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Research of PMU Optimal Placement in Power Systems

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Abstract: The analysis of power system observability and the rules of PMU placement are concisely presented. Several algorithms of PMU placement as well as their differences and relations are discussed in details: a graph-theoretic procedure based on Depth First Search can find the optimal placement the most quickly; Simulated Annealing Method can help the algorithm converge to global optimum. Base on these two algorithms, an improved algorithm: Minimum Spanning Tree Method is proposed. It keeps good balance of quality and efficiency of the optimal placement, and improves the multiformity of the results by improving the optimization rule of Depth First Search Method.

Key-Words: Phasor Measurement Unit (PMU); Observability; Depth First Search; Simulated Annealing Method; Minimum Spanning Tree; Optimal PMU Placement (OPP)

1 Introduction

GPS-based synchronized phasor measurement is a new technique developed in resent years, which is called one of three front subjects in power systems [1]. The power angles and voltage phasors of key buses are very important state variables for power system stability, so they must be monitored accurately. Synchronous Phasor Measurement Unit (PMU) is a new type of measure unit with high precision, which can succeed in watching the entire system state in real-time directly by monitoring phasors of voltage of the buses.

In modern times, power systems connect each other on a quite big scale, it is uneconomical as well as unnecessary to place one PMU in one node. So researching of PMU optimal placement to realize one purpose has a very far-reaching meaning. Because of increasingly augmentation of power network capacity and complexity of nation power network interlinkage, monitoring and controlling of power system stability becomes more and more important at present. Research how to use the least PMU to realize the stability of the whole power network has the inestimable essentiality and imminency. Paper [2] regards the PMU placement problem as the multiple targets programming issue, one is the least number of PMU, and the other is the most system redundancy of observation after one fault happens (one line stop working). It gets good effect by using descendible algorithm to acquire the Pareto optimal result of the more aim programming issue. Paper [3] studies that the least number of PMU in the status of power flow equation is solvable directly. This paper studies the PMU optimal placement in the precondition of the observability of the whole power system.

How to select the placement bus comes down to optimal combined problem. How to solve this problem mostly has three methods: enumeration method, elicitation method and search method. The efficiency of enumeration method is a little low. Elicitation methods need to find the given rule to different problem, which is not strongly general-purpose. Search method uses some rules to find the optimal result at random, which can keep good balance of quality and efficiency of the optimal placement, and be wide applied in many optimal algorithms [4].

This paper introduces three optimal algorithms: depth first search (DFS) method, simulated annealing (SA) method, and minimum spanning tree all belong to search arithmetic. After introducing their characteristic and purpose in detail, the simulation results of PMU placement are analyzed, and prove the algorithms are effective and profitable.

2 Power system Observability analysis and PMU placement rules

2.1 Observability analysis

The estimate principle of PMU optimal placement in power system mostly is power system observability. After place a new PMU, whatever method is used, the observability of power system must be checked. If the system is observable, then the placement stops, else the placement must be continued. For the random inputs, during the limitary time, if the output can ascertain the input uniquely, then call the state of power system is observability. As paper [5] mentions, to review whether the system is observable, we can analyze the system from two points, namely algebra observability and topology observability.

(1) Algebra observability

It can describe one power system with N nodes and m measurements by using the linear metrical equation as follows.

$$z=Hx+v \tag{1}$$

Where z is the metrical vector of m dimensions

H is the Jacobian matrix of m×(2N-1) dimensions

x is the voltage vector of 2N-1 dimensions

v is the metrical noise vector of m dimensions

If the metrical Jacobian matrix is nonsingular and well-conditioned, which satisfies Rank（H）=2N-1,then call the system is algebra observability.

(2) Topology observability

From graph theory, power system can take as one graph with N apexes and b margins G = (V，E), V is the set of the apexes, E is the set of the margins, which separately correspond the sets of buses and branches of the system. The metrical network compose a metrical subgraph $G'=(V', E')$, and $V'\subseteq V$, $E'\subseteq E$. If the metrical subgraph G′ and graph G satisfy the relation $V\subseteq V'$, meaning the subgraph G′ contains all the apexes of the graph G, and then call the system is topology observability.

