Unit Response Matrix Coefficients Development;

ANN Approach

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***Abstract**:* The ANN methodology, inspired by neurobiology theories of massive interconnection and parallelism has been successfully employed in variety of optimization problems. In ground water management models, either governing equations are embedded into the management model or unit response matrixes are employed .Unit response matrixes development requires huge amount of data and/or simulation runs. In this paper , ANN is employed to develop unit response matrix coefficients to be later used in the management model .To do it , a ANN model has been trained to predict the outcome of the flow code , which results in unit response matrix coefficients for the aquifer under consideration. To train the ANN model different realizations from pumping well co-ordinates, distance between pumping and observation wells, and hydraulic conductivities of pumping wells were used, it was concluded that pumping well co-ordinates may be successfully employed for developing unit response matrix coefficients to be later used in management models .To test the performance of the proposed approach, the hypothetical aquifer was assumed. The aquifer response to different pumping stresses were compared using a well-defined simulation model and those resulted from unit response matrixes developed by (1) ANN approach and (2) direct data from groundwater simulation runs. It was concluded that ANN may be successfully employed for development of unit response matrix with limited data from field study or ground water simulation runs.

Key Words: Unit Response Matrix, Artificial Neural Networks, Ground Water

1 Introduction

Using mathematical models for simulation and optimization of groundwater systems, has received increasing attention in recent two decades. Since the governing equation of flow in porous media is a partial differential equation (i.e. Bossinesque eq.), it cannot be directly included in any management optimization model. Therefore, one of the following two methods is usually employed: 1-Embedding method (EM), and 2-Unit response matrix method (URM) [3].

In EM differential equations, describing the flow in porous media is converted into a set of instantaneous algebraic equations, via numerical methods such as finite differences of finite elements. Then these equations are directly embedded into the optimization model in the form of model constraints. The resulted schemes often are very large and nonlinear and solution of them is very difficult. On the other hand in URM method the aquifer's response to any excitation will be estimated and generalized for the whole aquifer. To do it the aquifer may be simulated with a distributed parameter model, such as MODFLOW [8], then unit response coefficients (URCs) of aquifer are generated by running the model repeatedly.

Assuming linear behavior for aquifer, and the principle of superposition, the water table fluctuations may be estimated employing the developed excitation-response equation. Since these equations are linear the management model may be formulated as a linear programming (LP), provided objective function and other constraints are linear. The resulted model may then be solved by any standard package of LP solvers.

The assumption of linear relation between excitation (pumping, recharging) and aquifer response (drawdown or rise of water table) is quite valid for confined aquifers, and it must be verified for other types of aquifers. However if the drawdowns (rises) compared to the thickness of the saturated zone is small, the method is applicable with fair accuracy [1].

The simplicity of the URM method is the main reason for its extensive use in groundwater management models, compared to the EM. However generating the URCs is a very time consuming procedure. After calibrating the simulation model, one must run it repeatedly to develop URCs. In practical applications, the number of excitations needed for URCs development is much more than the number of source or sink terms.

Artificial Neural Networks (ANNs) present a tool by which URCs may be developed in a relatively shorter time period. In this paper the capabilities of ANNs for generating URCs are presented. After description of the method, the URCs of a hypothetical aquifer are generated via ANN approach and the results are compared with those obtained from repeatedly running the simulation model, and those of a multivariate regression model.

2 Unit Response Matrix Method

The URM method was initially developed for oil used in oil fields [6]. Later it was employed in groundwater modeling and management too. The Bossinesque of a two dimensional flow in a heterogeneous anisotropic aquifer has the following form:

$$\frac{\partial}{\partial x}\left(T_x \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(T_y \frac{\partial h}{\partial y}\right) = S\frac{\partial h}{\partial t} + W \tag{1}$$

In which:

Tx and *Ty*: Transmissivity of aquifer in *x* and *y* directions respectively,

h: water table elevation,

S: Storativity of aquifer,

W: Sink and sources term,

x, y: spatial coordinates,

t: time coordinate,

Considering similarity between flow in porous media and that of heat transfer in a solid body, the following relation between pumping rate and drawdown in water table may be obtained [10], [4]:

$$s(k,n) = \sum_{t=1}^{n}\sum_{j=1}^{J} \beta(k,j,n-t+1).q(j,t) \tag{2}$$

In which:

s(k,n): drawdown in well *k* at the end of time period *n*,

$\beta(k,j,n-t+1)$: unit response coefficient, that is the (unit) drawdown in well k at the end of time period n due to unit pumping at well *j* (*j* may equal *k*) during time period *t*.

q(j,t): pumping rate at well *j* during time period *t*

J: total number of pumping cells.

Maddock [7] developed this method for obtaining the optimum discharge of an aquifer system with three wells. He applied the term "Algebraic technological functions" for URCs. Morel-Seytoux [10] employed the same concept for stream-aquifer systems using "Discrete Kernels" term for URCs. Heidari [4], Yazicigil [14], and Reichard [11] used URCs in LP management models to define the optimum management strategies of aquifers. Barlow et al. [1], and Miller et al. [9], are among other researchers who employed the URCs to investigate the interaction of surface and groundwater bodies in large-scale basins.

3 Artificial Neural Networks

ANNs as a computational method, was originally presented by Rosenblatt as Perceptron nets and Widrow as ADALINE nets. The method is based on the complicated theory of parallel process of biologic neuron systems. The basic elements of an ANN are the artificial neurons which may be referred as nodes, units or processing elements. Figure 1 presents the main elements of a biological neuron and corresponding elements in an ANN.

The input pattern to a node is similar to a dendrite of a biological cell, which can be presented by a vector with N elements. $X=(x_1, x_2, \dots x_N)$ Now the scalar quantity S may be defined as:

$$S=\sum_{n=1}^{N} w_n . x_n = W^T . X \tag{3}$$

In which, $W= (w_1, w_2 \dots w_N)$, is vector of weights. The scalar S then enters to a nonlinear transition function f, to yield output y, as

$$y = f(S) \tag{4}$$

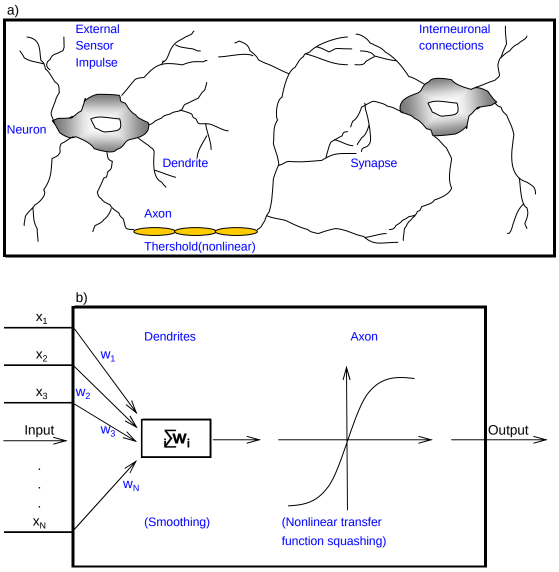

Fig. 1- Neural networks: a) structure of a biologic neuron, b)structure of an artificial neural network

The function f often takes the form of sigmoid or hyperbolic tangent, that the former is more common. The sigmoid function is defined by the following relation,

$$f(S) = (1+\exp(-s))^{-1} \tag{5}$$

The output y either can be the model result or be treated as an input to the next layer in multi-layer networks. Figure 2 shows a general neural network structure. As it has seen, each net has formed of an input layer, one or more hidden layer, and one output layer. There are many algorithms that were developed for computing optimum weights, among them the back propagation algorithm has been used extensively. In this algorithm, at first, nodes give small weights randomly. Then in a repetitive procedure, these weights are improved based on comparison between observed and computed outputs.

Most of the ANN-based studies in the field of water resources deal with rainfall-runoff modeling with limited number being in the field of groundwater systems.

