Proceedings of the 5th WSEAS/IASME Int. Conf. on SYSTEMS THEORY and SCIENTIFIC COMPUTATION, Malta, September 15-17, 2005 (pp50-55)

A Variable Parameters Linear Programming Method And Its Application In Power System

Zhou Jian, Jiang Chuanwen, Wang Chengmin, Song Yufei, Wang Yu, Chen Xi
Department of Electrical Engineering, Shanghai Jiaotong University
Huashan Road 1954, PR China, 200030

***Abstract*:-** The linear programming method has widely applied in practice engineering due to universality in the mathematical model and validity in the simplex method. While constants in the constrains partly changed with variable coefficients in the objective function, it is improved effect that using a new search technology to solve the changed parameters linear programming problem. This technology is used in economic operation of electric power system.
***Keywords*:** -linear programming, electric power system,economic operation

1. Introduction

The linear programming method is a kind of widely applied optimization methods in practical engineering, a lot of problems can be solved with the linear programming method. Generally speaking, while solving the practical problem with the linear programming method need the following several stages: put forward the question; analyses the situation; search for the materials and relevant data; abstract and summarize the question then form the mathematical model; get the optimum solution to support the decision of the decision-maker; put into practice finally. But the practical situation often change, the parameters of the mathematical model which has already set up is not fixed.

To this kind of question, some literature [1][2] have proposed the sensitivity analysis method of the linear programming. The sensitivity analysis method of the linear programming analyses the influence to optimum solution when one or several parameters in the mathematical model of the linear programming change. But the sensitivity analysis method most literature have proposed is aim at the situation when a parameter in the mathematical model of the linear programming changes. In practical engineering, a kind of question is often met, this kind of question can be described into a mathematical model of the linear programming, its control and decision variable usually are bounded, and the state variable coefficient changes along with the change of variable value scope, such as the economic operation of electric power system. This paper proposes an iteration and search algorithm to solve this kind of problem in practical engineering, and apply it to economic operation of electric power system.

2. Question Description

A lot of questions in the practical engineering can be described into a linear programming problem of the bounded variable, its standard mathematical model as follows:

$$\min z = \sum_{i=1}^{N} c_i x_i \qquad (1)$$

$$a_{i,j}x_i = b_j \quad (2)$$
$$L_i \le x_i \le U_i \quad (3)$$

Where, $i = 1,2,\cdots N$ is the number of the state variable; $j = 1,2,\cdots M$ is the number of constrains; c_i、$a_{i,j}$、b_i are parameters; L_i、U_i is respectively upper bound and lower bound of state variable x_i.

When the value scope of state variable x_i changes, coefficient c_i usually changes too, as Fig.1 shows.

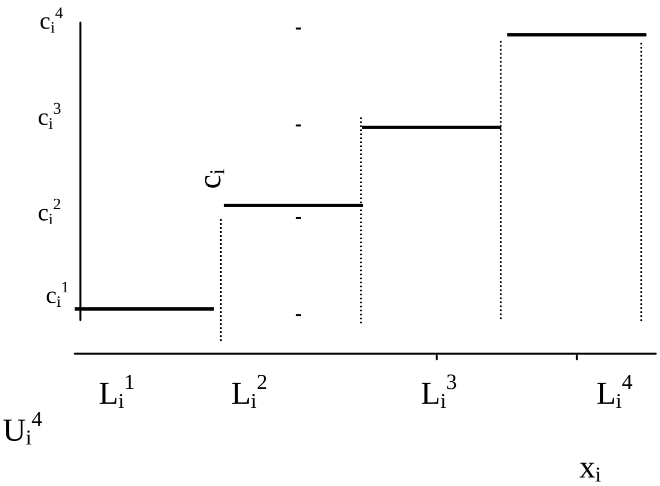

Fig. 1 Fig of changed parameters linear programming method

In Fig.1, the value scope of state variable x_i is divided into 4 sectors, the lower bound of the first sector, for the minimum value scope of the state variable x_i; The upper bound of the last sector, for the maximum value scope of the state variable x_i; $L_i^2 = U_i^1, L_i^3 = U_i^2, L_i^4 = U_i^3$. Along with the value scope of state variable x_i changes from low to the high, the corresponding coefficient c_i also as changes from low to high (as black line in Fig. 1 shows). It is difficult to solve this kind of question with ordinary sensitivity analysis method of the linear programming for some big systems which have a lot of state variable. This paper proposes an iteration and search technology to solve this kind of problem.

