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Wireless HDTV Channel Estimator using LMS Criterion to Yacoub's Distributions

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Abstrac: This paper presents an estimator to wireless HDTV (High Definition Television) using the LMS (Least Mean Square) Criterion. This method evaluates the parameters $\kappa - \mu$ or $\eta - \mu$ for Yacoub Distribution and the parameter m for Nakagami Distribution from simulated data. These distributions model the fast fluctuation of envelope of a received signal composed of clusters of multipath waves propagating in a non-homogeneous environment. The procedure to estimate these parameters uses the pdf (probability density function) obtained from received signal. The signal is generated using two fading channel simulator development by author. The results show that the Yacoub´s Distributions provides a best fitting to simulated data than Nakagami-m Distribution. The Yacoub's distributions have two parameters to estimate, while the Nakagami distribution have just one parameter. The analysis of the received signal and the estimated curves confirm that there is a very good agreement between the estimated parameters from signal and the parameters used to generated the correspondent signal.

key-words: $\kappa - \mu$ Distribution, $\eta - \mu$ Distribution, Nakagami-m Distribution, fading channel.

1 Introduction

Traditionally, the performance of communications systems was take using the additive noise, such AWGN (Additive White Gaussian Noise). Unhappily, there are a phenomenon more complicated and complex, that limits the performance drastically: the multipath. This phenomenon is know by "ghost". The multipath phenomenon results in two degradation in the receiver signal. When the multipath delay profile is characterized by standard deviation that is minor that the symbol duration, the receiver's antenna catches many signals from same symbol transmitted. The result is the fluctuation of amplitude of signal received, occurs an intra-symbol interference, characterizing the effect denominated fading. We can't resolve this effect, but we can mitigate using an AGC (Automatic Gain Control) and diversity techniques. In other way, when the standard deviation is greater that the symbol duration, the receiver's antenna catches many signals from different symbols. This phenomenons is know as ISI (Inter Symbol Interference), the receiver signal is a linear combination of many symbols. This last effect can be mitigated using adaptive equalization. Then, we can characterize fading although of reception of an infinity number of waves from reflection, diffraction and scattering of transmitter signal in propagation environment. An deterministic analysis is very difficult because the phenomena involved is complex and the characterization of all them is a hard target. Many papers in the literature show that the stochastic approach is more adjusted and fits perfectly to experimental data.

The well-know distributions are Rayleigh (1889 by John William Strutt) [1], Hoyt (1947 by R. Hoyt) [2], Rice (1948 by S. O. Rice) [3] and Nakagami-m (1960) [4]. In all this model the phases of a cluster of waves are random and have equal delay time, characterizing a frequency non-selective channel. This characteristic allows one to use the Center Limit Theorem. Then, the summation of various waves that arrive at receiver results in two complex Gaussian processes with in-phase and quadrature Gaussian distributed variables and with equal means and standard deviations. This set of waves is denominated of cluster. Admitting that the in-phase and quadrature Gaussian distributed variables have means equal to zero and equals standard deviations, then it allows to model an environment of propagation without a dominant component over the scattered waves (a situation of Non-Line of Sight), then the Rayleigh distribution is obtained. With a little modification, the Hoyt model can be obtained adopting two complex Gaussian processes with in-phase and quadrature components with different standard deviations.

If the components in-phase and quadrature Gaussian are distributed variables with equal standard deviations, but means different of zero, it allows to model an Rice environment. Is obvious that the Rayleigh distribution is a particular case of Rice distribution.

From experimental data Nakagami-m signal can be seen as combination of clusters from different multipath waves with no dominant components within any cluster. These four distribution models consider the propagation in a homogeneous environment.