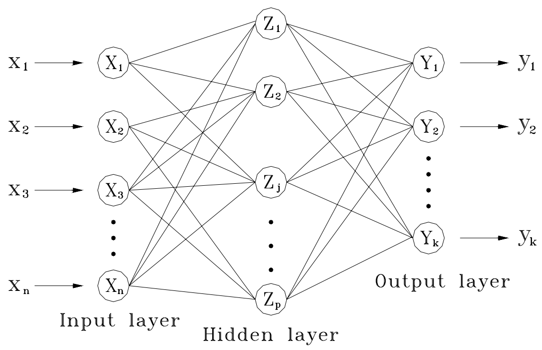

Fig. 2- General structure of an ANN

Rizzo and Dougherty [12], combining ANNs and Kriging develop Neural-kriging method for estimating spatial distribution of hydrodynamic properties of aquifers. Rogers and Dowla [13] used an ANN combined with a solute transport model and Genetic Algorithms for computing optimum remediation strategy of a polluted aquifer. Johnson and Rogers [5] in a similar work used ANNs with Solute transport model and Simulated Annealing for an aquifer optimum remediation. Coulibaly et al. [2], Employed different ANNs, for simulation of water table fluctuation in Burkina Faso. This paper presents an ANN based method to estimate unit response coefficient of an aquifer with limited data, comparing the capability of the proposed algorithm with design of an ANN system calls for determination of input parameters, the extent of training data, structures of ANN system, transition function and training algorithm, as well as network selection criteria.

4 Designing and Training of ANNs

Determination of input parameters requires some knowledge about nature of the problem under consideration. For example in aquifer drawdown problem, the drawdown in each point could be a function of distance from pumping well (d), pumping rate (Q), boundary condition and aquifer properties (k or T and S).It is worth to note that however the drawdown in a certain aquifer, ultimately is a two dimensional function of the position of the point of interest from the position of the pumping well (s).

From several transition functions, sigmoid function has been used extensively in various fields in engineering and has been used in this study. For training algorithm the back propagation or delta rule scheme has proved to be efficient in various problems. The same scheme has been employed in this study as training algorithm.

To select an appropriate network, root of mean square error (RMSE), and correlation coefficient of training and testing has been employed in this investigation. Training algorithm the back propagation or delta rule scheme has proved to be efficient in various problems. The same scheme has been employed in this study as training algorithm.

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5 Work Example

To illustrate the performance of the proposed ANN scheme, a hypothetical aquifer is considered (Fig.3). The aquifer is 1800m×1800m long in dimension, which has been discretized in 324 (100m×100m) meshes. East and the west of the aquifer are bounded by no flow boundaries, and north and south of it has constant head boundary. The aquifer is completely heterogeneous in hydraulic conductivity and the storativity of it, is assumed to be 0.2. The bedrock depth is 100 m and the initial water table is 90m for all cells. The purpose of the example is to develop URCs for the observation well (cell) located at the central cell for unit pumping rate in pumping wells (cells) as shown in figure 4.