3. A New Changed Parameters Linear Programming Algorithm

First of all, we can know that by the theorem of the bounded variable linear programming[2]

If question (1) ~ (3) have optimum solution, then certainly solved the problem in the basic feasible solution place. The solution is the basic feasible solution, when the non- base variable value is the upper bound or the lower bound value solution. So the above-mentioned question solution must have part of variables values to the upper bound, part of variables values to the lower bound, another part of variables value is situated between the upper bound and lower bound. Write the question (1) ~ (3) as the form of matrix:

$$\min \quad cx \quad (4)$$
$$AX = b \quad (5)$$
$$L \le x \le U \quad (6)$$

where L is lower bound set of the state variable; U is upper bound set of the state variable.

Matrix A in the equation (5) is decomposed into (B, N_1, N_2) ,where B is m step full order square matrix of the corresponding base variable, N_1 is corresponding lower non- base variable, N_2 is corresponding upper non- base variable, correspondingly:

$$x = \begin{bmatrix} x_B \\ x_{N1} \\ x_{N2} \end{bmatrix} \qquad c = \begin{bmatrix} c_B \\ c_{N1} \\ c_{N2} \end{bmatrix}$$

$$L = \begin{bmatrix} L_B \\ L_{N1} \\ L_{N2} \end{bmatrix} \qquad U = \begin{bmatrix} U_B \\ U_{N1} \\ U_{N2} \end{bmatrix} \qquad (7)$$

Then,

$$x_B = B^{-1}b - B^{-1}N_1 x_{N1} - B^{-1}N_2 x_{N2} \qquad (8)$$

$$f_0 = c_B x_B - (c_B B^{-1} N_1 - c_{N1}) x_{N1} - (c_B B^{-1} N_2 - c_{N2}) x_{N2} \qquad (9)$$

Where, f_0 is value of the objective function.

In equation (9), if $c_B B^{-1} N_1 - c_{N1} \le 0$ and $c_B B^{-1} N_2 - c_{N2} \ge 0$, Because variable x_{N1} reaches the low bound , x_{N2} reaches the upper bound, so the objective function value f_0 can't be reduced again, it is the optimum value. In variable coefficient situation, change the upper bound of x_{N1} and lower bound of x_{N2} as states above, then x_{N1} becomes lower non-base variable, x_{N1} becomes upper non- base variable in the new value scope. Suppose c_{N1} and c_{N2} do not change, because of $c_B B^{-1} N_1 - c_{N1} \le 0$ and $c_B B^{-1} N_2 - c_{N2} \ge 0$ so the objective function value can also drop, it has not reached optimum value. But along with the change of upper or lower bound of x_{N1}, x_{N2}; c_{N1} and c_{N2} also change, when the change of c_{N1} and c_{N2} cause $c_B B^{-1} N_1 - c_{N1} \ge 0$ and $c_B B^{-1} N_2 - c_{N2} \le 0$, the new optimum solution has produced.

We can know from above that mentioned, can use iteration search algorithm to solve this kind of changed coefficient linear programming question, the step to be as follows:

1. Give definitely the initial value of c, L, U, solve the linear planning problem with the simplex method using equation (4) ~ (6).
2. check-up the received optimum solution whether the value of the state variable reaches the upper bound or the lower bound.
3. If the state variable value reaches the upper bound, and there is no maximum of reaching of this state variable, then replace the coefficient and value scope of the corresponding variable as the parameter of the previous sector, return to 1; If the state variable value reaches the lower bound, and there is no minimum of reaching of this state variable, then replace the coefficient and value scope of the corresponding variable as the parameter of the previous sector, return to 1;
4. If all variables value had not reached the upper bound or the lower bound, or the objective function value no longer drops, then stop calculation, the computation ended, the solution at this moment is an optimum solution;

4. Application In Economic Operation Of Electric Power System

The economic operation of electric power system is the economic power distribution among every generation unit in certain operation cycles, meet various kinds of operation conditions, in order to reach the minimum of the total operating cost. At present, a lot of algorithms, λ-dispatching method[3] , linear programming method[4][5] and so on, have already proposed in solving the economic operation of electric power system, these method all more or less have some limitations.