In 2001, Professor Michel Daoud Yacoub got two general model that considers the received signal composed by summation of n clusters (Raleigh or Hoyt) [5] [6]. This distribution includes Rayleigh, Hoyt, Rice and Nakagami-m as special cases. These model are called Yacoub's Distributions or, for simplicity, $\kappa - \mu$ or $\eta - \mu$ - the physical parameters of distribution. Professor Yacoub considers your distributions ”*composed of clusters of multipath waves propagating in a homogeneous environment. Within any cluster, the phases of the scattered waves are random and have similar times delay with delay-time spreads of different clusters being relatively large. The clusters of multipath waves are assumed to have the scattered waves with identical powers but within each cluster a dominant component is found that presents an arbitrary power* [5]”. In this case, the summation of clusters results in the $\kappa - \mu$ or $\eta - \mu$ Distributions and the Center Limit Theorem is not valid. Thus, we have a non-homogeneous propagation model. Physically, the κ parameter represents the ratio between the total power of the dominant components and the total power of the scattered waves. The parameter μ represents the number of cluster that form the received signal. And the η is the ratio between in-phase and quadrature variations. The Table 1 shows the abstracts about this six models.

This paper is organized as follows: Section 2 presents an overview about Yacoub's ($\kappa - \mu$ and $\eta - \mu$) Distributions and Nakagami-m Distribution. Section 3 introduces the Least Mean Square algorithm and shows the partial differentiation equations to $\kappa - \mu$, $\eta - \mu$ and Nakagami-m Distributions. Section 4 will show the curves for PDF and CDF using fading generator for Yacoub's Distributions and the curves estimated using LMS algorithm. Finally, Section 5 presents the conclusions.

TABLE 1
THE MODELS FOR FADING.

Models	Clusters	Mean	Standard Variations
Rayleigh	1	0	$\sigma_1 = \sigma_2$
Hoyt	1	0	$q = \sigma_1/\sigma_2$
Rice	1	A	$\sigma_1 = \sigma_2$
Nakagami-m	m	0	$\sigma_1 = \sigma_2$
$\kappa - \mu$	μ	p, q	$\sigma_1 = \sigma_2$
$\eta - \mu$	2μ	0	$\eta = \sigma_1^2/\sigma_2^2$

2 The models to $\kappa - \mu$, $\eta - \mu$ and Nakagami-m Distributions

2.1 The $\kappa - \mu$ Distribution

The $\kappa - \mu$ Distribution was presented by Yacoub in [5]. The envelope for Yacoub's $\kappa - \mu$ Distribution, r, can be written in terms of the in-phase, x_i , and quadrature, y_i , components of the fading signal as [5]

$$r^2 = \sum_{i=1}^{n}(x_i + p_i)^2 + \sum_{i=1}^{n}(y_i + q_i)^2, \qquad (1)$$

where x_i and y_i are mutually independent Gaussian processes with mean $E[x_i] = E[y_i] = 0$ and variance $Var[x_i] = Var[y_i] = E[x_i^2] = E[y_i^2] = \sigma^2$ [5]. The p_i and q_i values are the mean value of the in-phase and quadrature components of the i^{th} cluster, respectively. The $\kappa - \mu$ model can be understanding by summation of μ Rayleigh clusters.

The $\kappa - \mu$ probability density function of the envelope signal can be written as [5]

$$p(\rho) = \frac{2\mu(1+\kappa)^{\frac{\mu+1}{2}}}{\kappa^{\frac{\mu-1}{2}}\exp(\kappa\mu)}\rho^{\mu}\exp\left(-\mu(1+\kappa)\rho^2\right) \qquad (2)$$
$$\mathrm{I}_{\mu-1}\left(2\mu\sqrt{\kappa(1+\kappa)}\rho\right),$$

and represent the small scale variation of the fading signal with envelope r and normalized envelope $\rho = \frac{r}{\hat{r}}$, where $r \geq 0$ and $\hat{r} = \sqrt{E[r^2]}$.

By definition [5], κ is given by

$$\kappa = \frac{\sum_{i=1}^{n}(p_i^2 + q_i^2)}{2n\sigma^2}, \qquad (3)$$

and is the ratio between the total power of the dominant components and the total power of the scattered waves. The mean squared value, variance of power and κ physical parameters are continuous and n is discrete. This limitation can be made less stringent by defining a parameter μ being the real extension of n, given by [5]

$$\mu = \frac{E^2[r^2]}{Var[r^2]} \times \frac{1+2\kappa}{(1+\kappa)^2}. \qquad (4)$$

Note that $\mu = n$ is the number of clusters.