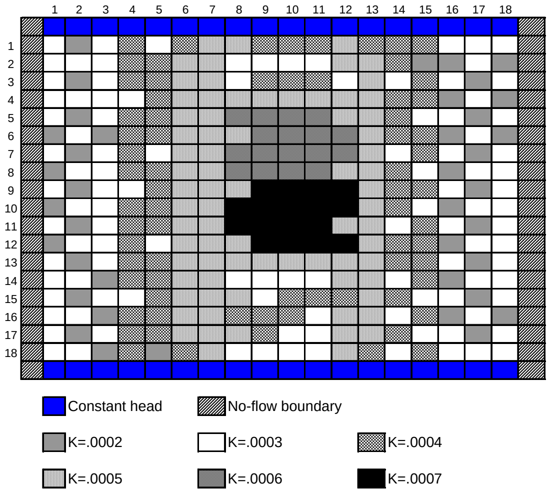

Fig. 3-Hypothetical aquifer and boundary conditions

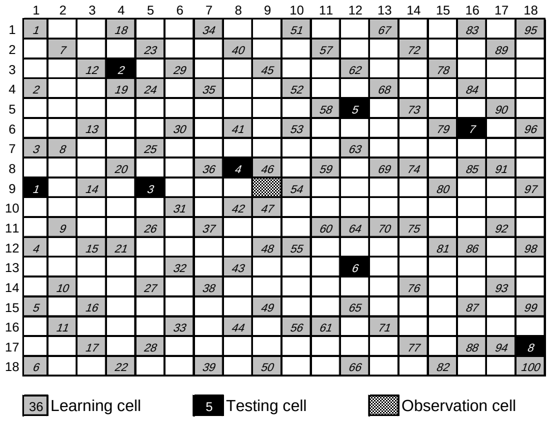

Figure 4-Location of different cells in aquifer

With a constant pumping rate of 1000, the test was carried out for a 10 days observation period. Different combination of input parameters was considered, among which the following 4 schemes were reelected for presenting the results:

ANN1, with only one input node: d;

ANN2, with two input nodes: d and k;

ANN3, with two input nodes: x and y;

ANN4, with four input nodes: d,k,x,y;

In with, d =the distance between pumping and observation cells; k =the pumping cell hydraulic conductivity; and x,y = the coordinates of pumping cell related to the origin. The required data (response of observation cell to pumping in training and testing cells) were generated with applying the well-known MODFLOW package [8]. Several combinations of hidden layers and the number of nodes in each layer have been tested. Among them the best one was a network with 2 hidden layers each with 10 nodes. Figures 5-8 show the result of training and testing data for 4 different networks as defined earlier.

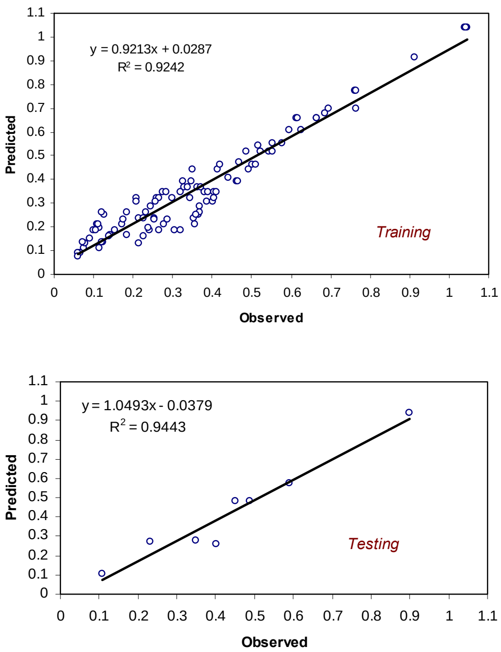

Fig. 5- Correlation between observed and predicted drawdowns in training and testing of ANN1

With only 1 input node (d), the ANN1 estimates the response in a moderate level of accuracy (Fig.5). However, for planning purposes the errors are high. On the other hand, adding the hydraulic conductivity of the pumping cells as an input data has insignificant effect on outputs (Figs. 6 and 8). It was concluded that, the third model performs better than the other models (Fig.7). In fact, by selecting x, y of pumping cells as input parameters the spatial properties of aquifer, as well as the effects of boundary conditions will be considered.