2.2 PMU Placement Rules

(1) Assign one voltage measurement to a bus where a PMU has been placed, including one current measurement to each branch connected to the bus itself.

(2) Assign one voltage pseudo- measurement to each node reached by another equipped with a PMU.

(3) Assign one current pseudo- measurement to each branch connecting two buses where voltages are known. This allows interconnecting observed zones.

(4) Assign one current pseudo-measurement to each branch where current can be indirectly calculated by the Kirchhoff current law (KCL). This rule applies when the current balance at one node is known, i.e. if the node has no power injections (if N-1 currents insert to the node are known, the last current can be computed by difference).

The pseudo-measurement proposed doesn't mean measure directly, but calculate the require measurement indirectly by the KCL, KVL, using the correlative measurements. It can improve the convergence of the result by applying this rule in a variety of algorithms, so that it can reduce the number of the PMUs, which has the widespread economic practicability.

3 Introduction of Algorithms

3.1 Depth First Search method

Depth First Search method (DFS) is applied extensively in earlier time, which is one of the tree search methods of PMU placement. This method uses only Rules from 1 to 3 (it does not consider pure transit nodes). The first PMU is placed at the bus with the largest number of connected branches. If there is more than one bus with this characteristic, one is randomly chosen. Following PMUs are placed with the same criterion, until the complete network observability is obtained.

The essence of this method expands from the nodes placed the PMU to the pseudo-measurement voltage nodes through the measurement or pseudo-measurement current branches, and then to all the nodes. The expanded nodes create a metrical tree, if the tree contains all the node of the system, then the system is topology observability, if some node is not contained in the metrical tree, then the system as well as these nodes is not complete observability. DFS only consider the "depth" through the process of expanding, which makes the observational topologies lay over each other unavoidably and increases the unwanted redundancy.

3.2 Simulated Annealing Method

Simulated annealing method is put forward by Metropolis in 1953. It solves the problems of combination optimization by simulating the physical anneal process of the solid matter (such as metal). In the physical anneal process, usually heat the metal up to melt first, making the particles move freely, namely, the high energy stage, then lower the temperature gradually, making the particles form the crystal lattice of low energy stage. As long as the temperature round the freezing point drops as slow as possible, the matter can break away from the local stress to form the ground state crystal of the lowest energy stage. Make the crystal to the optimal result, the cooling process and the optimization process correspondence; accordingly engender the simulated annealing algorithm. The procedure can be subdivided into several main steps, as follows.

(1) Select an initial condition $\times_0$ from the approve solution space randomly, calculate its target function value $f(\times_0)$, and select the initial control temperature T_0 and the length of Markov Chain;

(2) Engender a random disturbance in the approve solution space, then gain a new state $\times_1$, calculate its target function value $f(\times_1)$;

(3) Judge whether it satisfy $f(\times_1) < f(\times_0)$, if yes, then accept the new state $\times_1$ as the current state; if not, then judge whether it satisfy $\times_1$ according to Metropolis rule, if yes, then accept the new state $\times_1$ as the current state, else accept the state $\times_0$ as the current state;

(4) Judge whether the sample process ends according to some convergence rule, if yes then turn to (5); if not, then turn to (2);

(5) Lower the control temperature T according to some annealing project;

(6) Judge whether the annealing process ends according to some convergence rule, if yes then turn to (7); if not, then turn to (2);

(7) Output the current solution as the best solution.