2.2 The $\eta - \mu$ Distribution

This model represent the summation of n clusters of propagation. where each cluster is represent by Hoyt model. Then, the envelope for Yacoub's $\eta - \mu$ Distribution, r, can be written in terms of the in-phase, x_i , and quadrature, y_i, components of the fading signal as [6]

$$r^2 = \sum_{i=1}^{n} x_i^2 + \sum_{i=1}^{n} y_i^2, \qquad (5)$$

where x_i and y_i are mutually independent Gaussian processes with mean $E[x_i] = E[y_i] = 0$ and variances given by $Var[x_i] = E[x_i^2] = \sigma_x^2$ and $Var[y_i] = E[x_y^2] = \sigma_y^2$, of the i^{th} cluster [6].

The probability density function for $\eta - \mu$ envelope r, $p(r)$, can be found using the standard, but long and tedious procedure of transformation of variables [7]. Here, we just list the equation for $p(\rho)$ with normalized envelope signal for $\eta - \mu$ distribution:

$$\begin{aligned} p(\rho) =& \frac{4\sqrt{\pi}\mu^{\mu+\frac{1}{2}} h^{\mu} \rho^{2\mu}}{\Gamma(\mu) H^{\mu-\frac{1}{2}}} \exp(-2\mu h \rho^2) \times \\ & \mathrm{I}_{\mu-\frac{1}{2}}\left(2\mu H \rho^2\right), \end{aligned} \qquad (6)$$

where $\rho = \frac{r}{\hat{r}}$ $(r \geq 0)$, $\hat{r} = \sqrt{E[r^2]}$ is the rms value of r, and the parameters h and H are written as

$$h = \frac{2 + \eta^{-1} + \eta}{4} = \frac{(1+\eta)^2}{4\eta} \qquad (7)$$

$$H = \frac{\eta^{-1} - \eta}{4} = \frac{1-\eta^2}{4\eta} \qquad (8)$$

By definition [6], η and μ are, respectively, given by

$$\eta = \frac{\sigma_x^2}{\sigma_y^2} \quad \text{and} \quad \mu = \frac{E^2[r^2]}{Var[r^2]} \times \frac{1+\eta^2}{(1+\kappa)^2}. \qquad (9)$$

In $\eta - \mu$ distribution $n = 2\mu$, where n is the number of clusters.

2.3 The Nakagami-m Distribution

By definition, the Nakagami-m signal is composed by summation of m Rayleigh signals [8]. This way, the signal envelope, r , modeled by Nakagami-m Distribution, is given by

$$r^2 = \sum_{i=1}^{m} r_i^2, \qquad (10)$$

where each component r_i , $i = 1, 2, ..., m$, corresponds to one Rayleigh envelope. The pdf for Nakagami-m Distribution and normalized envelope signal is given by [4]

$$p(\rho) = \frac{2m^m}{\Gamma(m)} \rho^{2m-1} \exp(-m\rho^2). \qquad (11)$$

where

$$m = \frac{E[r^2]}{Var[r^2]} \qquad (12)$$

3 Least Mean Square(LMS) Criterion

The Least Mean Square(LMS) Criterion can be used to estimate the $\kappa - \mu$, $\eta - \mu$ or Nakagami-m parameters. This distributions allows has better results in model real channel behavior. The channel estimation is performed by the adjust of the κ , η , μ and m parameters in each distribution. It is interesting to use this general fading distributions because it includes the well know fading distributions [5] [6]. This is performed by setting the appropriated parameter value that adjust the pdf theoretical curve with the real (simulated) curve.

The LMS criterion is used in many problems of physics and engineering because the error surface obtained from this formulation have a concave geometry. The idea is find the equation to update the parameters that drive the error to global minimum.

