

Analysis on OPF and Operational Economy of Electricity Market -- Compositive Impacts of Slack Bus Voltage

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Abstract: -OPF is an essential tool to study interconnected systems and electricity markets. One of the main goals of PX and electric utilities is to find out the scheme with minimum generation cost. When grid topology and load are determined, the change of slack-bus voltage will affect the operational results of OPF and economic performances of electricity market. For the first time, this topic is presented and quantitatively studied in the paper by optimizing tool. Some important and useful conclusions are obtained through test examples. The results show that using optimizing approaches can remarkably reduce the total generation cost; keeping appropriate slack-bus voltage is helpful to reduce the generation cost and MW losses, and bring utilities extra benefits; To the grid with negative MVAR losses, voltage stability and security of power systems must be paid more attentions.

Keywords:-Economy - Electricity Market - Optimal Power Flow - Slack Bus Voltage

1. Introduction

With rapid development of restructuring of power systems, interconnecting grids and constructing regional or even national electricity market will become more and more popular. In an interconnected system, Power Exchange (PX) and electric utilities are facing many challenges in techniques. One of the main motives of PX and utilities is to use the advanced tool of optimal power flow (OPF) and to find out the operational scheme with minimum generation cost.

To a given system, adoption of optimizing approach can reduce the total generation cost. However, the impacts of voltage of balance node (marked as V_s) on economy of electricity market are generally neglected. In fact, balance node is usually located at the bus of large plant that answers for frequency regulating. The voltage of this node can be a value within a certain range, saying 0.98-1.10 p.u. Tests and analysis show that the magnitude of V_s has important impacts on the solution of OPF

program and economy of electricity market. The author of document [1] had made some researches on this issue in the field of specific provincial network, and pointed out the importance of this problem. However, only a simple and conventional program of load flow was used in his research, and the conclusion is not universal. Further to the study, this paper designs an optimization procedure (OPF) and a conventional BX-based program of power flow, and has done many tests in IEEE 11, 14 and 30-bus systems. For the first time, a more thorough study on the impacts of V_s has been made on the total generation cost, MW and MVAR losses. Some important conclusions are obtained, they are useful to the competitive electricity markets.

2. Description for Test System Models

IEEE 11, 14 and 30-bus test systems are used. The level of load and economic parameters of generator in each system are carefully determined. The data are listed in table 1. Where the meaning of coefficients a_i , b_i , c_i is same as in equation (1).

Table 1 Economic Parameters of units and Loads in Test Systems

System	Level of loads		Node of unit	Node type	Economic parameters of units		
	MW	MVA R			a _i	b _i	c _i
11-bus	47.08	46.0	1	Balance	500	300	0
			8	PV	1250	100	0
14-bus	259.0	73.5	1	Balance	63	200	0
			2	PV	350	175	0
			3	PV	1250	100	0
			6	PV	166.8	325	0
			8	PV	500	300	0
30-bus	283.4	126.2	1	Balance	75	200	0
			2	PV	350	175	0
			5	PV	1250	100	0
			8	PV	166.8	325	0
			11	PV	500	300	0
			13	PV	500	300	0

3. Optimization Tool and Flowchart of Analysis

3.1. Optimization Tool

The optimization tool is a set of program, i.e. Newton OPF. The purpose of using OPF is to search the optimal solution of objective and power flow under present constraints of power system. The objective is minimum generation cost of whole system [2]. The mathematic description of OPF problem is as following:

$$\left. \begin{aligned} \text{Min } F(\mathbf{y}) &= \sum_{i=1}^{N_g} (a_i P_{g,i}^2 + b_i P_{g,i} + c_i) \\ \text{s.t. } \mathbf{g}(\mathbf{y}) &= \mathbf{0} \\ \mathbf{h}(\mathbf{y}) &\leq \mathbf{0} \end{aligned} \right\} \quad (1)$$

Where, \mathbf{y} is the vector of system variables. $F(\mathbf{y})$ is the objective, being total generation cost of all generators, in \$/hr. $P_{g,i}$ is MW output of i_{th} generator (including balance unit), in p.u.. N_g is the number of generators. a_i , b_i and c_i are coefficients in cost function of i_{th} generator. $\mathbf{g}(\mathbf{y}) = \mathbf{0}$ is the set of equation constraints, e.g. balance of nodal injection of power; $\mathbf{h}(\mathbf{y}) \leq \mathbf{0}$ is the set of inequality constraints that are composed of operational variables.

In this paper, Newton method and skills to deal with sparse matrixes are utilized to solve the problem. Through a series of transform, the solution of OPF can be achieved from following equations [3]:

$$\left. \begin{aligned} \mathbf{W}^0 \Delta \mathbf{z} &= -\mathbf{d}^0 \\ \text{or} \quad \Delta \mathbf{z} &= (-\mathbf{W}^0)^{-1} \mathbf{d}^0 \end{aligned} \right\} \quad (2)$$

Where $\Delta \mathbf{z} = \begin{bmatrix} \Delta \mathbf{y} \\ \Delta \boldsymbol{\lambda} \end{bmatrix}$ is the increment vector of variable \mathbf{y} and $\boldsymbol{\lambda}$. $\boldsymbol{\lambda}$ is the vector of Kuhn-Tucker multipliers [4]. $\Delta \mathbf{y}$ is the increment vector of variable \mathbf{y} ; $\Delta \boldsymbol{\lambda}$ is the increment vector of K-T multipliers; \mathbf{W} is called for \mathbf{W} matrix; \mathbf{d} is the column gradient vector.