Apply simulated annealing method in the PMU placement practically; the codes are as follows [6]:

begin
evaluate coverage of PMU placement set S
E : = N — number of buses in the observed region
T : = 15
M : = $\min\{0.002\binom{N}{v_{test}}, M_{\max}\}$
Where v_{test} is the size of search place.
for i : = 1 **to** 40 **do**
for j : = 1 **to** M **do**
randomly select a PMU, save the bus location of the selected PMU
randomly select a non-PMU bus, evaluate coverage of the modified placement set
E_{new} : = N - number of buses in the observed region
if $E_{new} = 0$ **then**
Return with 'system observable' and the modified placement set
if
ΔE : = E_{new} - E
if $\Delta E > 0$ **then**
generate a random accept/reject value with a probability $\exp(-\Delta ET)$
if reject **then**
return selected PMU to previous bus location
if
if
do
T : = 0:879T
do
return with 'system not observable'
end

3.3 Minimum Spanning Tree Method

Minimum Spanning Tree method (MST) is a modified depth first approach. The procedure can be subdivided into three main steps:

(1) Generation of N minimum spanning trees: Fig. 1 depicts the flow chart of the minimum spanning tree generation algorithm. The algorithm is performed N times (N being the number of buses), using as starting bus each bus of the network.

(2) Search of alternative patterns: The PMU sets obtained with the step (1) are reprocessed as follows: one at a time, each PMU of each set is replaced at the buses connected with the node where a PMU was originally set, as depicted in Fig. 1. PMU placements which lead to a complete observability are retained.

(3) Reducing PMU number in case of pure transit nodes: In this step it is verified if the network remains observable taking out one PMU at a time from each set, as depicted in Fig. 1. If the network does not present pure transit nodes, the procedure ends at step (2).

The placement sets, which present the minimum number of PMUs, are finally selected. This method uses the graph theoretic principle more than depth first method. It finds the bus which maximizes the coverage of the network with the existing PMU's, then set a PMU at the bus. It can reach the complete convergence, and also make the optimization best.

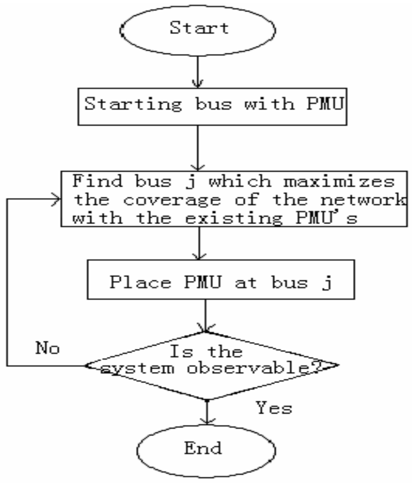

Fig.1 Flow chart of MST method

4 Simulation and analysis

4.1 Simulation example 1

IEEE14-bus system is depicted in Fig.2. Where node 7 is pure transit node, we also call it no load node. There are no loads to consume the power, no generators to inject the power either. The power injected by node 8 and node 4 transmits to node 9 completely, that is why it named pure transit node. According to PMU placement rule (4), for node 7, as long as two current branches are known, the left current branch can be calculated by pseudo-measurement.

DFS, SA and MST methods are used separately to complete the simulation. The results are shown in Tab.1.

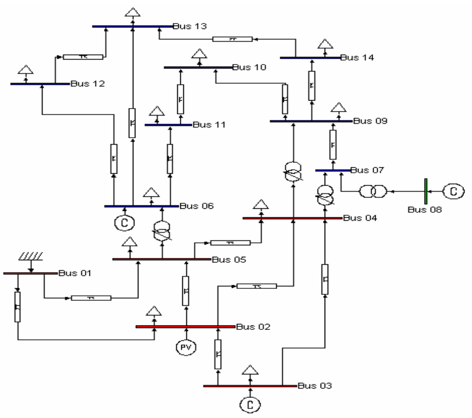

Fig.2 IEEE14-bus system

Table 1 Results of IEEE14-bus system

Method	Elapsed time	Number of PMUs	Placement
DFS	0.701s	6	1、4、6、8、10、14
SA	2.423s	4	4、5、6、9
MST	4.226s	3	2、6、9

The simulation results are obtained in the microcomputer with 1.4 G Hz, 256 M EMS memories. The elapsed time is different for the different computers, but proportion of time is the same. From Table 1, the speed of DFS method is the fastest, but it needs the most PMUs. There is no difference between SA method and MST method when the number of the system' bus is small, but their optimization is also demonstrated.