First, we must defined the error function. We work with the pdf obtained from practical data (simulated data) and the theoretic pdf curves. The LMS Criterion consists in minimize the following equation

$$\frac{\partial}{\partial \xi} \sum_{i=1}^{N} [p(\rho_i) - \hat{p}(\rho_i)]^2 \rightarrow 0 \qquad (13)$$

where $\frac{\partial}{\partial \xi}$, $p(\rho)$, $\hat{p}(\rho)$ corresponds the partial differentiation in relation to ξ (parameters to each distribution), the simulated pdf and the estimated pdf, respectively. The LMS criterion is based on mean square error difference between the simulated pdf, $p(\rho)$, and the estimated pdf, $\hat{p}(\rho)$. The LMS algorithm updates the ξ parameters to decrease the error between the pdf's. The partial differentiation equations in relation to ξ used to drive the algorithm to minimum in Yacoub's ($\kappa - \mu$ and $\eta - \mu$) and Nakagami-m Distribution are:

$\kappa-\mu$ Estimation

$$\frac{\partial}{\partial\kappa}p(\rho_i)=\frac{\rho_i^{\mu}exp(-\mu(1+\kappa)\rho_i^2)}{exp(\kappa\mu)}\Bigg\{\Bigg[\frac{(\mu^2+\mu)(1+\kappa)^{\frac{\mu-1}{2}}}{\kappa^{\frac{\mu-1}{2}}}-\frac{2\mu^2(1+\kappa)^{\frac{\mu+1}{2}}}{\kappa^{\frac{\mu-1}{2}}}+\frac{(\mu-\mu^2)(1+\kappa)^{\frac{\mu+1}{2}}}{\kappa^{\frac{\mu+1}{2}}}\Bigg]\times$$
$$\mathrm{I}_{\mu-1}(Z)-\frac{2\mu^2(1+\kappa)^{\frac{\mu+1}{2}}}{\kappa^{\frac{\mu-1}{2}}}\rho_i^2\,\mathrm{I}_{\mu-1}(Z)+\frac{(1+2\kappa)}{\sqrt{\kappa(1+\kappa)}}\frac{\mu^2(1+\kappa)^{\frac{\mu+1}{2}}}{\kappa^{\frac{\mu-1}{2}}}\rho_i\Bigg[\mathrm{I}_{\mu-2}(Z)+\mathrm{I}_{\mu}(Z)\Bigg]\Bigg\} \tag{14}$$

$$\frac{\partial}{\partial\mu}p(\rho_i)=\frac{(1+\kappa)^{\frac{\mu+1}{2}}\rho_i^{\mu}exp(-\mu(1+\kappa)\rho_i^2)}{\kappa^{\frac{\mu-1}{2}}exp(\kappa\mu)}\Bigg\{\Big[2+\mu\ln\Big(\frac{1+\kappa}{\kappa}\Big)-2\mu\kappa\Big]\,\mathrm{I}_{\mu-1}(Z)+$$
$$2\mu\Big[\ln(\rho_i)-(1+\kappa)\rho_i^2\Big]\,\mathrm{I}_{\mu-1}(Z)+2\mu\rho_i\sqrt{\kappa(1+\kappa)}\Big[\mathrm{I}_{\mu-2}(Z)+\mathrm{I}_{\mu}(Z)\Big]\Bigg\} \tag{15}$$

where $Z=2\mu\sqrt{\kappa(1+\kappa)}\rho$.

$\eta-\mu$ Estimation

$$\frac{\partial}{\partial\eta}p(\rho_i)=\frac{4\sqrt{\pi}\mu^{\mu+\frac{1}{2}}h^{\mu}}{\Gamma(\mu)H^{\mu-\frac{1}{2}}}\rho_i^{2\mu}\exp(-2\mu h\rho_i^2)\Bigg\{\Big(\frac{-\eta^{-2}+1}{4}\Big)\,\mathrm{I}_{\mu-\frac{1}{2}}(Z)\Big[\frac{\mu}{h}-2\mu\rho_i^2\Big]+$$
$$\Big(\frac{-\eta^{2}-1}{4}\Big)\Bigg[\frac{-(\mu-\frac{1}{2})}{H}\,\mathrm{I}_{\mu-\frac{1}{2}}(Z)+\mu\rho_i^2\Big[\mathrm{I}_{\mu-\frac{3}{2}}(Z)+\mathrm{I}_{\mu+\frac{1}{2}}(Z)\Big]\Bigg]\Bigg\}. \tag{16}$$

