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Dynamical Behaviour of Funicular Railways: a First Approach

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Abstract - The research mainly aims to study the dynamical behaviour of the railroad cars motion when these last are moved by a wire, during the start and stop transients.
In this first step the mathematical model and the equation of motion were defined; by means of MatLab code the laws of motion were computed for both the cars and the winch, supposing that on this last acts a moment that increases (decreases) linearly with time during the start (stop) transients.
The first computed results show that non negligible railroad cars oscillations occur during the start and stop transients, as it is observed in existing funicular railways.

Key-Words: - Transients dynamics, funicular railways, non constant coefficient systems

1 Itroduction

It is well known that in all those systems in which a mass is moved from an actuator through a non rigid link, the mass itself doesn't move with the same law of motion of the actuator as relative motions between actuator and mass occur. In some case, due to dynamical effects, these relative motions can reach significant amplitudes. This occurs all the times in which the transmission between an actuator and the mechanical part that receives the motion can not be assumed as "rigid"; in this case the mass law of motion can be very different from the planned one.
In particular, when a car is moved by a wire (e.g. funicular railways, cableways, elevators etc.), because of the elasticity of the wire the car itself will not move as the winch moves. What take place, instead, mainly during the start and stop transients, are the non negligible cars oscillations.
These oscillations are obviously undesirable as they are uncomfortable for passengers and can give trouble to the transmission and to other mechanical parts (i.e. backlash, fatigue etc.)
In order to reduce significantly the vibrations of non rigid mechanical system, several techniques have been proposed and developed (see e.g. [1-6]) ; in any case it is necessary to start from the knowledge of a suitable mathematical model of the system and its dynamical behaviour.
For the reasons above, it seems to us interesting to start investigations on the dynamical behaviour of railroad cars moved by means of a steel wire.
This would result in a first step to find adequate motor laws of motion for the motor to consent the reduction of the undesired railroad cars oscillations that occur during the start and stop transients.

2 The mathematical model

These initial investigations were conducted on a simple 3 d.o.f. damped model. The model is shown in fig.1

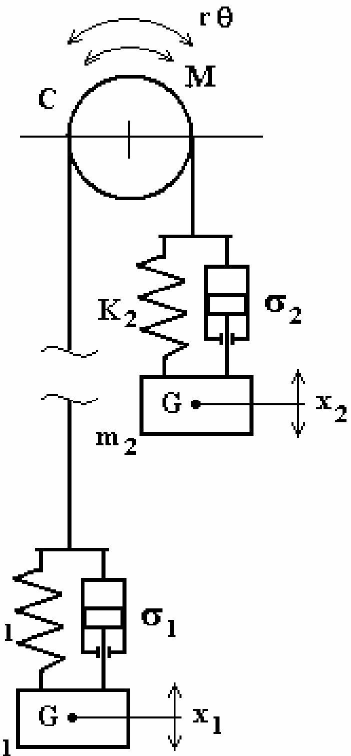

Fig.1 – Scheme of the system

The motor moves a capstan C that by means of a transmission that is supposed "rigid"; consequently the capstan pulleys will move with the law of motion given by the motor (eventually linked through a gearbox). Both the railroad cars are moved by steel wires, having internal damping, whose coefficients of stiffness change, during the motion, as the wire length changes. The railroad cars are supposed to be rigid (as the stiffness is considered in the wire) and their mass is the car mass plus the passengers mass and one half of the wire mass; as the wire length changes during the motion, also the car mass is non constant.
The friction between rails and wheels is considered as a constant force whose sign is opposite to the car speed sign; the aerodynamic friction has been assumed to be negligible.
What reported above allows to obtain the following equation of motion.