4.2 Simulation example 2

IEEE30-bus system is shown in Fig.3, where nodes 6、22、25、27、28 are pure transit nodes. Apply DFS, SA and MST methods respectively to complete the simulation. The results are shown in Table 2.

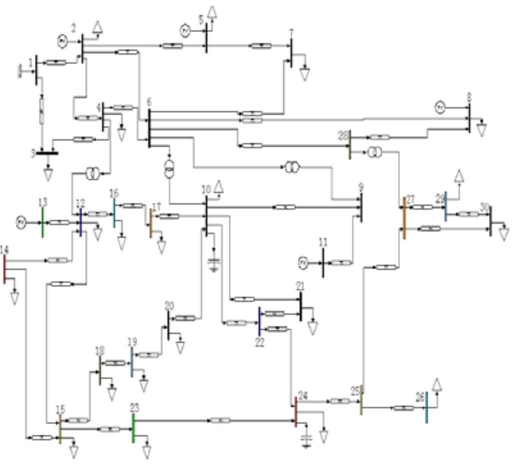

Fig.3 IEEE30-bus system

Along with the increase of the system' buses number and the link' complexity, the defects of DFS method and SA method are gradually evident. In the process of optimization it must use PMU placement rule (4) frequently because of the increasing of the system' nodes number especially of the no load nods number. DFS method does not use the criterion, it just finds the buses which connect with the most branches in the system, that make its result not very ideal and brings unwanted redundancy as well as poor economical efficiency. The number of PMUs that DFS method needs is almost twice which accounts for 40% of the total buses comparing to the two other methods.

Table 2 Results of IEEE30-bus system

Method	Elapsed time	Num. of PMUs	No. of projects	Placement
DFS	0.471s	12	1	3、5、6、11、12、17、18、20、21、24、26、27
SA	3m 44.032s	7	1	2、4、10、12、19、24、27
MST	17.846s	7	9	——

The convergence of SA method' result has the close relation with the initial value. It must calculate the whole nodes after setting a PMU because it may engender a new disturbance during the calculation process which makes the parameter matrix huge, thus the speed of placement is restrict. The elapsed time of it is 3 minutes and 44seconds, as 560 times as DFS method and 13 times as SA method.

Furthermore, the diversity of the solution is very poor for DFS method and SA method; they only have one setting project. MST method can overcome these shortcomings and hold their excellencies at the same time. In the process of optimization, MST method assimilates the thought of DFS, but differing from this method, it finds the bus which maximizes the coverage of the network with the existing PMU's not the bus with the largest number of connected branches. When the system reaches a complete observability, it can reduce PMU number in case of pure transit nodes, which can reduce the redundancy and increase economical efficiency. At the same time, MST method saves each bus which has the same coverage of the network during the process of optimization, then find the best setting project separately, which leads to the multiformity of the result.

Tab.3 Projects of optimal placement of MST

Serial number	Sets
1	1、7、10、12、19、23、27
2	1、5、10、12、19、24、30
3	3、5、10、12、19、23、27
4	1、5、10、12、19、24、29
5	1、5、10、12、15、20、27
6	1、2、10、12、19、23、27
7	1、5、10、12、18、23、27
8	1、5、10、12、15、19、27
9	1、5、10、12、19、23、27

5 Comparison of Algorithms

The optimization problem of PMU placement means the minimum number of PMUs n_p and the most appropriate place S(n_p) which lead to the maximal observability of the network and biggish redundancy of data.

$$J = \min\{\max R(n_p, S(n_p))\}$$
$$s.t. \quad f_{obs}(n_p, S(n_p)) = 1 \qquad (2)$$

From the point of view of optimization, the system' observability lies on two variables, one is the number of PMUs n_p, the other is the aggregation of measure place S(n_p). Because there is no means to obtain the minimum number of PMUs $n_{p.\min}$ directly up to the present, it depends on iterative algorithm to plough around this kind of problem. The search place of n_p is [0, b], while S(n_p) is $C_n^p b$. Expressions (2) is a kind of non linear combination optimization problem which is high dimensional and discontinuous. It is impossible to apply the routine of optimal methods to find the complete optimal solution for it always has a great number of local extreme. It must use modern optimal approaches especially DFS and SA methods which have the capability of overall optimization and independent on the information such as grads to solve this kind of complex problem.

