# **A Study of Effect of Different Static Load Models and System Operating Constrains on Static Voltage Stability**

CHENG-HONG GU, QIAN AI, JIAYI WU Department of Electrical Engineering Shanghai Jiao Tong University 1954, HuaShan, Road , Shanghai, 200030 **CHINA** 

*Abstract:* This paper uses continuation power flow to simulate P-V curves to analyze the effects of different load model and system operating constrains on power system static voltage stability. It is found that the critical points with ZIP and exponential function load model are relatively similar. While to restrict generators' reactive power output, it would be unfavorable for voltage stability. Transmission line active and apparent power limits have quite similarly influences on voltage stability. But current limits are different, which are much serious. In addition, nodal voltage amplitude low limits are better among 0.90-0.95p.u and upper limits almost have no influences.

*Key-Words:* power systems; voltage stability; CPF; static load model; operating constrains

# **1 Introduction**

During the past few years, lots of reports about voltage instability have appeared in different countries. Thus studies on more precise and powerful tools and models for large power system voltage stability analysis are quite monumental[1, 2]. The continuation power flow (CPF), which was presented in 1992, remains well- conditioned at and around the critical point [3]. Continuation method is a kind of techniques to trace nonlinear dynamic system solution locus at equilibrium points [4]. It renews equations of power flow to make them be convergent in all possible load situations. So divergence due to ill conditioning is not encountered at the critical point [5,6]. Because of the practicability and robustness of its models, numerous studies are done on CPF and it becomes one of the basic computing tools in Energy Management System (EMS). Lots of different CPF appear lately. Paper [7] presents a new kind of CPF for investigating the nonlinear effects of varying multiple branch parameters. And according to paper [7], paper [8] also presents a new contingency parameterization continuation power flow model for faulty steady analysis.

The accuracy and correctness of the results of voltage stability are directly related to the load models used in analysis [9]. The paper [10] analyzes effects of different static characteristics of load models on system static voltage stability and presents a practical criterion and voltage operating conditions considering load static voltage characteristics. Paper [11] summarizes the usage of different load models in voltage stability studies, introducing present situation of load models.

Besides, there are lots of system operating constrains, greatly affecting system voltage stability. Generator reactive power output has its limits. Considering loads stability and their longevity and transmission lines heat limits and stable limits, nodal voltage amplitudes, transmission line power flow also have their limits [12]. However, there are few systematic studies about how power system operating limits affect static voltage stability, since many papers only takes one or two limits into consideration while analyzing. Paper [13] finds that when a generator of a heavily loaded electric power system reaches a reactive power limit, the system can become immediately unstable and a dynamic voltage collapse leading to blackout may follow.

This paper uses CPF to simulate system's P-V curves to analyze the effects of different static load models and system operating constrains on system static voltage stability. It is found that, by using ZIP and exponential load models, the critical points are quite similar. Besides, the basic theory used in case analysis is also deduced. Through simulation, it is

clear that generators' reactive output limit weakens voltage stability. Transmission line active and apparent power limits similarly affect voltage stability, but current limit is different and much serious. In addition, considering load stability and its longevity and social benefits, nodal voltage amplitude low limits between 0.90-0.95p.u.are satisfying. But upper limits almost have no influences at all.

## **2 Static Load Models** [1]

Different load models would greatly affect power voltage stability analysis. In this paper, two specific static load models are studied.

#### **2.1 Exponential Load Model**

exponential fuction model of load is depicted by equation (1) and (2), which are for active and reactive power respectively.

$$
P = P_0 \left(\frac{U}{U_0}\right)^{\alpha}
$$
  
\n
$$
Q = Q_0 \left(\frac{U}{U_0}\right)^{\beta}
$$
\n(1)

Where:  $U_0$  is referenced or rated voltage,  $P_0$ ,  $Q_0$  are powers consumed in rated voltage and indexes  $α$ ,  $β$ change according to different load types.

The following three load models are vital: constant impedance model (Z Load), when  $\alpha = \beta = 2$ ; constant current model (I Load), when  $\alpha = \beta = 1$ ; and constant power model (P Load), when  $α=β=0$ .

#### **2.2 ZIP Load Model**

ZIP load model is composed of three different parts: constant impedance model, current model, and power model. Its active and reactive power models are given below:

$$
P = P_0[a_P(\frac{U}{U_0})^2 + b_P(\frac{U}{U_0}) + c_P]
$$
 (3)

$$
Q = Q_0 [a_Q(\frac{U}{U_0})^2 + b_Q(\frac{U}{U_0}) + c_Q]
$$
 (4)

Where  $Q_0$ ,  $P_0$ are power consumed by load at referent voltage, then we can get the following equations:

$$
a_p + b_p + c_p = a_Q + b_Q + c_Q = 1
$$
 (5)

Through changing the coefficients, we may realize a host of ZIP loads.

