# Law of Motion Influence on the Start-Stop Transients of Funicular Railways

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*Abstract* - The main aim of this research is to analyse the influence of the actuator's law of motion on the railroad cars motion when these last are moved by a wire.

The first results show that, during the start and stop transients, the moment versus time law (hence the shape of the law of motion given by the winch) significantly affects the railroad cars motion. This suggests that, by means of a suitable actuator's law of motion, the railroad cars oscillations occurring during the start and stop transients can be possible significantly reduced.

Key-Words: - System dynamics, funicular railways, laws of motion influence

### **1** Introduction

As it was observed in many investigations, if a mass is moved from an actuator through a non rigid link, the mass itself doesn't move according to the same law of motion of the actuator and relative motions between actuator and mass occur. Hence the mass law of motion can be very different from the planned one.

Such oscillations and the (variable) inertia forces caused by the accelerations of the system components can cause several undesirable effects as backlash in the transmission components, fatigue phenomena and, in funicular elevators and cable-railways, lack of the passengers comfort.

Several techniques have been developed in order to obtain a reference signal, to control the servomotors, that can reduce significantly the vibrations of this non rigid mechanical system.

In [1] a pulse shaped technique is employed, based on a short finite Fourier series expression for the forcing function. The command signal is shaped by selecting the coefficient in a sine series definition of a forcing function.

In [2] the end point vibrations of a flexible system are reduced by means of shaped command inputs.

In [3, 4] the suppression of the vibration is obtained by a train of impulses that modifies the command signal.

In [5,6] a technique is proposed to compute the appropriate servomotors laws of motion to obtain the planned end effector trajectories in a robots with non rigid transmission.

For the reasons above, it seems to us interesting to start investigations on the influence of the law of

motion, given from the motor, on the dynamical behaviour of railroad cars moved by means of a steel wire (e.g. funicular railways, cableways, elevators etc.). This will allow to find adequate motor laws of motion by which it will be possible to reduce the undesired railroad cars oscillations that occur during the start and stop transients.

## 2 The mathematical model

For initial investigations, was considered a simple 3 d.o.f. damped model that was reported in a previous investigation [7].

# **3** Computed results

As reported in [7], the equations of motion have been solved by means of MatLab code, by using the function ODE45 of the II order Runge Kutta algorithm with variable step of integration.

It was considered that, on the winch itself, acts a moment during a transient 20 seconds long. The moment versus time, during the start transient, is reported in fig.1; as shown in the figure, three cases have been considered:

Case 1 - the moment is increased from M = 0 Nm at t = 0 to  $M = 1.5 \cdot 10^5$  Nm at t = 20 s, following a linear law. This last moment value represents the value which results in the same car displacement of case 1 if the wire was rigid.

Case 2 - the moment is constant ( $M=10^5$  Nm) during the 20 seconds

Case 3 - the moment is increased from M = 0 s at t = 0 to  $M = 1.5 \cdot 10^5$  Nm at t = 20 s , following a sinusoidal law.



A stop transient was also considered. During this stop transient, the moment versus time laws have the same shape but, obviously, the motor acts as a brake.

The results of the numerical integration are reported below.

### 3.1 - Start transient.

#### 3.1.1 – Case 1

In the figures 2a, 2b and 2c are reported the laws of motion (respectively: rotation, angular velocity and acceleration) of the winch when a linearly increasing moment is applied to the winch itself.



Fig. 2 – Winch law of motion, start

The solid lines refer to the damped system with non rigid wire; to allow comparison the figures show, also, the behaviours with non damped and non rigid wire (dashed lines) and the behaviour with rigid wire (dashed-dotted lines).

In the figures 3a, 3b and 3c are reported the laws of motion of car 1 (respectively: rotation, angular velocity and acceleration).



Fig. 3 - Car 1 law of motion, start

In the figures 4a, 4b and 4c are reported the laws of motion of car 2.

From the figures above, from a qualitative point of view, almost the same observations reported in [7] can be made:

- The cars laws of motion significantly differ from the planned ones, as oscillations with significant amplitude take place. In the scale adopted for the diagrams, this is particularly evident if velocity and acceleration are considered. These last ones can probably be un-acceptable for the passengers.

- The winch laws of motion, too, differ consistently from the expected ones, as high amplitude accelerations occur. This phenomenon can cause undesirable effects on the gears and on the transmission between motor and winch

- The damping seems to have poor effects on both the cars and the winch behaviour.



Fig. 4 - Car 2 law of motion, start

#### 3.1.2 - Case 2

In the next figures, we will limit our reports just the description of acceleration diagrams of the system components (winch, car 1 and car 2) as accelerations are the most significant ones.

