 1, \ldots, k, \end{array} \tag{3}$$

where f_i are the original multiple objective functions of problem (1), S is the original feasible region, z_i^* are predefined *design goals* associated with the objective function f_i, $w_i \in \mathbf{R}$ are predefined weighting coefficients, and γ is an auxiliary variable making the new single objective function.

The method requires $2k$ input parameters (k goals z_i^* and k weights w_i), but only uses $2k-1$ degrees of freedom. This becomes obvious from the geometrical representation, again demonstrated on the 2D case in Figure 4. The goal vector $\mathbf{z}^* = (z_1^*, z_2^*)^T$ represents a goal point in the objective space, either feasible ($\mathbf{z}^* \in Z$, as in the shown case) or infeasible. The weight vector $\mathbf{w} = (w_1, w_2)^T$ defines the direction of movement from the goal point (when $\gamma = 0$) to the unique solution point $\mathbf{z}_\mathrm{S} = (z_{\mathrm{S}1}, z_{\mathrm{S}2})^T$ achieved by minimizing γ. Thus, as in the case of WSS, only the direction of the vector $\mathbf{w}$ is important; any changes in its magnitude would be compensated by changes in the values of γ.

Similarly to that in WSS, the weighting vector $\mathbf{w}$ in GAM enables the designer to express a measure of the relative trade-offs between the objectives. For instance, setting w_i equal to z_i^* ensures that the same percentage under- or overachievement of the goals z_i^* is achieved.

However, unlike WSS, GAM allows access to any noninferior solution, regardless of convexity of the Pareto front. Also the curvature of the Pareto front has no adverse effect on the relation between a chosen weighting vector and the corresponding obtained noninferior objective vector. And, by setting some of the weights w_i to zero, hard constraints can easily be incorporated into the design.

Among disadvantages of GAM we can count the fact that there are more parameter values to be given by the designer than in WSS. Also the increased number of constraints makes the resulting scalar optimization problem computationally more difficult.

3 Application Steps

The application of either method consists in performing the following series of steps:

1. Division of requirements to objective functions and constraints. For each requirement we decide, whether we want it to be represented by an objective function f_i to be minimized, or by an (in)equality constraint. We also specify the vector of decision variables $\mathbf{x}$.

2. Construction of the objective functions. We compose suitable objective functions f_i. For an easier orientation, it is a good idea to apply some kind of a unifying system. For example, the authors have used a rule requiring that the objective value of 0 represent an ideal, only theoretically attainable solution, the value 1 be still practically well acceptable, while the value 10 be an already totally unacceptable solution.

3. Choice of constraints and bounds. We specify the exact form for any equality constraints, inequality constraints and bounds on decision variables x_i defining the feasible region S.

4. Choice of multiobjective optimization method. Based on our knowledge about the multiobjective problem to be solved and our requirements, we choose a suitable multiobjective optimization methods. While WSS might be preferable if the the problem is known to be convex, or if a smaller number of controlling parameters is advantageous, GAM is likely to provide a better control over the accessed parts of the Pareto front, but only at the cost of higher demands on the designer as well as on the used computational power.

5. Exploring the Pareto front. Now we repeatedly choose the values of the required parameters (like w_i in case of WSS and GAM and possibly z_i^* in case of GAM) and solve the corresponding scalar optimization problem using a suitable scalar method to obtain sample solutions from all parts of Pareto front that might be of interest. This step can usually be (at least to some extent) automated.

6. Evaluation of the obtained data. In this stage the designer has a number of various noninferior solutions, from which, based on his/her judgement and experience, he/she can select one representing the best trade-off available.