$$\begin{cases} \ddot{x}_1 = \dfrac{EA}{m_I}\dfrac{R\theta - x_1}{L_{10} - x_1} - f_{1C}\, g\cos\alpha\, sign(\dot{x}_1) + \\ .+\dfrac{\sigma_{S1}}{m_I}\left(R\dot{\theta} - \dot{x}_1\right) - \dfrac{\sigma_{S1}}{m_I}\dot{x}_1 \\ \\ \ddot{x}_2 = \dfrac{EA}{m_{II}}\dfrac{R\theta - x_2}{L_{20} + x_2} - f_{2C}\, g\cos\alpha\, sign(\dot{x}_2) + \\ +\dfrac{\sigma_{S2}}{m_{II}}\left(R\dot{\theta} - \dot{x}_2\right) - \dfrac{\sigma_{S2}}{m_{II}}\dot{x}_2 \\ \\ \ddot{\theta} = \dfrac{\Delta M}{I} + \dfrac{EAR}{I}\left(\dfrac{x_1 - R\theta}{L_{10} - x_1} + \dfrac{x_2 - R\theta}{L_{20} + x_2}\right) + \\ +\dfrac{R}{I}\left[\sigma_{S1}\left(\dot{x}_1 - R\dot{\theta}\right) + \sigma_{S2}\left(\dot{x}_2 - R\dot{\theta}\right)\right] \end{cases} \quad (1)$$

where:

$$\begin{cases} m_{1f} = \rho A\left(L_{10} - x_1\right) \\ m_{2f} = \rho A\left(L_{20} + x_2\right) \\ m_f = m_{1f} + m_{2f} \\ m_I = m_C + m_{1P} + \dfrac{1}{2}m_{1f} \\ m_{II} = m_C + m_{2P} + \dfrac{1}{2}m_{2f} \\ I = I_0 + \dfrac{1}{2}m_f R^2 \end{cases}$$

The meaning and the values of the quantities in eq. (1) are reported in the appendix.
In the equations above, the d'Alembert dynamics has been employed as, in our opinion, in this way each of the terms of the equations has more physical evidence.
As in eq. (1) masses and stiffness coefficients are non constant, the equations are non constant coefficient derivative equations, hence the system own frequencies change with the car positions.
In fig 2 are reported the eigenvalues of the system ω_2 and ω_3 (for the system considered in fig.1 it is always $\omega_1 = 0$), computed during the first 20 seconds of the car run that will be considered.

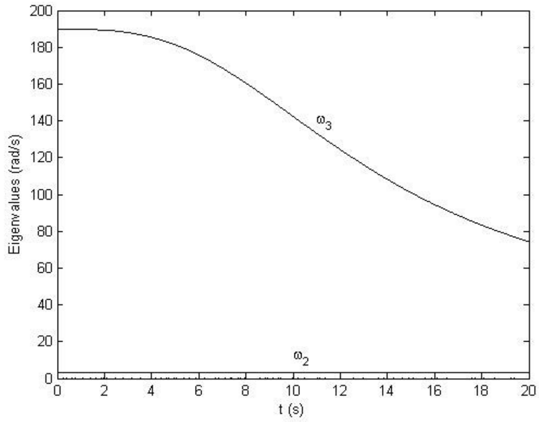

Fig. 2 – System eigenvalues

It must be observed that, during the run, the wire length's changes in percentage (hence the stiffness) is significant for the wire of car 2 while is negligible for that of car 1 ; thus one of the own frequency is quite constant while the other changes significantly.

3 Computed results

The equations of motion have been solved by means of MatLab code, by using the function ODE45 of the II order Runge Kutta algorithm with variable step of integration. The maximum absolute error (AbsToo) and the relative one (RelToo) didn't exceed 10^{-4}; for each step of the integration, the error e(i) for each of the components y(i) of the solution vector y satisfies the condition:

|e(i)|<=max[RelTol*abs(y(i)),AbsTol(i)]

As for the dynamical behaviour of the cars during the starting (and stop) transients, the computed results have been obtained considering an existing funicular railway; the data (i.e. masses, lengths, stiffness, damping etc.) are reported in the appendix.