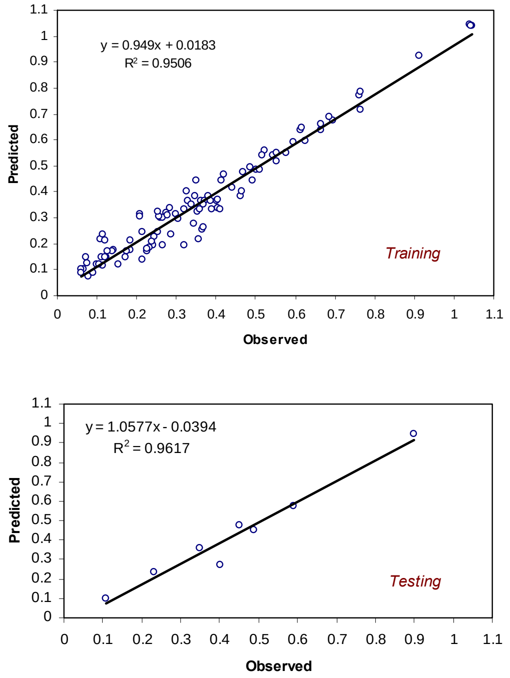

Fig. 6- Correlation between observed and predicted drawdowns in training and testing of ANN2

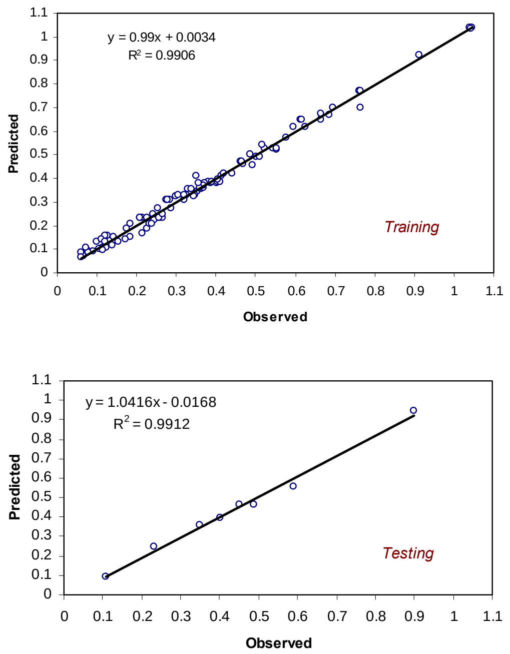

Fig. 7- Correlation between observed and predicted drawdowns in training and testing of ANN3

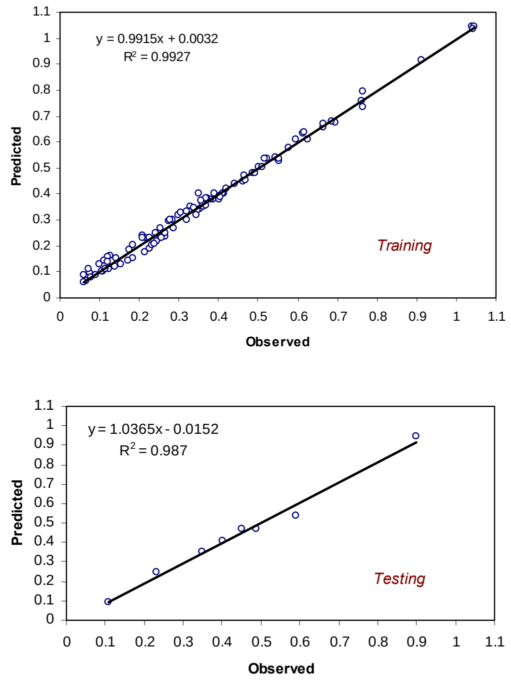

Fig. 8- Correlation between observed and predicted drawdowns in training and testing of ANN4

For the purpose of assessing the effect of the number of training wells, three alternative sets of training data were considered for the selected as:

ANN5, with 50 training cells,

ANN6, with 25 training cells, and

ANN7, with 13 training cells.

Location of training cells were selected randomly, while testing cells remaining the same. Performance of the model is highly dependent on the relative position of the training and testing cells. The best results may be expected when the testing cells are surrounded by the training cells. For the scheme presented in Figures 9 to 11 results for different models are presented in table 1. As of table 1, 78% reduction in the number of training cells (i.e. reducing 50 to 13), increases the training error by less than 2%. However, this result may only be valid for this specific problem and may not be generalized for other problem settings.