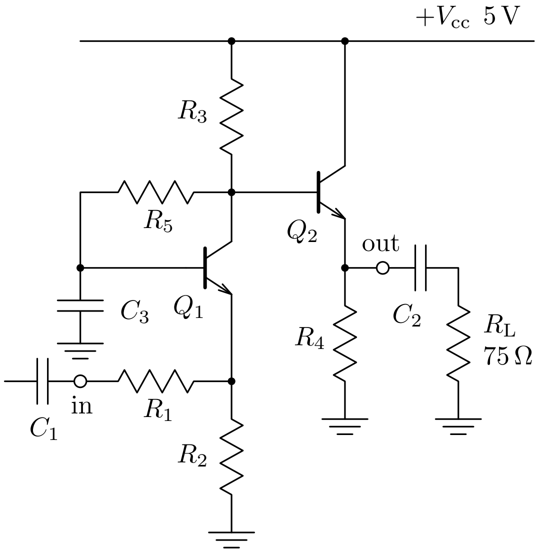

Figure 5: Amplifier schematic

4 Example Design Application

Let us now demonstrate the use of optimization methods on a practical example of high frequency electronic circuit [3, pp. 913–915]. We are to design a transistor video amplifier with an input matched to a source impedance $75\,\Omega$, with its output able to drive a $75\,\Omega$ load and with $1\,V_{\rm pp}$ output capability. The $3\,$dB roll off frequency $f_{\rm m}$ should be as high as possible, the low-frequency voltage gain $A_{\rm v}$ should be positive and of the biggest possible value, and the total DC supply current $I_{\rm cc}$ should be as low as possible. A nice way to achieve the noninverting gain is to use a common-base input stage with an emitter follower output, as suggested in Fig. 5. As the decision variables we will use the resistances R_1–R_5.

The assignment formulation intentionally does not define any preferences among the four extreme requirements on the circuit. Therefore, multiobjective methods are used to explore the four-dimensional Pareto front of the feasible region.

We assume that the capacitors C_1–C_3 have sufficiently large capacitances not to influence the low frequency gain. All high-frequency gain characteristics are thus determined only by the transistors' inner capacitances. The type 2N5179 was prescribed for both transistors Q_1 and Q_2. The standard Gummel-Poon bipolar transistor model is used.

As a measure of impedance matching, a low-frequency voltage standing wave ratio SWR will be used:

$$\mathrm{SWR} = \frac{1+|\varrho|}{1-|\varrho|}, \quad \varrho = \frac{R_{\rm i} - 75\,\Omega}{R_{\rm i} + 75\,\Omega}. \tag{4}$$

The considered multiobjective optimization problem can be written as follows

$$\begin{array}{ll} \text{minimize} & \mathrm{SWR},\ I_{\rm cc} \\ \text{maximize} & A_{\rm vdB},\ f_{\rm m} \\ \text{subject to} & V_{\rm out} \le 3.5\,\mathrm{V}, \end{array} \tag{5}$$

where the constraint condition concerning the output voltage $V_{\rm out}$ ensures the required $1\,\mathrm{V_{pp}}$ output capability.

Before we start the proper multiobjective optimization, it is a good idea to examine the best values $z_i^{\rm o}$ attainable by the four optimized characteristics if optimized alone. We *a priori* know that $\mathrm{SWR}^{\rm o} = 1$, because with a suitable value of R_1 the input resistance $R_{\rm i}$ can be made exactly equal to $75\,\Omega$. It is also clear that $I_{\rm cc}^{\rm o} \rightarrow 0\,\mathrm{mA}$. For $A_{\rm vdB}$ the independent maximum value was found to be $A_{\rm v}^{\rm o} = 40.72\,$dB, for which $R_{\rm i} = 18.41\,\Omega$, $\mathrm{SWR} = 4.073$, $I_{\rm cc} = 1.346\,\mathrm{mA}$ and $f_{\rm m} = 350.7\,\mathrm{MHz}$; and the maximum $f_{\rm m}$ found is $f_{\rm m}^{\rm o} = 860.3\,\mathrm{MHz}$, for which $R_{\rm i} = 546.1\,\Omega$, $\mathrm{SWR} = 7.282$, $A_{\rm v} = 4.281\,$dB, and $I_{\rm cc} = 7.532\,\mathrm{mA}$.