3.2. Flow chart of calculation and analysis

Steps for solving OPF and analyzing the impacts of voltage of balance node are simply described as following:

- 1) Set a value of balance node voltage, saying $V_s = 0.98$;
- 2) Put a set of original values to vector \mathbf{y}^0 and $\boldsymbol{\lambda}^0$;
- 3) Solving \mathbf{W}^0 and \mathbf{d}^0 , then \mathbf{z}^0 ;
- 4) Get the value of $\Delta \mathbf{z}^0$ from equation (2), and further $\mathbf{z}^1 = \mathbf{z}^0 + \Delta \mathbf{z}^0$;
- 5) Taking \mathbf{z}^1 into next interactive process, i.e. step 3, therefore $\Delta \mathbf{z}^1$, \mathbf{z}^2 , ..., till the condition of convergence is met, for example, all elements of gradient vector \mathbf{d} are less than 5×10^{-4} . Thus, we can conveniently calculate the total generation cost, branch flows, grid losses, node voltages, tap ratio of regulated transformer under this voltage of balance node.
- 6) Increase V_s with step length of 0.01, then go back to the first step, till $V_s < 1.10$ p.u..

4. Experimental Analysis

4.1. Impacts of slack-bus voltage on the total generation cost

Two problems will be discussed in this case. One is what benefit the optimization tool brings to electricity market, i.e., what functions the tool plays in the market? Another one is to what extent the optimization results can be affected by voltage of balance node?

4.1.1. The functions of optimization tool

To observe the action of optimization program in electricity market, two separate programs have been designed. They are respectively BX based conventional program and that of optimal power flow, which is just described above.

Given a value of V_s , under same conditions and state of system, initial power flow and the relative generation cost of whole system can be calculated by BX based program. Meanwhile, optimal power flow and the relative generation cost can be also obtained by OPF program. Based on these results, the difference of generation costs between two situations will be known. In the same way, we can get other necessary data from the flow chart of analysis demonstrated above, and observe the changes of generation cost before and after optimization to the different voltage of balance node. The results are shown in table 2. Due to the page limited, only data of 11- and 30-bus systems are listed.

Table 2 Total Generation cost under different voltage of balance node (in \$/hr)

V_s (p.u.)	11-bus system		30-bus system	
	BX program	OPF	BX program	OPF
0.98	152.339409	148.608678	911.1793	807.1682
0.99	152.638654	148.608740	908.5784	806.1391
1.00	152.990477	148.608740	906.3422	805.4074
1.01	153.394569	148.608740	904.4728	804.7118
1.02	153.850651	148.608739	902.9718	804.0494
1.03	154.358497	148.608739	901.8411	803.4177
1.04	154.917939	148.608739	901.0825	802.8060
1.05	155.528888	148.608739	900.6977	802.2112
1.06	156.191348	148.608738	900.6884	801.6327
1.07	156.905431	148.608738	901.0563	801.0697
1.08	157.671375	148.608738	901.8032	800.5220
1.09	158.489565	148.608738	902.9308	792.9889

It can be seen that the optimal generation costs by OPF are obviously less than the initial ones, no matter what value is the voltage of balance node or what scale the system. In other words, application of optimizing method could reduce the total generation cost and obtain reasonable benefits. This is an important economic signal to electricity market.

4.1.2. Impacts of slack-bus voltage on the optimization results

Firstly, let us analyze the results related to BX based program. From the data in table 2. The generation cost in 11-bus system has a trend to simply and non-linearly increase with the raise of voltage of balance node, i.e. from 152.339409 when $V_s=0.98$ to 158.489565 when $V_s=1.09$, the relative rate of difference is about 4.4%. In the sense of economy, it seems that the system should be run at the point with

lower voltage of balance node. Unfortunately, this viewpoint is wrong and might mislead the system operator. In the other hand, the value of total generation cost in 30-bus system changes from high to low, and then from low to high once more. Apparently, the voltage of balance node has great affects on the results of conventional power flow program, and no consistent conclusion exists. This is a positive answer.

Now let us consider the situation of OPF. Although nearly no changes exist to 11-bus system, the total generation cost in 30-bus systems (including 14-bus system, not shown here) decreases simply with the increase of voltage of balance node. Concretely, in 30-bus system, the optimal generation cost decreases from 807.1682 ($V_s=0.98$) to 792.9889 ($V_s=1.09$), the absolute difference is 14.18 \$/hr that is about 1.79% of the total generation cost when $V_s=1.09$). Provided the situation does not change, the total difference of annual generation costs will be up to $\$124.21 \times 10^3$ merely from the change of V_s . To a greater system, this benefit should not be neglected. Evidently, in the viewpoint of pure economy, the system should run at the position with as higher voltage of balance node as possible. Generally, if a system has enough reserve in voltage stability and system security, this purpose is not difficult to be completed.