3.1 – Start transient

As for the start transient, it was supposed that on the winch acts a constant moment that grants the equilibrium of the cars and a variable moment that linearly increases from 0 Nm to $1.5 \cdot 10^5$ Nm within a time of 20 seconds as shown in fig 3.

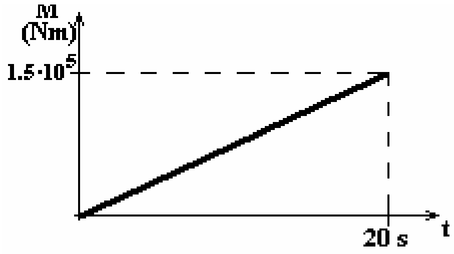

Fig.3 – Moment versus time

In the figures 4a, 4b and 4c are reported the laws of motion (respectively: rotation, angular velocity and acceleration) of the winch. The solid lines refer to the damped system with non rigid wire and, for comparison are also reported the behaviours with non damped and non rigid wire (dashed lines) and, also, for comparison, the behaviour with rigid wire (dashed-dotted lines).

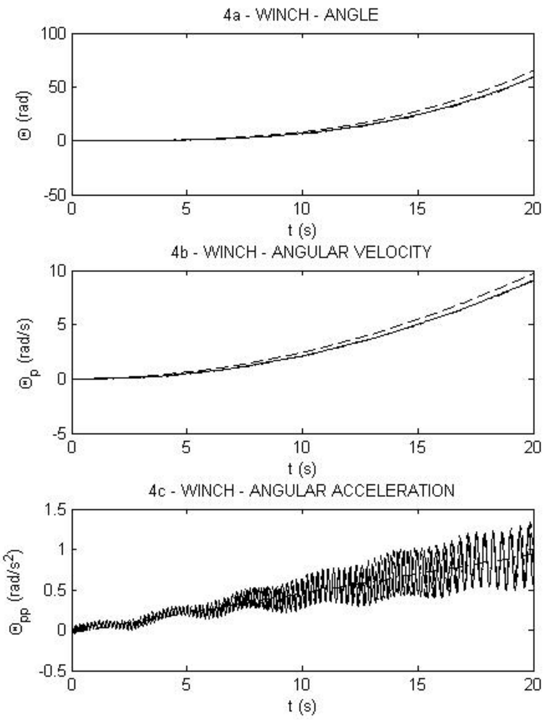

Fig.4 – Winch law of motion, start

In the figures 5a, 5b and 5c are reported the car 1 laws of motion (respectively: displacement, velocity and acceleration) as it was made for the winch.

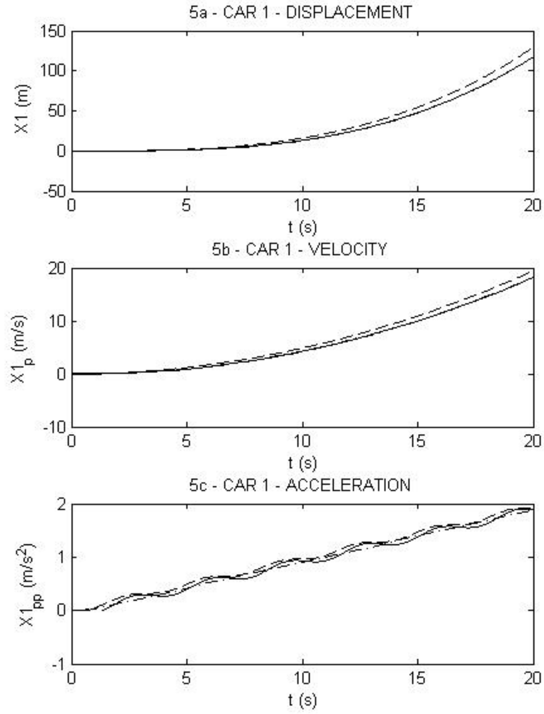

Fig.5 – Car 1 law of motion, start

In the figures 6a, 6b and 6c are reported car 2 laws of motion.