Both WSS and GAM were used to solve problem (5).

4.1 Weighted Sum Strategy

The following choice of objective functions was made in accordance with the recommendation in Step 2 of Section 3:

$$\begin{array}{rcl} f_1 &=& 10(\mathrm{SWR}-1), \\ f_2 &=& A_{\rm vdB}^{\rm o} - A_{\rm vdB}, \\ f_3 &=& \dfrac{I_{\rm cc}}{1\,\mathrm{mA}}, \\ f_4 &=& \log \dfrac{f_{\rm m}^{\rm o}}{f_{\rm m}}. \end{array} \tag{6}$$

A simple penalty function method is used to convert the constrained optimization problem into an unconstrained one. In this method, constraints are enforced by means of additive components called penalty functions, increasing the resulting objective function, and which are progressively dependent on the amount of the violation of the constraints. Only one constraint applies in our case, and is expressed by a penalty function of the form:

$$c_1 = \max\left(\frac{V_{\rm out} - 3.5\,\mathrm{V}}{3.5\,\mathrm{V}}, 0\right) \times q, \tag{7}$$

where q is a coefficient controlling how much the constraint component is emphasized over the usual minimized components in the objective function, $q = 100$.

The resulting single objective function used is

$$f_{\mathrm{P}}(x) = \sum_{i=1}^{4} w_i f_i^2(x) + c_1^2(x), \tag{8}$$

with the usual normalizing condition

$$\sum_{i=1}^{4} w_i = 1. \tag{9}$$

The objective function f_{P} is minimized using the Levenberg-Marquardt method. The needed derivatives of f_{P} are determined by finite differences for relative changes of 10^{-2}. The iterations end when the maximum relative change in the decision variables between iterations is less than 10^{-4} or after reaching a maximum allowed number of iterations.

In order to diminish the danger of finding only a local minimum different from the global one, the computation for each set of weights is divided in two stages. In the *first stage*, up to 15 iterations are performed from each of a chosen set of 162 starting points. These 162 starting points are generated by 5 nested loops combining the following values of resistances R_1–R_5: $R_1 \in \{20\,\Omega, 50\,\Omega\}$, $R_2 \in \{500\,\Omega, 1\,\mathrm{k}\Omega, 2\,\mathrm{k}\Omega\}$, $R_3 \in \{2\,\mathrm{k}\Omega, 5\,\mathrm{k}\Omega, 10\,\mathrm{k}\Omega\}$, $R_4 \in \{1\,\mathrm{k}\Omega, 2\,\mathrm{k}\Omega, 5\,\mathrm{k}\Omega\}$, and $R_5 \in \{10\,\mathrm{k}\Omega, 20\,\mathrm{k}\Omega, 50\,\mathrm{k}\Omega\}$. Ten best results of the first stage are printed in the order of increasing objective function values. The result with the smallest objective function value is used as the starting point in the *second stage*, where up to 100 iteration of the same algorithm are performed to further increase the accuracy of the found minimum.

4.2 Goal Attainment Method

To allow comparisons, the same set of objective functions f_i is assumed as that used with WSS (6). The constraint penalty function c_1 (7) remains in action, but we also have four new penalty functions related to functions f_i, $q = 100$:

$$\begin{aligned}
g_1 &= \max\left(\frac{10(\mathrm{SWR}-1) - w_1\gamma - P}{P}, 0\right) \times q,\\
g_2 &= \max\left(\frac{A^{\mathrm{o}}_{\mathrm{vdB}} - A_{\mathrm{vdB}} - w_2\gamma - P}{P}, 0\right) \times q,\\
g_3 &= \max\left(\frac{I_{\mathrm{cc}}/1\,\mathrm{mA} - w_3\gamma - P}{P}, 0\right) \times q,\\
g_4 &= \max\left(\frac{\log(f^{o}_{\mathrm{m}}/f_{\mathrm{m}}) - w_4\gamma - P}{P}, 0\right) \times q,
\end{aligned} \tag{10}$$

