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Model Reduction of Uncertain Discrete Systems Having an Interval Structure Using Genetic Algorithms

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Abstract: - In this paper, an evolutionary approach is proposed to obtain a reduced-order discrete interval model for uncertain discrete-time systems having interval uncertainties based on resemblance of discrete sequence energy between the original and reduced systems. System performance of the discrete interval model obtained by using the proposed evolutionary approach is then verified based on time responses of the resulting model in comparison to existing methods to demonstrate the effectiveness of the proposed approach.

Key-Words: - Model reduction, discrete-time systems, genetic algorithms, uncertain systems, interval plant, discrete sequence energy.

1. Introduction

Most practical systems, such as flight vehicles, electric motors, and robots, are formulated in continuous-time uncertain settings. The uncertainties in these systems arise from unmodelled dynamics, parameter variation, sensor noises, actuator constraints, etc. These variations do not follow any of the known probability distributions in general, and are most often quantified in terms of amplitude and/or frequency bounds. Hence, practical systems or plants are most suitably represented by continuous-time parametric interval models [1,2], instead of deterministic mathematical models.

In many situations, it is also desirable to replace the high-order system by a lower order model. Typical methods for model reduction include aggregation method, moment matching technique, Pade approximation, and Routh approximation, etc. Recent developments of model reduction have been made toward the direction to handle uncertain interval systems [3]-[6] based on variants of the Routh approximation methods, where interval arithmetic is performed to derive Routh $\alpha-\beta$ or $\gamma-\delta$ canonical continued-fraction expansion and inversion [7]. It has been shown, however, that some interval Routh approximants may not be robustly stable even though the original interval system is robustly stable [6,7]. Furthermore, the reduced models obtained via these methods are generally not suitable for robust controller design because of their poor frequency responses in comparison to their original counterpart [7]. As far as model reduction of discrete interval systems is concerned, very limited discussions have been found in literature [17]-[24]. Among them, there was a method of model order reduction using Pade approximation to retain dominant poles [8], where the denominator of the reduced model is formed by retaining the dominant poles of the given discrete interval system, while the numerator is obtained by matching the first r moments of the model with that of the system. Based on multipoint Pade approximation [9], a reduced model was obtained as a Pade approximant of the original system about $2r$ points. However, system performances of the reduced-order models via the above-mentioned methods are generally not satisfactory. More effective approaches need to be developed so that characteristics of the reduced model suitably approximates those of its original system.

Recent developments of evolutionary algorithms [10,12] have provided a promising alternative to address the above-mentioned problems and difficulties because of their capabilities of directed random search for global optimization [13,14]. This motivates the use of genetic algorithms to derive a reduced discrete interval model with a desired order for the original system based on the degree of resemblance of the discrete sequence energy. System performance of the discrete interval model obtained by using the proposed evolutionary approach is then verified based on time responses of the resulting model in comparison to existing methods to demonstrate the effectiveness of the proposed approach.

2. Problem of model reduction of discrete interval systems

Consider the stable discrete-time system with interval uncertainties:

$$G(z,\boldsymbol{a},\boldsymbol{b}) = \frac{[b_0^-,b_0^+]z^{n-1}+[b_1^-,b_1^+]z^{n-2}+\ldots+[b_{n-2}^-,b_{n-2}^+]z+[b_{n-1}^-,b_{n-1}^+]}{[a_0^-,a_0^+]z^{n}+[a_1^-,a_1^+]z^{n-1}+\ldots+[a_{n-1}^-,a_{n-1}^+]z+[a_n^-,a_n^+]} = \frac{B(z)}{A(z)} \tag{1}$$

The problem can now be formulated to determine a desired rth-order stable model

$$G_r(z,\boldsymbol{c},\boldsymbol{d}) = \frac{[d_0^-,d_0^+]z^{r-1}+[d_1^-,d_1^+]z^{n-2}+\ldots+[d_{r-2}^-,d_{r-2}^+]z+[d_{r-1}^-,d_{r-1}^+]}{[c_0^-,c_0^+]z^{r}+[c_1^-,c_1^+]z^{r-1}+\ldots+[c_{r-1}^-,c_{r-1}^+]z+[c_r^-,c_r^+]} = \frac{D(z)}{C(z)} \tag{2}$$

so that the performance of the reduced model $G_r(z,\boldsymbol{c},\boldsymbol{d})$ suitably approximates that of the original system $G(z,\boldsymbol{a},\boldsymbol{b})$ in term of resemblance of discrete sequence energy.