In the figures 5a, 5b and 5c are reported the accelerations of the winch, the car 1 and the car 2 respectively, when a constant moment is applied to the winch itself. This scenario was studied as it represents the worst condition.

The lines style has the same meaning as previously described.

If these results (that refer to a constant moment applied to the winch) are compared to those related to case 1 (linear increasing moment), it is evident that, with an linearly increasing moment, the undesirable oscillation amplitudes are lower than those with a constant moment. This suggest that the moment derivative (and hence the jerk of the winch law of motion) plays a significant role on the phenomenon.



Fig. 5 – Winch, Car 1 and Car 2 acceleration, start

#### 3.1.3 - Case 3

As for this case, also, just the accelerations diagrams of the system components (winch, car 1 and car 2) will be reported.

In the figures 6a, 6b and 6c are reported the accelerations of the winch, the car 1 and the car 2 respectively, when to the winch is applied a moment that increases with sinusoidal law.

As it can be observed by comparing these figures with the previous ones, if the moment is increased with a sinusoidal law of motion, the oscillation amplitudes of all system components are the lower ones.

This seems to confirm that the moment derivative (and hence the jerk of the winch law of motion) plays a significant role on the phenomenon: the lower is the moment derivative near the beginning and the end of the transient phase, the lower are the system components oscillations.



Fig. 6 - Winch, Car 1 and Car 2 acceleration, start

### 3.1 - Stop transient.

As previously told, the stop transient was also considered. The moment laws applied to the winch have the same shapes than those of the start transient but they decrease from the maximum value to zero in 20 seconds.

In the figures 7a, 7b and 7c are reported the accelerations of -respectively- the winch, the car 1 and the car 2, when the moment applied to the winch decreases from  $1.5 \cdot 10^5$  Nm at t = 0 to 0 Nm at t = 20 s, following a linear law.

In the figures 8a, 8b and 8c are reported the accelerations of -respectively- the winch, the car 1 and the car 2, when the moment  $(10^5 \text{ Nm})$  applied to the winch is a brake constant moment.

In the figures 9a, 9b and 9c are reported the accelerations of -respectively- the winch, the car 1 and the car 2, when the moment applied to the winch itself decreases from  $1.5 \cdot 10^5$  Nm at t = 0 to 0 Nm at t = 20 s following a sinusoidal law.

The behaviour that can be observed during the stop transients confirms that, also in this case, the lower the moment derivative near the beginning and the end of the transient phase, the lower are the system components oscillations.



Fig.7 - Winch, Car 1 and Car 2 acceleration, stop



Fig.8 - Winch, Car 1 and Car 2 acceleration, stop



Fig.9 – Winch, Car 1 and Car 2 acceleration, stop

As already observed in [7], also if different law of moment variation are considered, both during the stop and start transients, in all the considered cases, it can be observed that the dynamical behaviour is essentially composed by two components: a low frequency component (whose period is about 3.2 s) and an higher frequency one whose period ranges from 0.16 to 0.39 seconds; this is particularly evident if the diagrams of the acceleration are considered.

In fig.10 is reported the Fast Fourier Transform of car 2 acceleration (se fig.8) during the stop transient with a constant (brake) moment applied to the winch.



Fig.10 – FFT of car 2 acceleration

This aspect can be connected with the eigenvalues of the system that have been reported in [7] : one of the own frequency of the system is about constant, while the other changes during the run, due to the changes of the wire length and stiffness. From the diagrams reported in the present paper, what observed in [7] seems to be confirmed, in fact during the start transient the higher frequency component decreases its frequency, while during the stop transient the higher frequency component decreases its frequency. It must be observed that as the length of the shorter wire increases (start transient), it's stiffness decreases and the mass of the sub-system increases; the opposite happens during the stop transient.

### **4** Conclusions

In this report we presented a first study on influence of the law of motion on the behaviour during the start and stop transients of cablerailway systems, such as funicular railways, cableways, elevators etc.

The first results show that the shape of the moment (applied to the winch) versus time law seems to affect to a great extent the behaviour of the system components (winch and cars) during the start and stop transients.

The results show that, as already told, the lower is the moment derivative (and hence the law of motion jerk) near the beginning and the end of the transient phases (both start and stop), the lower are the system components oscillations. This suggests that, by a suitable motion control, the undesirable effects connected with the system components oscillation during the start and stop transients could be significantly reduced.

The effect of the wire damping influence is observable but doesn't seem to play a decisive effect on the system behaviour.

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