#### **3 Theory analysis** [12][14]

It is knew to all that system operating voltage is

decided according to reactive power balance theory and transmission line end voltage is much concerned with line power flow and its parameters. In this section, we use basic theory to display and deduce some useful equations for case analysis.

#### **3.1 Reactive Power Balance Theory** [14]

System operating voltage depends on reactive balance. If the output of reactive sources can not satisfy the load demand and the loss, voltage may decrease. From the power equations of non salient pole generator, equation (6) can be obtained, which is the reactive characteristic equation of non salient pole generator. 2

$$
Q = \sqrt{\left(\frac{EV}{X}\right)^2 - P^2} - \frac{V^2}{X}
$$
 (6)

where, *P*, active power, *Q*, reactive power, *E*, potential behind *X*, *X*, reactance of the line.



 Fig1. Decide Operating Voltage Using Power Balance

Usually, load demand for reactive power is much related to voltage level, the higher the voltage is, the more the demand. Plot the load and generator reactive power characteristic curves based on equation and load characteristics together. According to reactive power balance between load and generator principle, we can decide the system operating voltage, which is the intersection point of the two curves.

Apparently, once load demand increases, its characteristic curve will move upward; demand decrease, and its curve will move downward. It is also true of generator reactive power output. Consequently, system operating voltage changes with the movement of intersection points.

#### **3.2 Equivalent Circuit Voltage computation**

Let us pay attention to the simple circuit in Fig.2. Here, *R*, resistance, *S*, apparent power, *I*, current, *X* and *V* are the same as those in equation (6).



Fig2. Simple Equivalent Circuit

Using vector theory in steady state, replacing current by power, and supposing the head end voltage is knew, we will obtain (7) and (8).

$$
\Delta V_1 = \frac{P_1 R + Q_1 X}{V_1}
$$
\n
$$
\delta V_1 = \frac{P_1 X - Q_1 R}{V_1}
$$
\n(3)

So the voltage of tail end can be deduced, which is depicted by equation (9)

1

$$
V_2 = \sqrt{(V_1 - \Delta V_1)^2 + (\delta V_1)^2}
$$
 (9)

As it is knew to all, in high voltage power system, R<<X. Therefore, R can be considered as 0. Then take  $(7)$  and  $(8)$  into  $(9)$ , and set R=0 and the next equation (10) can be got:

$$
V_2 = \sqrt{V_1^2 + \frac{(P_1^2 + Q_1^2)X^2}{V_1^2} - 2Q_1X}
$$
 (10)

If replace current by power indexes, then we finally obtain equation (11):

$$
V_2 = \sqrt{V_1^2 + I_1^2 X^2 - 2Q_1 X}
$$
 (11)

In the following sections, it will be found that equations  $(7)$   $\sim$  (11) are quite useful for case analysis.

#### **4 Case Analysis**

In this paper, we use IEEE 39-bus system as an example to analyze its voltage stability in different load model and different system operating constrain conditions. CPF parameterized method is perpendicular intersection, step is 0.5, and maximal iteration is 50. Once maximal iteration is over50 and error is bigger than set value, we think the algorithm is divergent.

The following load increasing mode is used to make system reach its limitation: increasing active and reactive load power simultaneously, with power factor unchanged, as follows:

$$
P_G = P_{G0} + (\lambda + \gamma k_G) \tag{12}
$$

$$
P_L = P_{L0} + \lambda P_D \tag{13}
$$

$$
Q_L = Q_{L0} + \lambda Q_D \tag{14}
$$

Where:  $P_{G0}$ ,  $P_{L0}$ ,  $Q_{L0}$  are base generator and load powers  $P_s$ ,  $P_p$ ,  $Q_p$  are the generator and load power directions;  $\lambda$  is loading parameter



Fig.3. IEEE 39-Bus System Diagram

## **4.1 Different Static Load Models' Effect on Voltage Stability**

Use constant impedance, current, and power loads and  $Z\overline{IP}$ <sup>1</sup>loads to tie to bus 4 # respectively, to see how they affect voltage stability. Then Fig.4 is illustrated to reflect how bus 4# voltage changes, with loading parameter  $\lambda$  changing.



1: P Load; 2: I Load; 3: ZLoad; 4: ZIP Load 1; 5: ZIP Load 2

Fig.4. Nose-Curves Under Different Load Models

Table1. Maximal λ Under Different Load Models

	Z Load	I Load	P Load	ZIPL <sub>oad</sub> 1	ZIP Load2
	2.4885	2.4723	2.4398	2.5362	2.5531
$\tau$ ime	14.751	22.733	12.769	12.979	11.907

From Fig.4 and table1, it can be seen that upper sections of nose-curve are quite similar for all load models. It means when system structure and load increasing modes are decided, voltage critical points are analogous. However, we should notice that ZIP load' s loading parameter  $\lambda$  is the biggest; P load's is the smallest, Z load's and I Load's are intermediary. That means critical point of exponential load model is much related to the model's indexes: the bigger the indexes are, the bigger the critical points would get.