The complete economic operation of electric power system problem may describe as follows:
Objective function:

$$\min z = \sum_{t=1}^{T}\sum_{i=1}^{N} c_{i,t}\, p_{i,t} \qquad (10)$$

1. System power equilibrium constraints:

$$\sum_{i=1}^{N} p_{i,t} = P_{d,t} \qquad (11)$$

2. Rise and descent velocity of generator output:

$$R_{i,\min} \ge p_{i,t} - p_{i,t-1} \ge R_{i,\max} \qquad (12)$$

3. System spinning reserve constraints:

$$\sum_{i=1}^{N} P_{i,t}^{\max} - \sum_{i=1}^{N} p_{i,t} \ge S_{\max,t}, \sum_{i=1}^{N} p_{i,t} - \sum_{i=1}^{N} P_{i,t}^{\min} \le S_{\min,t} \qquad (13)$$

4. Constraints of generation unit output upper and lower bound:

$$P_{i,t}^{\min} \le p_{i,t} \le P_{i,t}^{\max} \qquad (14)$$

Where, , i =1, 2, …N, is the number of generation unit in power system, t =1, 2, …T, Is the number of time-block which comes from the division of plan cycle, $c_{i,t}$ is cost coefficient of i -th generation unit at t -th time-block; $p_{i,t}$ is output of i -th generation unit at t -th time-block; $P_{d,t}$ is system load at t -th time-block; $P_{i,t}^{\max}, P_{i,t}^{\min}$ are respectively maximum and minimum output of i -th generation unit at t -th time-block; $S_{\max,t}$ is upper spinning reserve of system load; $S_{\min,t}$ is lower spinning reserve of system load; $R_{i,\min}$ is descent power velocity maximum value of i -th generation unit:; $R_{i,\max}$ is rise power velocity maximum value of i -th generation unit.

This is a bounded variable linear programming question. The goal is to cause the system total operating cost to be lowest. The typical cost curve of generation unit as the curve in Fig. 2 shows. Generally we make this cost curve partition staging and linear processing as the straight line in Fig. 2 shows. At this moment the economic operation of power system is same as above question, so definitely can use the method which introduced above to get the solution.

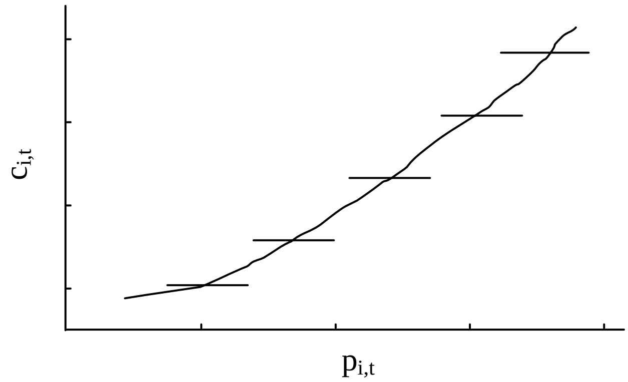

Fig. 2 Cost curve of generation unit

5. Experimental Results

The iteration and search technology this paper proposed is applied to economic operation of an actual electric power system which has 8 generation units. Take generation plan that one day in a cycle in electric power system for example, divide an entire day into 24 time-block, each time-block is an hour, (system data as follows: Table 1 is power system parameter; Table 2 is system load and spinning reserve data; Because unit's cost data is extremely huge, has not listed in here.) testing result as Fig.3 shows. Altogether we have carried on 5 iterative computations, production cost as Table 4 shows, the first iterative computation is a initialized value, final production total cost is ￥$4.9093+10^7$.

The computed results displayed that, various units output at various time-blocks satisfy each kind of constraints, also the operating cost is quite small, this explains the algorithm we proposes validity, can utilize in the actual system operation definitely.

Tab.1 Power system parameter

unit	output upper limit (MW)	output lower limit (MW)	output rise rate (MW/min)	output descent rate (MW/min)
1	130	20	130	130
2	130	20	130	130
3	460	100	460	460
4	265	100	265	265
5	160	100	160	160
6	455	100	455	455
7	455	100	455	455
8	470	100	470	470

Tab.2 System load and spinning reserve

system load (MW)	(1~8time-block)	2023	1942	2233	1943	1966	1946	1933	1993
	(9~16 time-block)	1999	2234	1883	1923	1912	2033	2034	2182
	(17~24 time-block)	1934	1846	2045	1934	1937	2230	1934	1913
Spinning reserve (MW)	(1~8 time-block)	50	50	50	50	50	50	50	50
	(9~16 time-block)	50	50	50	50	50	50	50	50
	(17~24 time-block)	50	50	50	50	50	50	50	50