3. Discrete sequence energy (DSE) of discrete interval systems

As a comparison basis, we need to evaluate the discrete sequence energy (DSE) for both the reduced and original interval models, respectively.

3.1 Discrete sequence energy (DSE) of deterministic systems

Discrete sequence energy of a sequence $h(nT)$, which plays a very important role in many control applications, is defined as:

$$\sum_{n=0}^{\infty} h^2(nT) \tag{3}$$

In evaluating Eq.(3), Parseval's theorem [15] states that sequence energy is related to a contour integration along the unit circle in a positive direction

$$\sum_{n=0}^{\infty} h^2(nT) = \frac{1}{2\pi j}\oint_c H(z)H(z^{-1})\frac{dz}{z} \tag{4}$$

where $H(z)$ is the z-transform of the sequence $h(nT)$. With a recursive algebraic algorithm [16], it is easy to compute the discrete sequence energy in Eq.(4). That is, for a discrete transfer function

$$H(z) = \frac{B(z)}{A(z)} \tag{5}$$

where $A(z)$ and $B(z)$ are polynomials with real coefficients

$$A(z)=a_0z^n + a_1z^{n-1} + \ldots + a_n\,,\ a_0 > 0 \tag{6}$$

$$B(z)=b_0z^n + b_1z^{n-1} + \ldots + b_n \tag{7}$$

and $A(z)$ has all its zeros inside the unit circle, we can obtain the discrete sequence energy as

$$\sum_{n=0}^{\infty} h^2(nT) = \frac{1}{2\pi j}\oint_c \frac{B(z)B(z^{-1})}{A(z)A(z^{-1})}\frac{dz}{z} = \frac{1}{a_0^{\,n}}\sum_{i=0}^{n}\frac{(b_i^i)^2}{a_0^i} \tag{8}$$

where coefficients $a_i^k\ and\, b_j^k$ are recursively defined [16] as:

$$a_i^{n-k} = \frac{a_0^{n-k+1}a_i^{n-k+1} - a_{n-k+1}^{n-k+1}a_{n-k+1-i}^{n-k+1}}{a_0^{n-k+1}},\ i = 0,1,\cdots,n-k,\ k = 1,2,\cdots,n \tag{9}$$

$$b_j^{n-k} = \frac{a_0^{n-k+1}b_j^{n-k+1} - b_{n-k+1}^{n-k+1}a_{n-k+1-j}^{n-k+1}}{a_0^{n-k+1}},\ j = n-k,\cdots,1,0,\ k = 1,2,\cdots,n \tag{10}$$

3.2 Discrete sequence energy (DSE) of interval systems

For a discrete interval transfer function

$$H_I(z,\boldsymbol{a},\boldsymbol{b}) = \frac{B(z)}{A(z)} \tag{11}$$

where $A(z)$ and $B(z)$ are polynomials with interval coefficients

$$A_I(z) = [a_0^-,a_0^+]z^n + [a_1^-,a_1^+]z^{n-1} + \cdots + [a_n^-,a_n^+] \tag{12}$$

$$B_I(z) = [b_0^-,b_0^+]z^n + [b_1^-,b_1^+]z^{n-1} + \cdots + [b_{n-1}^-,b_{n-1}^+]\,. \tag{13}$$

The DSE of the discrete interval system $H_I(z,\boldsymbol{a},\boldsymbol{b})$ of Eq.(11) can be obtained via interval arithmetic manipulations. That is, if $A(z)$ has all its zeros inside the unit circle and $h_I(nT)$ represents the impulse response of the discrete interval model $H_I(z,\boldsymbol{a},\boldsymbol{b})$, we can obtain the discrete sequence energy as:

$$\sum_{n=0}^{\infty} h_I^{\,2}(nT) = \frac{1}{2\pi j}\oint_c \frac{B(z)B(z^{-1})}{A(z)A(z^{-1})}\frac{dz}{z} = \frac{1}{[(a_0^n)^-,(a_0^n)^+]}\sum_{i=0}^{n}\frac{[(b_i^i)^-,(b_i^i)^+]^2}{[(a_0^i)^-,(a_0^i)^+]} = [\alpha^-,\ \alpha^+] \tag{14}$$

where coefficients $[(a_i^k)^-, (a_i^k)^+]$ and $[(b_j^k)^-, (b_j^k)^+]$ are recursively defined as:

$$[(a_i^{n-k})^-, (a_i^{n-k})^+] = \frac{[(a_0^{n-k+1})^-, (a_0^{n-k+1})^+][(a_i^{n-k+1})^-, (a_i^{n-k+1})^+] - [(a_{n-k+1}^{n-k+1})^-, (a_{n-k+1}^{n-k+1})^+][(a_{n-k+1-i}^{n-k+1})^-, (a_{n-k+1-i}^{n-k+1})^+]}{[(a_0^{n-k+1})^-, (a_0^{n-k+1})^+]},$$
$$i = 0,1,\cdots,n-k, k = 1,2,\cdots,n \tag{15}$$

and

$$[(b_j^{n-k})^-, (b_j^{n-k})^+] = \frac{[(a_0^{n-k+1})^-, (a_0^{n-k+1})^+][(b_j^{n-k+1})^-, (b_j^{n-k+1})^+] - [(b_{n-k+1}^{n-k+1})^-, (b_{n-k+1}^{n-k+1})^+][(a_{n-k+1-j}^{n-k+1})^-, (a_{n-k+1-j}^{n-k+1})^+]}{[(a_0^{n-k+1})^-, (a_0^{n-k+1})^+]},$$
$$j = n-k,\cdots,1,0, k = 1,2,\cdots,n \tag{16}$$

Similarly, the DSE of the reduced-order discrete interval model $G_r(z,\boldsymbol{c},\boldsymbol{d})$ of Eq.(2) can be obtained in exactly the same way.

4. Model reduction of discrete interval systems based on DSE

Figure 1 shows the framework of the GA-based model reduction scheme to derive a reduced model $G_r(z,\boldsymbol{c},\boldsymbol{d})$ for the original system $G(z,\boldsymbol{a},\boldsymbol{b})$ based on the closeness of discrete sequence energy (DSE) of the impulse responses between these two interval systems, in which $[\alpha^-, \alpha^+]$ and $[\beta^-, \beta^+]$ stand for the lower and upper bounds of the DSE of the impulse responses for the original and reduced discrete interval models, respectively.

The rationale of the proposed approach is to search for an optimal set of parameters for the upper and lower bounds of the uncertain coefficients of the reduced model $G_r(z,\boldsymbol{c},\boldsymbol{d})$ so that the objective functions

$$J_1 = |\alpha^- - \beta^-| \tag{17}$$

$$J_2 = |\alpha^+ - \beta^+| \tag{18}$$

are minimized. By doing so, the characteristics of the reduced-order discrete-time interval model $G_r(z,\boldsymbol{c},\boldsymbol{d})$ approximates those of its original system $G(z,\boldsymbol{a},\boldsymbol{b})$ if both objectives J_1 and J_2 are minimized.

With the multiobjective problem presented in Eqs. (17)-(18), the simplest and natural way is to have it reformulated as a mono-objective optimization problem by means of an aggregating function of the form

$$\min_{\substack{c\in C\\ d\in D}} J(c,d) = w_1|\alpha^- - \beta^-| + w_2|\alpha^+ - \beta^+| \tag{19}$$

where $w_i \geq 0$, i=1,2, are the weighting coefficients representing the relative importance of the objective functions [11]. Combining the objectives to obtain an optimized solution has the advantage of producing a single solution, because the design problem at hand requires no interaction with the decision making among the parameters derived by the proposed approach as far as the derivation of the reduced-order discrete interval model is concerned.

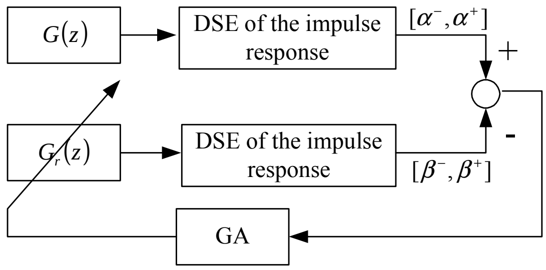

Fig. 1 Framework of the GA-derived reduced interval model.

5. Illustrated example

Consider the discrete interval system given by [9]

$$G_1(z) = \frac{[10.15,\ 10.35] + [8.5,\ 8.6]z}{[2.9,\ 3] + [8.7,\ 8.8]z + [6.5,\ 6.6]z^2}$$

The 1st order reduced model is

$$G_{r1}(z) = \frac{[3.7929,\ 3.8017]}{[1,\ 1] + [2.6280,\ 2.7452]z}$$

via multipoint Pade approximation as revealed in [9].