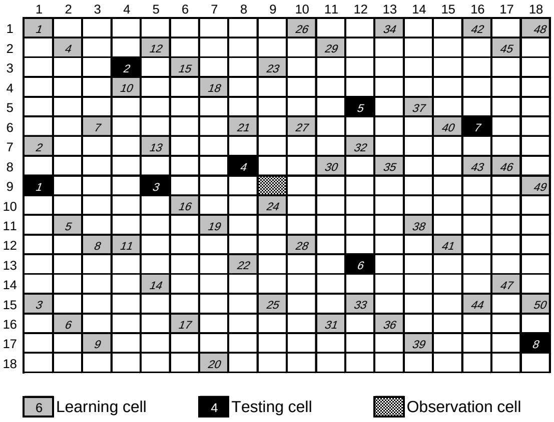

Fig. 9-Location of different cells in ANN5

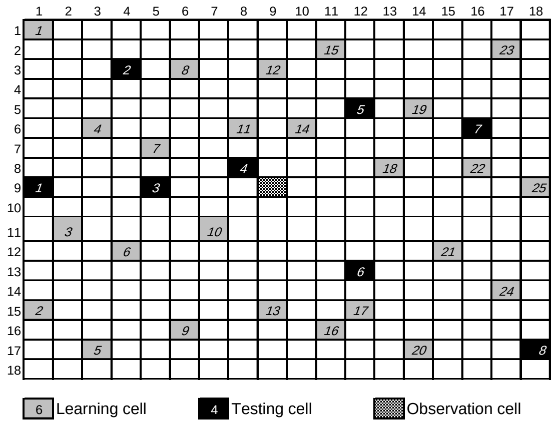

Fig. 10-Location of different cells in ANN6

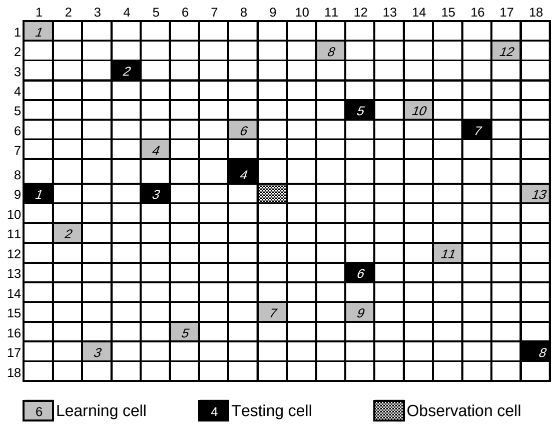

Fig. 11-Location of different cells in ANN7

Table 1-Results of Networks in training and testing

Network Name	Learning			Testing		
	No. of cells	R^2	Mean Error(%)	No. of cells	R^2	Mean Error(%)
ANN1	100	0.924	22.8	8	0.944	11.1
ANN2	100	0.951	17.7	8	0.962	8.2
ANN3	100	0.991	7.8	8	0.991	5.2
ANN4	100	0.993	7.3	8	0.987	5.6
ANN5	50	0.988	9.5	8	0.965	7.5
ANN6	25	0.981	8.1	8	0.990	5.8
ANN7	13	0.990	7.0	8	0.980	7.0

6 Comparison of Different Method of URM Generation

Based on the previous results, the URCs for the problem under consideration were generated and compared with two other common methods. To illustrate the capabilities of the proposed ANN model, a total of 32 cells were selected from which 16 were employed for training and the other 16 for testing process (Fig. 12). To be able to assess performance of the model, the exact drawdowns in observation cell for a ten day period, was determined employing MODFLOW.