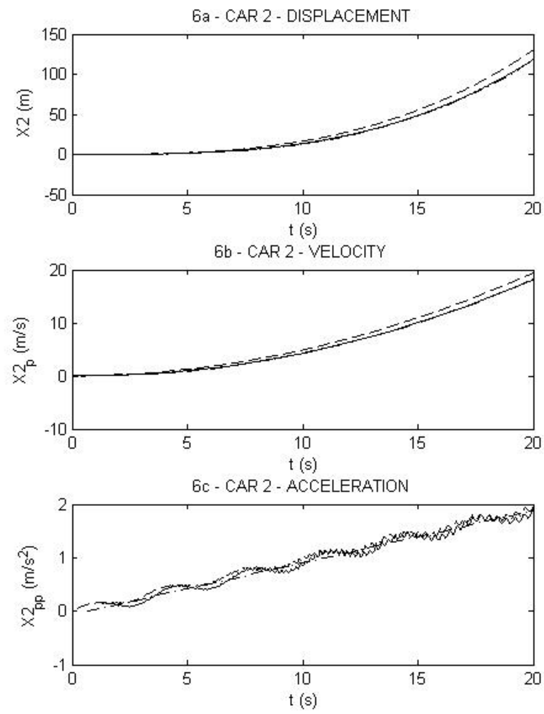

Fig.6 – Car 2 law of motion, start

From the figs above, it can be observed what follows:

- The cars laws of motion significantly differ from the planned ones, as oscillations with significant amplitude take place. In the scale adopted for the diagrams, this phenomenon is particularly evident if velocity and acceleration are considered. These can probably be non acceptable for the passengers.
- The capstan laws of motion also much differ from the expected ones, as high amplitude accelerations occur. This phenomenon can cause undesirable effects on the gears and on the transmission between motor and capstan
- The damping seems to have poor effects on both the cars and capstan behaviour.

3.1 – Stop transient

In order to compute the stop transient, it was considered that at t=0 both the cars move with the initial conditions (displacement and velocity) that were computed after the (20 seconds long) start transient; with this initial conditions, a moment, opposite to the motion, and decreasing linearly from 10^5 Nm to 0 Nm within a time of 20 seconds, is applied to the winch.

In fig.7 are reported the winch rotation, angular velocity and acceleration during the stop transient. The lines have the same meaning of those of the previous figures.

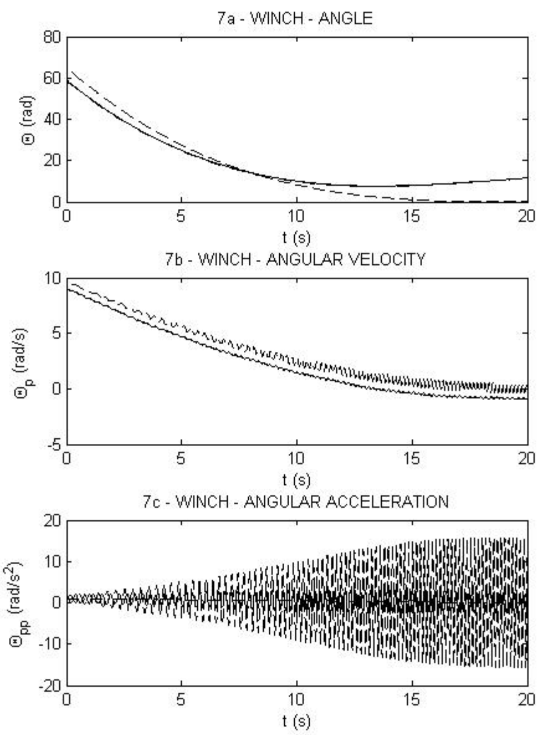

Fig.7 – Winch law of motion, stop

In fig.8 are reported car 1 displacement, velocity and acceleration.