By using the proposed GA-based approach, we obtain the reduced model

$$G_{rga1}(z) = \frac{[4.6474\ ,\ 4.6657]}{[2.8851\ ,\ 2.9096] + [3.8313\ ,\ 3.9350]z}$$

after 100 generations of evolution. The GA parameters adopted include a population size of 50, pc=0.8, pm=0.02. Tournament selection is used with a tournament size of 4. GA operators include two-point crossover and non-uniform mutation.

For comparison purpose, Table 1 shows the DSE of various interval models of this Example. It is clear that the reduced-order model $G_{rga1}(z)$ derived via the proposed approach possesses exactly the same DSE as that of the original system $G_1(z)$. The interval model $G_{r1}(z)$ derived via [9], however, has a serious deviation in terms of DSE from its original counterpart. Therefore, we expect better system performance from the interval model $G_{rga1}(z)$, in comparison to the interval model $G_{r1}(z)$. Figures 2-7 show impulse and step responses of the respective interval models, which confirm the effectiveness of the proposed approach.

To show the consistency of the evolution process, Fig. 8 illustrates the simulation results of the evolution process to derive the reduced-order interval model for 5 runs. As demonstrated in Fig. 8, satisfactory consistency of the evolution process can be obtained via the proposed GA-based approach.

Table 1 DSE of various interval models.

Discrete Interval Models	Discrete Sequence Energy
$G_1(z)$	[3.0162 3.5038]
$G_{r1}(z)$	[2.2010 2.4470]
$G_{rga1}(z)$	[3.0162 3.5038]

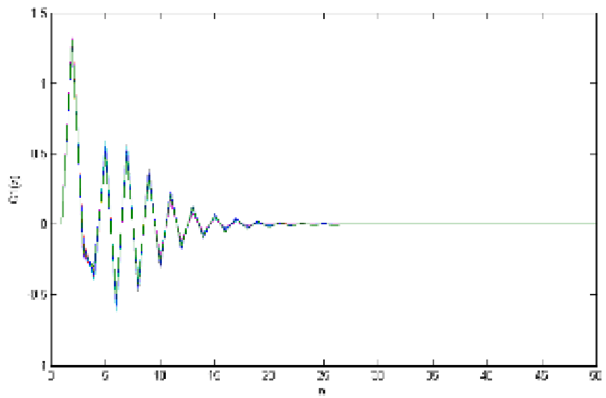

Fig. 2 Impulse responses of the original discrete interval system $G_1(z)$.

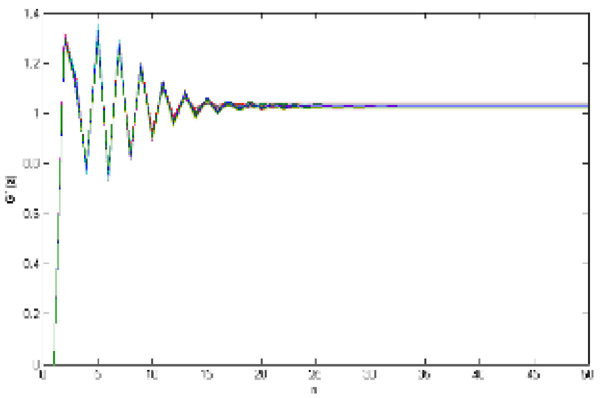

Fig. 3 Step responses of the original discrete interval system $G_1(z)$.

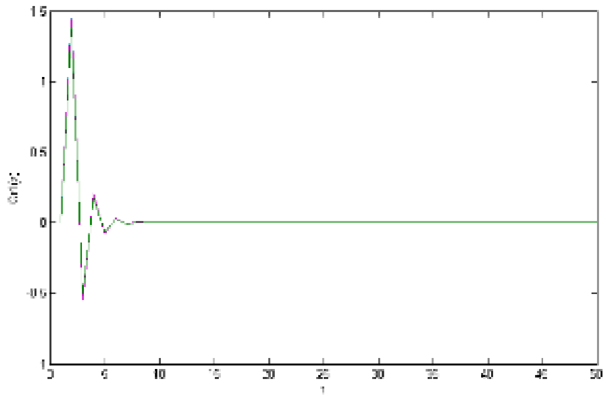

Fig. 4 Impulse responses of the reduced-order discrete interval model $G_{r1}(z)$ in [9].