with the single objective function to be minimized:

$$f(x) = \gamma. \tag{11}$$

Note that, for simplicity, we have set all design goals z_i^* equal to the same scalar value P. This is allowed by the special choice of objective functions f_i (6). The single minimized objective function is then formed as follows

$$f_{\mathrm{P}}(x) = \gamma^2 + \sum_{i=1}^{4} g_i(x)^2 + c_1(x)^2. \tag{12}$$

The same two-stage procedure based on the Levenberg-Marquardt method is used to obtain solutions for chosen values of w_i and P as that with WSS.

5 Obtained Results

5.1 Results by WSS (Table 1)

Nr.	w_1	w_2	w_3	w_4	SWR (–)	A_{v} (dB)	I_{cc} (mA)	f_{m} (MHz)	S_1 (%)	S_{t} (%)
1	0.25	0.25	0.25	0.25	1.016	40.05	0.4863	127.6	–	62.5
2	0.4	0.2	0.2	0.2	1.004	40.02	0.4849	126.9	75	65.6
3	0.2	0.4	0.2	0.2	1.019	40.10	0.4792	126.4	75	71.9
4	0.2	0.2	0.4	0.2	1.012	40.34	0.3755	112.1	50	71.9
5	0.2	0.2	0.2	0.4	1.015	40.04	0.4869	127.6	50	65.6
6	0.7	0.1	0.1	0.1	1.002	40.04	0.4775	125.4	75	68.8
7	0.1	0.7	0.1	0.1	1.066	40.23	0.4810	128.0	50	62.5
8	0.1	0.1	0.7	0.1	1.003	40.15	0.3106	94.3	50	65.6
9	0.1	0.1	0.1	0.7	1.027	40.07	0.4884	128.4	75	78.1
Single-run average correl.					62.5	62.5	50.0	75.0	62.5	–
Total average correlation					72.2	59.7	66.7	73.6	–	68.1

Table 1: Results obtained by WSS.

A number of optimization runs was performed, each with a different vector of weighting coefficients w_i. The final solution obtained by minimizing the objective function provides the four numeric characteristics of the resulting amplifier: SWR, A_{vdB}, I_{cc}, and f_{m}. They are given in the same order as their corresponding weights w_i. Thus we can see that, for example, increasing the value of w_1 in row Nr. 2 has decreased (i.e., improved) the value of SWR with respect to that in row Nr. 1. This is what is wanted and expected. We would also expect, that such an improvement will be reached only at the cost of deterioration in the remaining three characteristics. This happens with A_{v} and f_{m}, but not with I_{cc}, which is also lowered. Thus three of the four characteristics have changed in accordance with their weights and one has changed in the opposite direction. In the table this is expressed by the "success rate" S_1, describing the correlation between the directions of changes in the individual weights w_i and those of the corresponding results with respect to case Nr. 1.

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The expected directions of changes are actually derived from the changes of the ratio

$$\frac{w_i}{\sqrt{\sum_{j=1}^{4} w_j^2}}, \tag{13}$$

which is related to the "angle" between vector $\mathbf{w}$ and the ith axis in the four-dimensional space of vectors $\mathbf{z} = (z_1, z_2, z_3, z_4)^T$.

This concept of correlation can be generalized by an arbitrary choice of the reference row. If we take row Nr. i as the reference case (instead of Nr. 1), we obtain the respective correlation S_i. The last column in Table 1 then gives the total correlation rate S_t, defined for each row j as the average of S_i over all possible choices of reference rows:

$$S_{\mathrm{t}j} = \frac{1}{n-1} \sum_{\substack{i=1 \\ i \neq j}}^{n} S_{ij}. \tag{14}$$

Also individual column values of S_1 and S_t averaged over rows are given in % in the 2 additional rows of the table.