DFS method finds the first bus with the largest number of connected branches, and then continues to search in the solution place from the initial solution with the same criterion, until the complete network observability is obtained. The steps are as follows:

(1) Select the bus with the highest dimension to place PMU first;

(2) Search in the same way as step (1) and find the measurements, pseudo- measurements and extend-measurements, then estimate the degree of system observability;

(3) If the complete network observability is not obtained, do the steps (1) and (2) again to the unobservable area, until the complete observability is obtained.

DFS method only considers the criterion of 'depth' in the process, and doesn't do the repetitive operation, so its operand is minimal which leads to the highest speed comparing to the other two methods. But its result is not the optimum because its optimization criterion is rigescent and unitary, furthermore it doesn't consider that many measurements of network system can be obtained by pseudo-measurements. We can see that the number of PMUs DFS method needs is the most from the examples above, likely severalfold comparing to SA and MST methods when the number of power system' bus increase. It has poor universality only from the point of view of the economical efficiency.

SA approach has the complementary effect to DFS method. It is virtually a model of Markov Chain, which is a series of test solutions generated at one time. The result that every test solution generates is depended on the predecessor to a certain degree. This algorithm starts form a given initial solution, produces an aggregation of candidate solution. It accepts the whole candidate solutions, which are better than current solution in the process of optimization, and accepts inferior solution in a certain probability. This characteristic has the key effect on jumping out of local optimization in the search process, but it makes its operand and solution space increase leading to the low speed. The speed of SA method has something to do with the number of network buses, also with the number of branches each node connect. It must calls on each node n-1 times at least by SA method. From the simulation results, we can see that the elapsed time SA approach uses is the longest comparing to DFS and MST method, especially in the instance of that the number of system' buses is big and the connection of network is complex.

Anneal strategy that can get the ideal structure finally has the same important influence on the optimization, which also has the important effect on PMU optimal placement. SA algorithm can find the project, which can reach the system observability and need the least PMUs.

MST method is a modified DFS approach, it holds the excellence of high speed of DFS, improves the shortcomings of the poor systematization and complex arithmetic at the same time. MST method improves the optimization rule using pseudo-measurement, which can find manifold placement strategy in the context of ensuring the solution space. As Table 3 shows, it can find as many as 9 optimal setting projects when the system has 30 buses, which offers the selective multiformity in the actual work. The simulation results have demonstrated that MST method is excelling the other two methods in entirety and multiformity of the results.

Now compare the convergent speed, the whole convergence and the multiformity of the results of the three methods in Table 4.

Table 4 Comparison of three algorithms

Capacity / Algorithms	Convergent speed	Whole convergence	Multiformity
DFS	excellent	poor	poor
SA	poor	excellent	poor
MST	excellent	excellent	excellent

6 Conclusions

Compare several optimal methods from actual simulated examples, analyse their merits and demerits. On the basis of optimal principle of DFS method, we can obtain MST method by improving its optimization criterion. This approach overcomes the shortcomings of the poor optimization of DFS method and low speed of SA method, keeps good balance of quality and efficiency of the optimal placement, as well as improves the multiformity of the results. In the actual simulated examples, MST method can obtain the minimum number of setted PMUs on the premise of that the complete network observability is obtained.

The research of PMU is just in the stage of beginning at present. This paper studies the PMU optimal placement on the premise of system observability, which has guiding significance and realistic practicability for the stability control of our national power system network.

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