$$\frac{\partial}{\partial\mu}p(\rho_i)=\frac{4\sqrt{\pi}\mu^{\mu+\frac{1}{2}}h^{\mu}}{\Gamma(\mu)H^{\mu-\frac{1}{2}}}\rho_i^{2\mu}\exp(-2\mu h\rho_i^2)\Bigg\{\mathrm{I}_{\mu-\frac{1}{2}}(Z)\Big[\ln\Big(\frac{\mu h}{H}\Big)+\frac{\mu+\frac{1}{2}}{\mu}-$$
$$\Psi(\mu)+2\ln(\rho_i)-2h\rho_i^2\Big]+H\rho_i^2\Big[\mathrm{I}_{\mu-\frac{3}{2}}(Z)+\mathrm{I}_{\mu+\frac{1}{2}}(Z)\Big]\Bigg\} \tag{17}$$

where $Z=2\mu H\rho^2$.

Nakagami-m Estimation

$$\frac{\partial}{\partial m}p(\rho_i)=\frac{2m^m\rho_i^{2m-1}exp(-m\rho^2)}{\Gamma(m)}\Bigg\{\Big[(1+\ln(m)+2\ln(\rho_i)-\rho_i^2)\Big]-\Psi(m)\Bigg\} \tag{18}$$

where $\Psi(m)=\Gamma^{'}(m)/\Gamma(m)$.

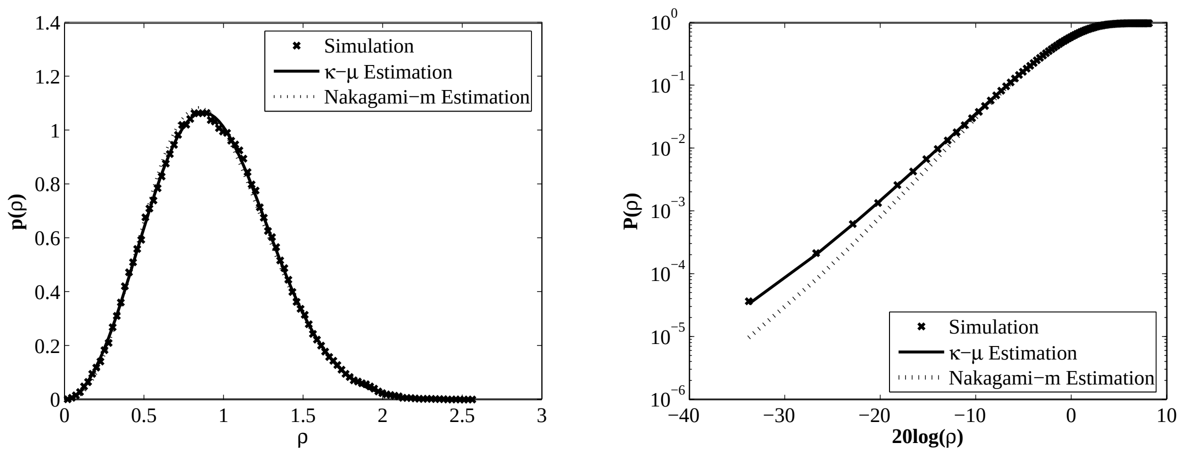

Fig. 1. The simulated and estimated pdf and cdf curves. Simulated signal with $\kappa = 0.75$ and $\mu = 1.50$. Estimated $\kappa = 0.72$ and $\mu = 1.51$. Nakagami parameter was estimated to $m = 1.74$