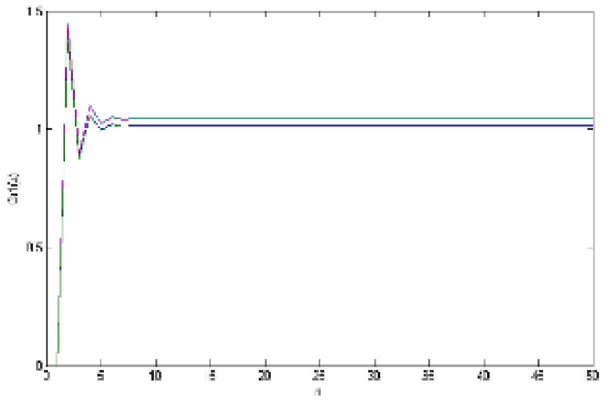

Fig. 5 Step responses of the reduced-order discrete interval model $G_{r1}(z)$ in [9].

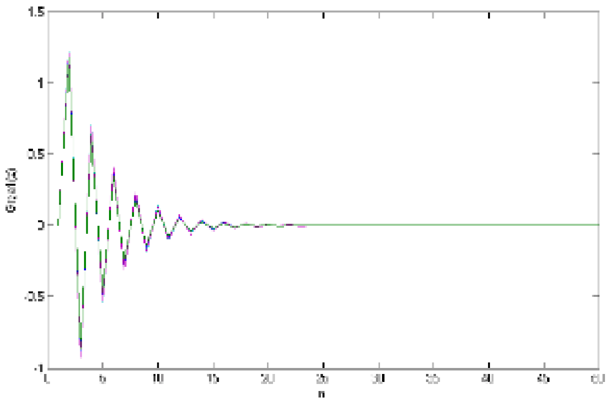

Fig. 6 Impulse responses of the reduced-order discrete interval model $G_{rga1}(z)$ obtained via the proposed approach.

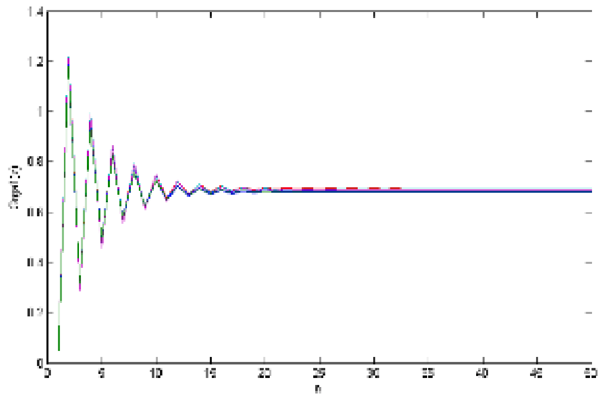

Fig. 7 Step responses of the reduced-order discrete interval model $G_{rga1}(z)$ obtained via the proposed approach.

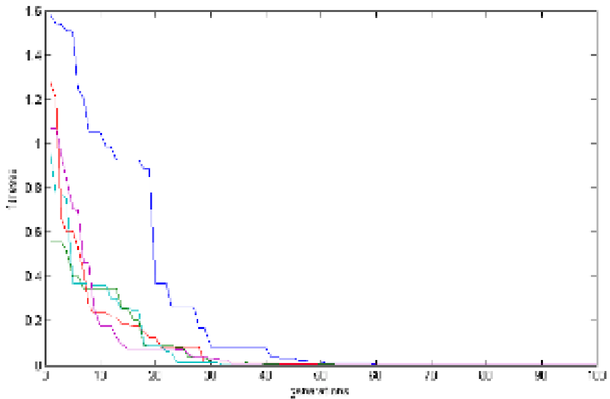

Fig. 8 Evolution process to derive the reduced model $G_{rga1}(z)$ for 5 runs.

6. Conclusions

There are very few discussions on the derivation of a reduced-order interval model for uncertain discrete-time systems having interval uncertainties. Existing approaches did not provide satisfactory results in general. To facilitate the derivation process, this paper has presented a GA-based approach as a computationally simple yet practical way to obtain an optimal reduced model for high-order interval systems with least deviation of DSE from its original counterpart. In comparison to the available techniques, the GA-based model reduction approach provides satisfactory performance. Simulation results have demonstrated that the reduced model obtained via the proposed approach out-performs the existing methods in terms of time responses and discrete sequence energy.

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