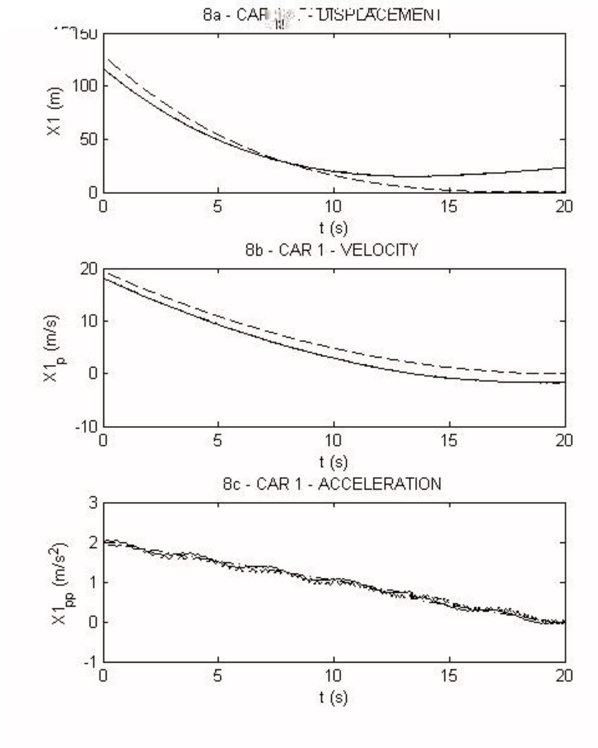

Fig.8 – Car 1 law of motion, stop

In fig.9 are reported car 2 displacement, velocity and acceleration.

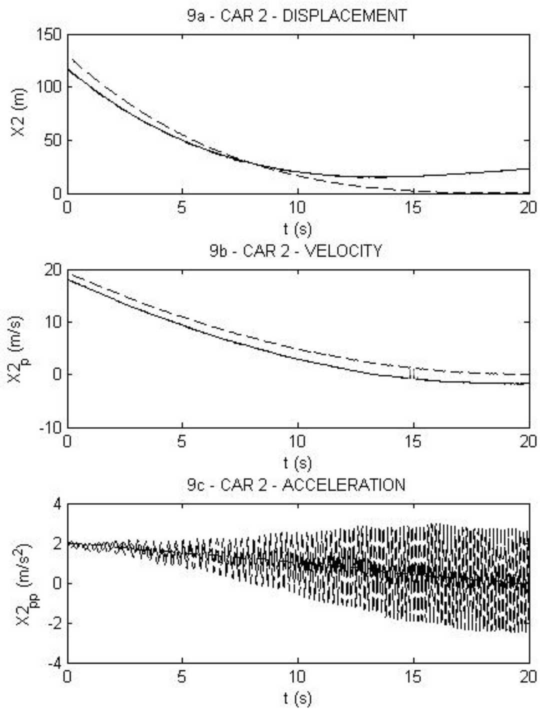

Fig.9 – Car 2 law of motion, stop

As it can be observed, from a qualitative point of view, the system behaviour during the stop transient doesn't differ to a great extent from the one observed during the start transient.

As for the displacement it can be observed that, if the wire damping is considered, the differences from the undamped behaviour are somehow more evident than those observed during the stop transient.
Also as far as the accelerations are concerned, it can be observed that the oscillations during the stop transient are higher than those during the start transients. This is in agreement wit what occurs in existing systems and suggests that the most important efforts for oscillation reduction must be done during the stop transients.