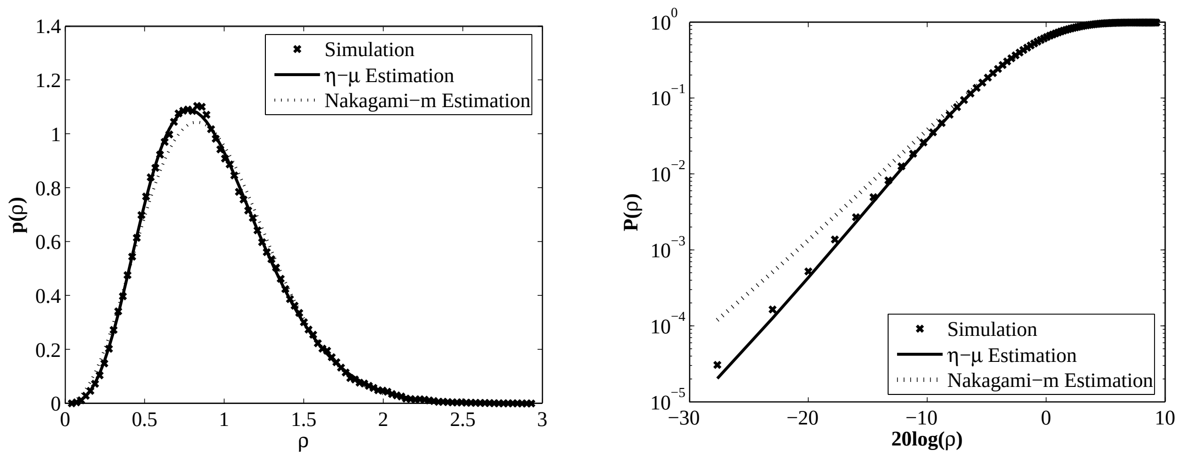

Fig. 2. The simulated and estimated pdf and cdf curves. Simulated signal with η=0.25 and μ=1.00. Estimated η=0.26 and μ=0.99. Nakagami parameter was estimated to m=1.60.

4 Simulations

The envelope of a received signal was obtained using a fading channel simulation for $\kappa - \mu$ and $\eta - \mu$ Distributions [9] [10]. The LMS algorithm estimates the $\kappa - \mu$, $\eta - \mu$ and m parameters that produce the pdf and cdf curves plotted in Figures 1 and 2. The signal Yacoub's $\kappa - \mu$ was modeled with fading for $f = 900$ MHz, $v = 60$ km/h, $\kappa = 0.75$ and $\mu = 1.50$. While the Yacoub's $\eta - \mu$ signal envelope has fading with $f = 900$ MHz, $v = 60$ km/h, $\eta = 0.25$ and $\mu = 1.00$, Figure 3.

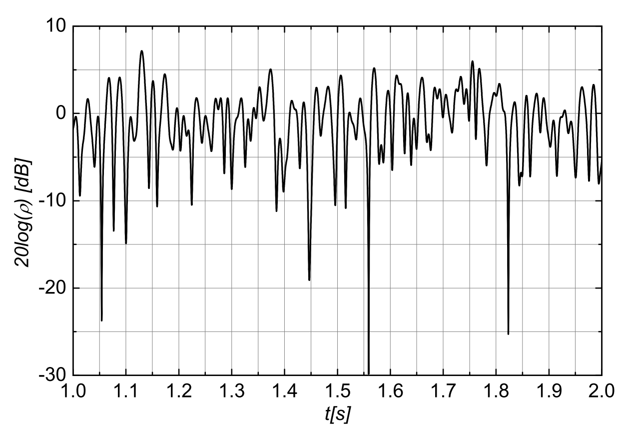

Fig. 3. The simulated envelope signal with η=0.25 and μ=1.

5 Conclusions

This paper proposed a method to estimate the fading of a received data using Yacoub's Distributions and Nakagami-m Distribution. The model shows that Yacoub's Distributions has a better attempt results to simulated data than Nakagami-m Distribution. The Yacoub's Distribution yield a better fitting tail to simulated data while Nakagami-m Distribution just produces a good fitting around the mean. The Yacoub's Distributions are easily adjustable because they use two estimated parameters. If the experimental data cdf is above Nakagami-m cdf, a best fitting is found using Yacoub's $\kappa - \mu$ Distribution. In the other hand, if the simulated data cdf is below Nakagami-m cdf a better attempt is found by Yacoub's $\eta - \mu$ Distribution. The next work is to develop the estimative for these parameters using the moments of distributions.

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