In conclusion, under the condition of system security, if operational mode of system can be determined by optimization tool and the voltage of balance node could be kept at a higher value, the benefits will reach maximum. This question is worthy to be considered and further researched in a wider field, especially in the situation when the objective of OPF problem is minimum cost of electricity-purchasing.

4.2. Impacts of slack-bus voltage on MW losses

The curves in Fig.1 shows the change of active power losses over voltage of balance node in 11, 14 and 30-bus systems. Two interesting phenomena can be seen. In one hand, the larger the system's scale is, the greater its active power losses. This is in

accord with what imagined. In the other hand, MW losses of system appear to simply and consistently decrease, the trend accords with the changes of total generation cost. Clearly, the voltage of balance node would affect the MW losses, especially to the large system. For example, MW loss of 14-bus system would reduce from 4.465743 to 3.699021MW, having a difference of 0.766722 MW; to 30-bus system, from 10.598491 to 8.867793 MW, with a difference of 1.730698 MW, dropping 16.33%. To the latter situation, if the price of electricity is 250 \$/MWh, the benefit from reduction of MW losses in one year will be up to 3.79×10^6 .

It is obvious that the right choice of voltage of balance node can reduce MW losses and operation cost in a large system, and bring electricity market extra benefits.

4.3. Impacts of slack-bus voltage on MVAR losses

Fig.2 illustrates the change of MVAR losses over V_s in 11, 14 and 30-bus systems. Other than MW losses of system that are certainly positive, the losses of reactive power might be negative in some networks. It is just the case in 14-bus system. As the voltage of balance node goes up, the absolute value of MVAR losses in 14-bus system increases too. The main reasons for this phenomenon include much compensating capacity, large charging power of lines to earth, and lower MVAR loads. In this system, the voltage level of whole system is usually high toward to up limit. It must be paid attention by system operator or market operator. Although MVAR pricing has not been brought into practice up to now in China, the overall and systematic scheme and planning of voltage stability and system security should be considered. On this basis, the value or the range of voltage of balance node in normal state of system can be rightly determined.

In contrast with 14-bus system, the MVAR losses in 30-bus system are positive and obviously decrease with increasing of voltage of balance node. It is because the flow of MVAR in transmission lines near balance node

becomes smaller. It can be seen that, in the same time, the impact of the voltage of balance node on MVAR losses is much greater than on MW losses. For an example of 30-bus system, the maximum difference is up to $(14.324971 - 3.202050) = 11.122921$ MVAR, with a drop of 77.65%. Therefore, it is advantageous to maintain a higher voltage of balance node.

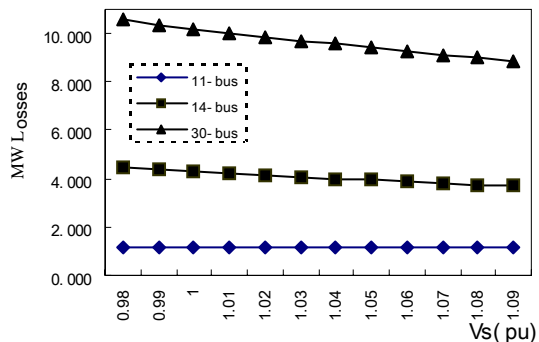


Fig.1 MW losses in three test systems

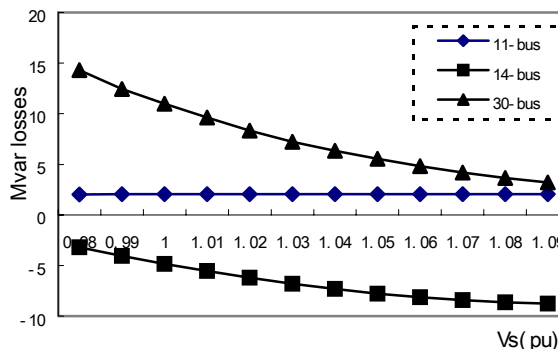


Fig.2 MVAR losses in three test systems

5. Conclusions

Economy plays a decisive role in electricity market. No matter what value is the voltage of balance node or what scale a system, application of optimizing method can reduce both the total generation cost and MW losses of the whole system, and achieve ideal benefits. This is an important conclusion to the competitive electricity market.

Impact of the voltage of balance node on MVAR losses is much greater than on MW losses. The system with positive MVAR losses should run at the position with as higher voltage of balance node as possible in order to get maximum benefit. The larger a system is, the more the benefit from that. To the system with negative MVAR losses, the voltage level of whole system is usually high toward to up

limit. It must be paid more attention by system operator or market operator.

In conclusion, the voltage of balance node has wide impacts on the total generation cost, and losses of active power or reactive power. It is an open subject and worthy to be further researched in a wider field.

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