Il all the considered cases (both during start and stop transient) it can be observed that the dynamical behaviour is essentially composed by two components: a low frequency component (which period is about 3.2 s) and an higher frequency one whose period is about 0.16 s at the beginning of the transient and about 0,39 s at the end of the transient; this is particularly evident if the diagrams of the acceleration are considered. So: while the frequency of the first one is almost constant, the frequency of the second decreases when the time is increased.
What above said is confirmed by a Fast Fourier Transform. In fig.10 is reported the FFT computed for the winch acceleration that is shown in Fig.4c.

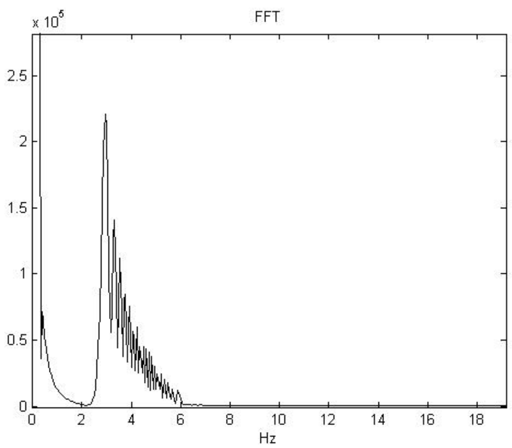

Fig.10 – FFT of the winch acceleration

As it can be observed from the figure above, the FFT shows the presence of a frequency of about 0.3 Hz ($T \cong 3.2$ s) and frequencies ranging from 2.5 Hz to 6.2 Hz ($0.4 \leq T \leq 0.16$ s).
This aspect can be connected with the eigenvalues of the system that are reported in fig 2: as already observed, during the run (due to the changes of the wire length and stiffness) one of the own frequency of the system decreases while the other is about constant.

4 Conclusions

A first study on a non constant coefficient model of a mechanical system has been proposed. The main aim was to examine the dynamical behaviour of cable-railway systems, such as funicular railways, cableways, elevators etc.; this in order to investigate on the possibility to compute winch (or more generally: actuator) laws of motion that can reduce the car oscillations during the start and stop transients.
These first results seem to agree, from a qualitative point of view with the behaviour that can be experimentally observed. Non negligible railroad cars oscillations occur during the start and stop transients; this can be unacceptable for the passengers comfort and can cause troubles to the system. To avoid this undesirable effects, it is useful to investigate on the possibility to compute a suitable law of the moment (versus time) that reduce these oscillations and accelerations.

Appendix

In Tab.1 the meaning and the amounts of the quantities in eq. (1) are reported.

Table 1

$\mathbf{X_1}$	Car 1 position	m
$\mathbf{X_2}$	Car 2 position	m
$\mathbf{\Theta}$	winch position	rad
$\mathbf{m_C}$	Car mass	20.000 kg
$\mathbf{m_{1P}}$	Car1 passeng. mass	20.000 kg
$\mathbf{m_{2P}}$	Car2 passeng. mass	10.000 kg
$\mathbf{m_{1f}}$	Car1 wire mass	variable
$\mathbf{m_{2f}}$	Car2 wire mass	variable
$\mathbf{m_f}$	Wire mass	5.052 kg
$\boldsymbol{\rho}$	Wire density	7.800 kg/mc
A	Wire section	$6.35 \cdot 10^{-4}$ m^2
$\mathbf{L_{10}}$	Wire 1 lenght at t=0	1000 m
$\mathbf{L_{20}}$	Wire 2 lenght at t=0	20 m
E	Wire elasticity	$1{,}1 \cdot 10^{11}$ N/m^2
α	Rail inclination	70 °
R	Winch radius	2 m
$\mathbf{f_{1C}}$	Friction coefficient	0.02
$\mathbf{f_{2C}}$	Friction coeff.	0.02
σ_{S1}	Damping coefficient	10 Ns/m
σ_{S2}	Damping coefficient	10 Ns/m
$\mathbf{I_0}$	Winch mass moment of inertia	400 kg m^2
g	gravity	9,81 m/s^2
$\mathbf{\Delta M}$	Variable moment	

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