Tab.3 Testing result for 8-machines 24-time_block system

unit	unit 1		unit 2		unit 3		unit 4		unit 5		unit 6		unit 7		unit 8	
T	output	unit price	output	unit price	output	unit price	output	unit price	output	unit price	output	unit price	output	unit price	output	unit price
1	60	974.4	20	1028	460	779.8	300	915.3	150	879.3	455	792.7	455	700.4	123	1017
2	40	853.3	20	940.8	100	1010	300	840.1	150	853.1	452	853.1	420	866.0	430	853.1
3	130	704.0	110	732.1	433	732.0	300	663.7	160	732.1	400	732.1	260	762.3	140	732.1
4	40	732.3	110	732.1	100	792.5	300	732.0	158	732.1	455	732.1	420	722.5	360	732.1
5	110	865.1	110	865.1	421	853.0	300	853.0	160	853.1	455	853.1	260	853.1	140	853.1
6	110	853.1	110	853.1	111	913.5	300	853.0	160	853.1	455	853.1	420	835.8	280	853.1
7	110	974.1	113	974.1	150	1034	300	974.0	160	974.1	400	974.1	420	974.1	280	974.1
8	103	1107	110	1107	150	1125	300	1095	150	1095	400	1095	420	1125	360	1095
9	110	1149	110	1149	380	1143	300	1149	150	1149	449	1149	360	1174	140	1149
10	110	1137	110	1137	454	1131	300	1137	160	1137	400	1137	260	1163	140	1137
11	110	1125	110	1125	403	1119	300	1125	160	1125	400	1125	360	1151	140	1124
12	110	1113	110	1113	388	1040	300	1113	160	1113	455	1113	360	1139	140	1113
13	110	974.1	110	974.1	432	913.5	300	974.0	160	974.1	400	974.1	260	997.3	140	974.1
14	110	853.1	123	853.1	150	913.5	300	853.0	160	853.1	400	853.1	420	853.1	360	853.1
15	110	986.1	114	986.1	150	1016	300	974.0	160	974.1	400	974.1	420	1004	360	974.1
16	100	1052	102	1052	150	1070	300	1052	150	1052	400	1052	420	1078	360	1052
17	110	1095	110	1095	399	1095	300	1095	160	1095	455	1095	360	1125	140	1095
18	110	1095	110	1095	150	1143	300	1095	156	1095	400	1095	420	1125	200	1095
19	110	1191	110	1191	410	1137	300	1191	160	1191	455	1191	360	1386	140	1191
20	110	1082	110	1082	454	1082	300	1082	160	1082	400	1082	260	1113	140	1083
21	110	1082	110	1082	402	1028	300	1082	160	1082	455	1082	260	1113	140	1083
22	110	974.1	110	974.1	450	961.9	300	974.0	160	974.1	400	974.1	260	1004	140	974.1
23	110	949.8	114	949.8	150	961.9	300	949.8	160	949.8	400	949.8	420	980.1	280	949.8
24	110	986.1	110	986.1	433	965.2	300	974.0	160	974.1	400	974.1	260	1004	140	974.1

Tab. 4 Production cost

iterations	1	2	3	4	5
Production cost	$4.82619+10^7$	$4.92627+10^7$	$4.90571+10^7$	$4.90507+10^7$	$4.90493+10^7$

6. Conclusion

This paper proposes an iteration and search technology to solve changed parameters linear programming problem, and apply it to economic operation of electric power system. After proof-testing by the actual system, this algorithm is simple, effective, and it is an convenient way to solve actual problems.

The linear programming is one of the best optimization algorithms at present, applying it to economic operation of electric power system is great suitable for the actual situation. This paper apply iteration and search technology to solve problem of load distribution in electric power system, describe the question into a linear programming mathematic model in the entire electric generation plan cycle, this can consider rise and descent rate restraint of the generation unit output, although question dimensions increase remarkably, this method can solve the large-scale electric power system economic operation problems definitely.

References

[1] Zhang Jianzhong，Xu Zhaojie. Linear Programming，Science Press，1990.

[2] James G Waight, Anjan Bose, Geral B Sheble. Generation Dispatch with Reserve Margin Constrains Using Linear Programming. IEEE Trans. on PAS, 1981,100(1).

[3] Waight J G, Bose A, and Sheble G B, Generation Dispatch with Reserve Margin Constrains Using Linear Programming. IEEE Transactions on PAS–100, 1981, No. 1：pp. 252–258.

[4] Piekutowski M, and Rose I A, A Linear Programming Method for Unit Commitment incorporating Generation Configuration, Reserve, and Flow Constrains. IEEE Transactions on PAS–104, 1985, No. 12: pp. 3510–3516.