On the other hand, when time consumption of simulation is taken into account, apparent differences appear and would jave great influence on voltage stability on-line analysis. Time used by I Load model is most and that of ZIP load model is least. Otherwise, ZIP load model can be considered as the integration of Z load, I Load and P Load, thus ZIP load model should earn much applicaton and is quite consistent with actual situations. So, it is superior to exponential load model.

## **4.2 Different System Operating Limits' Effect on Voltage Stability**

Usually, nodal voltage amplitude, generator output and line power flow all have their limits, which greatly influence on voltage stability. In this section, all of them will be considered.

## **4.2.1 Limits of Reactive Power Output of Generator**

Change reactive power output of generator 35# connected to bus 22# and the following table2 is obtained, where  $\lambda$  is bus 22# critical loading parameter.

## Table2. MaximalλUnder Different Reactive Power Output Limits



When the upper limit of reactive power output of generator 35  $(Q<sub>gMax</sub>)$  is set as 13 per unit (p.u), it begins to work and consequently, system value of  $\lambda$ at critical points decreases. Also, the smaller the upper limits become, the smaller critical value of  $\lambda$ grows. We can see from the Fig.1 that when generator reactive power output is set as a constant, its reactive power characteristic curve would stay still; while as for load characteristic curve, once load reactive power demand increases, its position will move form L1 to L2, which leads to the decrease of system operating voltage and consequently its critical point. That means nodal voltage amplitude would decrease and even reach critical points if there is no surplus reactive power support while reactive power demands continue to increase.

Therefore once nose-curve is used, reactive power output of generator has to be taken into consideration. Only in this way can it reflect the real situation.

maximal loading parameter  $\lambda$  is obtained:

### **4.2.2 Limits of Transmission Line Power Flow**  Next, to change upper limits of current, active power and apparent power on transmission line between bus7# and bus 8#, the following bus 7#

Table3. Maximal λ Under Different Current Limits

$I_{MAX}$		6.5				
	2.2109	2.2115	2.212	2.2134	2.2128	2.2116
	د. ۷					
	2.2133	2.2118				

Table4.Maximal  $\lambda$  Under Different Active Power Limits

$P_{MAX}$	4.3		3.5		2.8	2.5
	2.2109	2.2129	2.2134	2.2135	2.2114	2.2128
	2.3					
	2.2136	2.2120				

Table5.Maximal λ Under Different Apparent Power Limits



By using data in the above three tables and making use of curve fitting, three different curves in Fig.5 are obtained:





Fig.5. Fitting Curve of  $\lambda$  Under Different Limits

From above three tables and Fig.5, it is evident that when the line current upper limit  $(I_{MAX})$  is set to 7p.u. it begin to affect system voltage stability. As for active power limits  $(P_{MAX})$  and apparent power limits  $(S_{MAX})$ , they are round 4p.u and 4.3p.u. Thus, line

power flow limits might heavily influence voltage stability. And considering Fig.5, the critical points appear irregular oscillation. From curves 2 and 3, we find that how they affect voltage stability is quite similar, especially when upper limits are relatively small. Great difference appears when upper limit is big. As to current limits, the curve is different with the other two curves and it is comparatively flat, which means whatever the upper limit is big or small, they might similarly affect voltage stability.

Since we can learn from equations (10) and (11) that voltage of transmission line tail end is much concerned with line reactance *X*, head end voltage and power. When line power flow limits, current, active and apparent power limits are taken into consideration, the amplitude of  $V_2$  decreases. But the last two situations are quite similar, since in the second term of equation (10),  $X<1$  and  $O\ll P\le S$  for a transmission line. But as for the first situation, usually  $V \neq 1$  computed by power flow. So it is different form the others. Therefore, different conclusions may be drawn by using different constrains.

Here there are some reasons for oscillation. In mostly of situations, transmission line power flow limits may deteriorate system voltage stability. However IEEE 39-bus New England system is a multiple-generator power system, when transmission line power flow has limits, the system's power flow might change, and is no longer the natural one. As a result, once any area lacks of reactive power and it can not get ample reactive power support because of line power limit, critical loading parameter will decrease and critical point move left. On the other hand, if it can absorb reactive power support, nodal voltage amplitude can keep at high level and critical point also is bettered.

#### **4.2.3 Limit of Nodal Voltage Amplitude**

As we know, to sustain system stability and load stability, nodal voltage amplitude has to keep in a certain range. First, considering nodal voltage amplitude lower limit  $(V_{MIN})$ , the results are illustrated in table 6, that is the maximal loading parameters of bus 4.