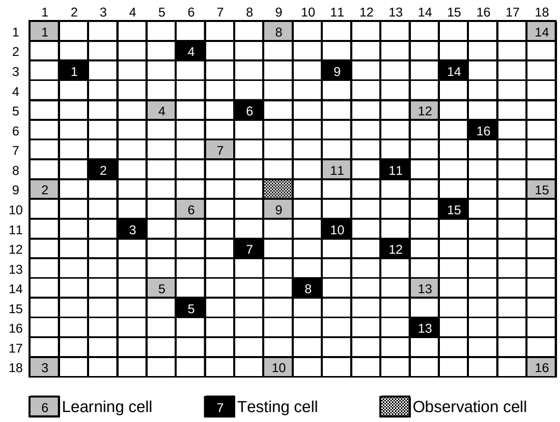

Figure 9-Location of cells for generating URCs

Except ANN method we use two other methods for generating URCs. The first one is the use of MODFLOW repeatedly (common method) and the second one is the based on a multivariate regression model. We use the same input data is regression model as we use for ANN. The general form of regression equation is:

$$s(t) = a_0(t) + a_1(t).x + a_2(t).y \qquad (6)$$

In which s(t) is unit drawdown in observation cell at the end of day t. a_0, a_1, and a_2 are constant, and *x*,*y* are pumping (testing) cells coordinates.

After generation of URCs based on different method the drawdown in observation cell at the end of each day of ten days period have been computed based on equation 2. The results summarize in table 2. As seen in this table, the result of ANN and MODFLOW is very close to exact drawdown. For better comparing, the result of 3 methods with exact values is illustrated in figure 17. It is clear that the ANN model could be generating URCs, with the same accurate of using MODFLOW model repeatedly.

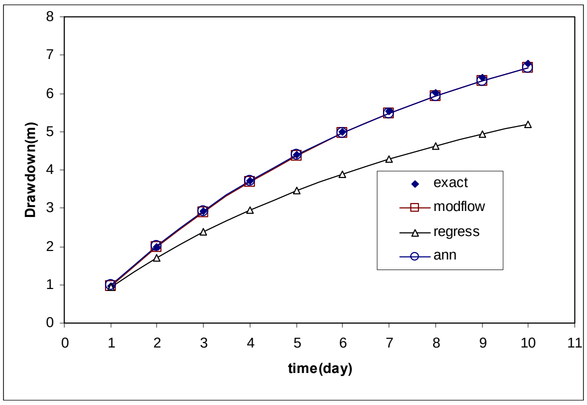

Fig. 10-Exact and predicted drawdowns with different models in observation cell

Table 2- Exact and predicted drawdown in different models and associated errors in observation cell

day	drowdown(m)				error(%)		
	exact	modflow	regress	ANN	modflow	regress	ANN
1	0.97	0.97	0.92	0.99	0.03	4.90	1.73
2	2.00	2.00	1.70	2.02	0.07	14.81	1.03
3	2.91	2.90	2.37	2.92	0.30	18.67	0.34
4	3.71	3.69	2.95	3.71	0.49	20.51	0.07
5	4.40	4.37	3.45	4.39	0.70	21.53	0.31
6	5.01	4.96	3.90	4.97	0.91	22.19	0.72
7	5.54	5.48	4.28	5.49	1.12	22.63	0.94
8	6.01	5.93	4.62	5.93	1.33	22.99	1.32
9	6.42	6.32	4.92	6.32	1.54	23.29	1.54
10	6.78	6.66	5.18	6.65	1.73	23.54	1.86
				mean	0.82	19.51	0.98

On the other hand since the location of testing cells was considered randomly, we could generate the URCs for more cells of the aquifer than generated here.

7 Summary and Conclusions

In this paper, the capability of ANNs in generating URCs was evaluated. The results show that with ANN one can generate URCs with a high level of accuracy. Comparison of the results with generated by using MODFLOW repeatedly and those of using multivariate regression, leads to the following major conclusions:

1-Based on x and y (coordinates) of training cells, one may develop an ANN model to generate URCs with acceptable accuracy.

2-With a relatively few number of training data points (cells); it is possible to generate the URCs for a relatively high number of cells.

3-For the case on there consideration, the URCs developed by the ANN model is as accurate as the results of simulation model with less execution time.

4-The trained ANN model may be utilized in the same aquifer, for different combination of testing cells.

5-The performance of multivariate regression model for generating URCs is weak, compared to the other two methods.

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