Considering the fact that a 50% average correlation means that exactly a half of the changes is in the opposite direction than required, the average S_1 of $62.5\,\%$ and the total average S_t of 68.1% look quite pessimistic. However, all the 37.5 and 31.9 % of "failures" occur only for the unemphasized characteristics; all those with an increased w_i have actually improved or (as f_m of Nr. 5) at least remained the same.

Another characteristic worth examining may be the *noninferiority test*. If an obtained solution is to be a valid candidate for noninferior solution, none of the find solutions in the same table may have all its characteristics better than those of the candidate solution in question. To perform this test for all n cases in a table, a total of $n(n-1)$ comparisons need to be done. All of the 9 solutions in Table 1 have passed.

5.2 Results by GAM (Table 2)

Even though the meaning of the weighting coefficients w_i here is somewhat different from that in WSS, each w_i still corresponds with a single objective. If the reference point $\mathbf{z}^*$ is located in the feasible objective region, then increasing w_i usually allows for larger changes in its objective. in the required direction. Thus increasing a w_i should lead to an improvement in the related objective value, and, conversely, a decrease in w_i should deteriorate the obtained objective value.

The structure of the table remains the same with the exception that an 'Inf.' column was added. It informs about whether in the remaining 22 rows there is a solution to which the one in question is inferior. This is the case for row Nr. 11, which is inferior to row Nr. 14. The average correlation rates of $80.7\,\%$ and $79.5\,\%$ look much better than those of the WSS results.

Nr.	w_1	w_2	w_3	w_4	SWR (–)	A_v (dB)	I_cc (mA)	f_m (MHz)	Inf.	S_1 (%)	S_t (%)
1	1	1	1	1	1.071	40.01	0.5051	133.6	no	–	80.7
2	1	0	0	0	1.000	32.38	0.8442	302.2	no	75	76.1
3	0	1	0	0	1.183	40.45	0.4769	129.1	no	75	75.0
4	0	0	1	0	1.630	30.72	0.4036	173.7	no	75	78.4
5	0	0	0	1	1.671	30.72	1.7975	583.1	no	100	83.0
6	1	1	0	0	1.058	40.11	0.4908	130.0	no	75	71.6
7	1	0	1	0	1.025	37.08	0.3521	128.1	no	100	86.4
8	1	0	0	1	1.025	30.71	1.2605	404.1	no	100	85.2
9	0	1	1	0	1.169	40.35	0.4767	129.2	no	100	88.6
10	0	1	0	1	1.957	40.16	1.0966	228.4	no	100	83.0
11	0	0	1	1	1.366	31.36	0.5877	237.8	yes	75	72.7
12	1	1	1	0	1.058	40.11	0.4908	130.0	no	100	81.8
13	1	1	0	1	1.071	40.01	0.5051	133.6	no	25	75.0
14	1	0	1	1	1.042	32.74	0.5143	253.5	no	75	80.7
15	0	1	1	1	1.121	40.04	0.5317	140.3	no	75	75.0
16	2	1	1	1	1.000	38.89	0.9729	130.8	no	100	85.2
17	1	2	1	1	1.183	40.45	0.4769	129.1	no	75	76.1
18	1	1	2	1	1.176	35.57	0.4625	154.0	no	75	85.2
19	1	1	1	2	1.476	35.53	0.8479	295.5	no	100	79.5
20	2	2	2	1	1.059	40.12	0.4908	130.0	no	100	78.4
21	2	2	1	2	1.045	40.00	0.4908	130.0	no	25	75.0
22	2	1	2	2	1.018	35.45	0.4999	203.3	no	100	78.4
23	1	2	2	2	1.119	39.97	0.4769	129.4	no	50	77.3
Single-run average					95.5	86.4	68.2	72.7	–	80.7	–
Total average					84.2	89.5	69.4	74.9	–	–	79.5

Table 2: Results obtained by GAM, $P = 10$.