Table 6. MaximalλUnder Different Nodal Voltage Limit

$\rm{V_{MIN}}$	0.97	0.96	0.95	0.94	0.93
	2.2118	2.2108	2.2134	2.2133	2.2131
	0.92	0.91	0.90		
	2.2122	2.2133	2.2132		

It is found that with nodal voltage amplitude lower limits varying, system critical points also change. When  $V_{MIN} \in [0.95, 0.90]$ , system voltage critical will move right, which means system voltage stability is bettered. As we can learn from Fig.1 that while loading nodal voltage amplitude declines, its characteristic will move downward and its reactive load demand also declines, with transmission line loss decreased. So system voltage amplitude will rise and its critical points also move right. Consequently, voltage lower limits in this range are much better for voltage stability. When it comes to upper limits, conclusions are changed. Changes of upper limits almost have no effect on voltage stability.

## **5 Conclusions**

From above-mentioned analysis, the critical points got using ZIP and exponential load model are quite similar, comparatively ZIP's closer right. Additionally, ZIP load model can be considered as the integration of Z load, I Load and P Load and from time-saving view, ZIP load model is much profitable.

While generators' reactive output is restricted in a certain range, it would be unfavorable for voltage stability, because of lacking reactive power. Moreover, we can see from the three tables and the curves got by fitting of curve, transmission line active and apparent power limit quite similarly affect voltage stability but current limit is different from the former two situations, which is much serious. Besides, the critical points appear irregular oscillation because of reactive power flowing in different areas.

While nodal voltage amplitude limits are taken into account, conclusions are quite different: nodal voltage amplitude low limits between 0.90-0.95p.u. are profitable and upper limits almost do not affect voltage stability. However, they have to be in a certain range for load stability and longevity favors.

*References*

- [1] ZHOU Shuang-xi, ZHU Zhi-ling, GUO Xi-jiu, WANG Xiao-hai. *Power System Voltage Stability and Its Control.* Beijing: China Electric Power Press. 2004.
- [2] Carson W. Taylor. *Power System Voltage Stability.* New York: McGraw-Hill. Inc.,1994.
- [3] Venkataramana, Ajjarapu, Colin Christy .The Continuation Power Flow: A Tool for Steady

State Voltage Stability Analysis. *IEEE, Trans on Power System,* 7(1),1992: 416-423.

- [4] SEYDEL R. *Practical Bifurcation and Stability Analysis-From Equilibrium to Chaos.* 2nd ed. New York: Spring-Verlag,1994.
- [5]ZHAO Jin-quan, ZHANG Bo-min. Summarization of Continuation Power Flow and Its Applications in Static Stability Analysis of Power System. *Automation of Electric Power Systems.* 2005,29(11):91-97.
- [6] FENG Z H, AJJARPU V, LONG B. Identification of Voltage Collapse Through Direct Equilibrium Tracing. *IEEE Trans on Power System,* 2000, 15(1):342-395.
- [7] FLUECK A J, DONDETIJ R. A New Continuation Power Flow Tool for Investigating the Nonlinear Effects of Transmission Branch Parameter Variation. *IEEE Tran on Power System ,*2000,15(1):223-227.
- [8] ZHAO Jin-quan, CHIANG Hsiao-dong, ZHANG Bo-ming. A New Contingency Parameterization Continuation Power Flow Model for Steady Stability Analysis. *Automation of Electrical Power Systems,* 2004, 28(14): 45-49
- [9] AN Wen-xia, CHENG Yun-ping, SHEN Zu-yi. A Survey of Voltage Stability Studies. *Power System Technology.* 2001,25(9).
- [10] Li Xin-ran, HE Ren-mu, ZHANG Jian. Effect of Load Characteristics on Power System Voltage Stability and The Practical Criterion of Voltage Stability. *Proceedings of the CSEE*.1999,19(4):23-30.
- [11] Li Xin-ran, CHEN Xin-yuan. Power Load Model and Its Modeling Method For Voltage Stability Analysis. *Proceedings of the EPSA.*  2000,12(6): 9-13.
- [12] FAN Xi-pu. *Electric Engineering in Power Plant.* Beijing: China Electric Power Press. 1995.
- [13] DOBSON I, LU L M. Voltage Collapse Precipitated by the Immediate Change in Stability when Generator Reactive Power Limits are Encountered. *IEEE Trans on Circuit and System :Fundamental Theory and Applications,*  1992,39(9):762-766.
- [14] HE Yang-zan, WEN Zen-yin, WANG Fu-ying, ZHOU Qin-hui. *Powe System Analysis.* Wuhan:Huazhong university of science and technology press.1996.