5.3 Infeasible Reference Point (Table 3)

When the reference point $\mathbf{z}^*$ is infeasible and "below" the Pareto front, it can only be made feasible by worsening one or more its coordinates. The relative amount of deterioration can be controlled by the weights w_i, as shown in Table 3. As all objective functions are scaled so that their typical values can be expected around 1, $P = 0.2$ should already position the reference point $\mathbf{z}^*$ outside of the feasible region. That this is probably the case can be seen from the first several rows of Table 3. Most of the emphasized objectives are significantly deteriorated (with row Nr. 4 being an exception). Therefore the "controlling logic" is inverse to that in the previous tables: increasing the weights w_i worsens the corresponding objective value and vice versa. Considering this fact, the overall performance is slightly worse than, but still comparable to, that of Table 2: 3 inferior cases (Nr. 1 is inferior to Nr. 15), and the average correlations worse by $6.8\,\%$ and $0.9\,\%$.

Nr.	w_1	w_2	w_3	w_4	SWR (–)	A_v (dB)	I_{cc} (mA)	f_m (MHz)	Inf.	S_1 (%)	S_t (%)
1	1	1	1	1	1.073	39.30	0.6192	129.9	yes	–	73.9
2	1	0	0	0	1.601	40.52	0.5872	165.7	no	100	75.0
3	0	1	0	0	1.020	28.45	0.4062	442.5	no	100	87.5
4	0	0	1	0	1.031	40.13	0.4809	127.1	no	50	51.1
5	0	0	0	1	1.020	40.35	0.3853	111.8	no	100	89.8
6	1	1	0	0	3.450	16.13	0.3731	502.4	no	100	86.4
7	1	0	1	0	1.973	40.34	0.8512	231.9	no	100	83.0
8	1	0	0	1	1.245	40.44	0.4118	115.7	no	100	86.4
9	0	1	1	0	1.019	27.86	1.5166	564.4	no	100	89.8
10	0	1	0	1	1.020	39.78	0.3292	100.6	no	75	81.8
11	0	0	1	1	1.025	40.18	0.4777	120.2	no	75	73.9
12	1	1	1	0	1.795	32.77	1.5167	541.1	no	100	81.8
13	1	1	0	1	1.098	39.95	0.2889	90.5	no	75	81.8
14	1	0	1	1	1.964	40.35	0.8473	230.8	no	75	76.1
15	0	1	1	1	1.020	39.45	0.4908	130.4	no	25	72.7
16	2	1	1	1	1.126	39.91	0.7124	132.4	no	75	77.3
17	1	2	1	1	1.082	39.31	0.5159	130.5	no	50	77.3
18	1	1	2	1	1.073	39.30	0.6192	129.9	yes	25	72.7
19	1	1	1	2	1.073	39.30	0.6192	129.9	yes	25	72.7
20	2	2	2	1	1.010	39.24	0.6769	130.6	no	75	68.2
21	2	2	1	2	1.064	39.53	0.4769	129.6	no	50	81.8
22	2	1	2	2	1.141	39.92	0.6224	129.5	no	100	80.7
23	1	2	2	2	1.064	39.53	0.4769	129.6	no	50	81.8
Single-run average					77.3	63.6	72.7	81.8	–	73.9	–
Total average					82.4	79.8	73.3	78.1	–	–	78.4

Table 3: Results obtained by GAM, $P = 0.2$.

Nr.	w_1	w_2	w_3	w_4	SWR (–)	A_v (dB)	I_{cc} (mA)	f_m (MHz)	Inf.	S_1 (%)	S_t (%)
1	1	1	1	1	1.071	40.01	0.5051	133.6	no	–	81.8
2	1	0	0	0	1.000	37.42	0.6208	154.7	no	75	80.7
3	0	1	0	0	1.183	40.45	0.4769	129.1	no	75	78.4
4	0	0	1	0	1.553	30.72	0.1629	131.1	no	100	86.4
5	0	0	0	1	2.001	30.71	1.8962	759.9	no	100	81.8
6	1	1	0	0	1.048	40.18	0.4812	127.5	no	75	72.7
7	1	0	1	0	1.020	39.86	0.3054	94.6	no	100	86.4
8	1	0	0	1	1.024	30.71	1.2347	405.2	no	100	85.2
9	0	1	1	0	2.000	40.39	0.4338	120.8	no	100	87.5
10	0	1	0	1	2.000	40.27	0.8697	237.6	no	100	84.1
11	0	0	1	1	1.470	30.71	0.5117	238.0	yes	75	76.1
12	1	1	1	0	1.049	40.18	0.4812	127.6	no	100	87.5
13	1	1	0	1	1.071	40.01	0.5051	133.6	no	25	76.1
14	1	0	1	1	1.011	32.80	0.5045	259.1	no	100	83.0
15	0	1	1	1	1.614	40.09	0.6595	186.5	no	75	78.4
16	2	1	1	1	1.000	38.89	0.9729	130.8	no	100	84.1
17	1	2	1	1	1.183	40.45	0.4769	129.1	no	75	84.1
18	1	1	2	1	1.486	39.08	0.2782	91.0	no	100	83.0
19	1	1	1	2	1.518	35.53	1.5207	324.7	no	100	78.4
20	2	2	2	1	1.059	40.12	0.4908	130.0	no	100	80.7
21	2	2	1	2	1.045	40.00	0.4908	130.0	no	25	73.9
22	2	1	2	2	1.057	35.42	0.5901	212.5	no	75	81.8
23	1	2	2	2	1.042	39.98	0.4908	130.1	no	25	72.7
Single-run average					90.9	86.4	68.2	81.8	–	81.8	–
Total average					84.4	89.5	69.4	81.0	–	–	81.1

Table 4: Results by GAM, $P = 10$, more iterations

5.4 Increased Iterations Limits (Table 4)

The last table was obtained for the same feasible reference point of $P = 10$ as Table 2, but with a significant increase in the limits on the numbers of Levenberg-Marquardt iterations. In the first and second stages, the values 100 and 1000 were used instead of 15 and 150, respectively. Even though it turned out that only in about a half of the cases any further iterations were performed in the second stage, a slight improvement of about $1\,\%$ is obtained in both correlation rates. Three rows of Table 4 exhibit improvement in S_1 over those in Table 2. Interestingly, the last two rows lost $25\,\%$ in S_1 each. A closer examination showed that even in these rows the achieved values of the minimized objective function f_{P} were below those of Table 2.

6 Conclusion

All four tables of results show that the objective values of obtained solution can be controlled by the choice of weighting coefficients w_i. However, even in the best case observed, about $20\,\%$ of changes in w_i have the opposite effect on the related goal then expected. Also, in a few cases, the obtained solutions were found to be inferior to solutions with a different weighting vector.

These problems can have several possible explanations. First, the iteration process of the Levenberg-Marquardt method may have, in some cases, converged too slowly and thus in the limited number of iterations provided a solution still too remote from the optimum. This might be caused by imprecise calculation of derivatives due to numerical noise of the embedded iteration process for determining quiescent operating point of the circuit. Another explanation may be that only local optima still far from the global one were reached in some cases. And thirdly, perhaps the chosen definition of correlation rate between the weighting vector and obtained objective values based solely on the signs of differences from a reference case is too simple, as it ignores the absolute values of the differences.

Therefore our future improvements will be aimed at trying a greater number of starting iterations and replacing the penalty function method by a more efficient alternative for solving the constrained optimization problem, e.g., Sequential Quadratic Programming [4].

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