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Privacy-Preserving Bayesian Network Parameter Learning

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Abstract: - Privacy is an important issue in data mining. Learning a Bayesian network (BN) from privacy sensitive data has been a recent research topic. In this paper, we propose to use a post randomization technique to learn Bayesian network parameters from distributed heterogeneous databases. The only required information from the data set is a set of sufficient statistics for learning Bayesian network parameters. The proposed method estimates the sufficient statistics from the randomized data. We show both theoretically and experimentally that, even with a large level of randomization, our method can learn the parameters accurately.

Key-Words: - Bayesian Network, Privacy-Preserving Data Mining, Distributed Heterogeneous Databases, Post Randomization

1. Introduction

Privacy-preserving data mining deals with the problem of building accurate data mining models over aggregate data, while protecting privacy at the level of individual records. There are two main approaches to privacy-preserving data mining. One approach is to perturb or randomize the data before sending it to the data miner. The perturbed or randomized data are then used to learn or mine the models and patterns [1]. The other approach is to use secure multiparty computation (SMC) to enable two or more parties to build data models without every party learning anything about the other party's data [3]. Privacy-preserving Bayesian network (BN) learning is a more recent topic. Wright and Yang [9] discuss privacy-preserving BN structure computation on distributed heterogeneous databases while Meng *et al*. [7] have considered the privacy-sensitive BN parameter learning problem. The underlying approach in both works is to convert the computations required for BN learning into a series of inner product computations and then to use a secure inner product computation method. The number of secure computation operations increases exponentially with the possible configurations of the problem variables. The current work on privacy-preserving BN learning focuses on the multiparty model, which requires that every party have some ability to compute. Besides this model, our paper considers a model where there is a data miner who actually does all the computations on behalf of the participating parties. SMC method has the following two drawbacks: (1) it assumes a semi-honest model, which is often unrealistic in the real world (2) it requires large volumes of synchronized computations among participating parties. Most of the synchronized computations are overheads due to privacy requirements. Post randomization overcomes the drawbacks of SMC method by some trade off between accuracy and privacy. A malicious party who does not obey the protocol in SMC method can easily get some private information of other parties while no party is able to get exact private information of other parties if post randomizations are implemented to individual data records.

2. Problem Formulation

The privacy-preserving BN learning involves distributed databases, where the database is owned by several parties. If the database is homogeneously distributed, privacy-preserving BN Learning is relatively easy since every party can send data miner (or other parties) the set of sufficient statistics from his part of the database. Privacy of individual records will not be breached by sending sufficient statistics to other parties or data miner. The problem of privacy-preserving BN learning from heterogeneous database is that several parties who each own a vertical portion of the database want to learn a global BN for their mutual benefits but they are concerned about the privacy of their sensitive variables. In this paper, we consider the BN parameter learning problem for the discrete variable case. Extensions to BN structure learning are possible and would be reported in a future publication. We consider the following two models.

Model I: There is no data miner; every party has to do some portion of learning computations, which corresponds to the multi-party model of SMC. Every party sends their randomized data to those parties who need those data.

Model II: There is a data miner who does all computations for the participating parties. Each party simply sends all its randomized data to the data miner.

3. Privacy Quantization

Consider a database D with n variables $\{X_1 \Lambda\ X_n\}$, where X_i takes discrete values from the set S_i. The post randomization for variable X_i is a (random) mapping $R_i : S_i \rightarrow S_i$, based on a set of transition probabilities $p^i_{lm} = p(\tilde{X}_i = k_m \mid X_i = k_l)$, where $k_m, k_l \in S_i$ and $\tilde{X}_i$ denotes the (randomized) variable value corresponding to variable X_i. The transition probability p^i_{lm} is the probability that a variable X_i with original value k_l is randomized to the value k_m. Post Randomization is so named because the randomization happens after data have been collected. Let $P^i = \{p^i_{lm}\}$ denote the $K_i \times K_i$ matrix that has p^i_{lm} as its $(l,m)th$ entry, where K_i is the cardinality of S_i. The condition that P^i is nonsingular has to be imposed if we want to estimate the frequency distribution of variable X_i from the randomized variables. In the following, we give out some simple but effective post randomization schemes on which our experiments are based. If variable X_i takes binary values, we can use Binary Randomization as shown in Fig. 1(a). If the variable is ternary, ternary symmetric channel as shown in Fig. 1(b) can be used.

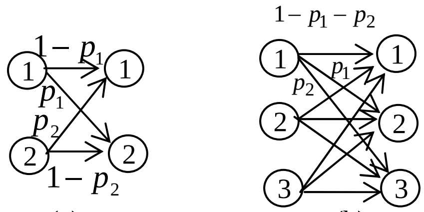


Fig 1. Randomization Schemes

We can apply the same randomization schemes independently to all of the variables: uniform randomization to the data set. Alternatively, we can use a non-uniform randomization where different post randomization schemes are applied to different variables independently. The non-uniform randomization is effective when different variables have different sensitivity levels. For example, we can choose different randomization parameters p_1 and p_2 to different binary variables for non-uniform randomization if the privacy requirements of the two variables are different. The non-uniform randomization includes the special case when there is no privacy requirement for some of the variables.

From the above, we can see that if variable X_i takes K_i values (or categories), the dimension of P^i will be $K_i \times K_i$. With larger K_i, more randomization is introduced into variable X_i in general. This is good from a privacy point of view. However, the variance of the estimator for frequency counts will also be lager under the same sample size. One solution for this problem is to partition the K_i categories into several groups such that a value in one group can only be randomized to a value in the same group. In this case, matrix P^i becomes a block diagonal matrix. The problem of how many groups should the K_i values be partitioned into is a matter of design choice.

The post randomization can also be implemented to several variables simultaneously. For example, the variables X_i and X_j can be randomized simultaneously according to transition probability $p(\tilde{X}_i = l_1, \tilde{X}_j = l_2 \mid X_i = k_1, X_j = k_2)$.

Randomizing variables simultaneously can avoid the possible inconsistency of the database caused by randomization.

We consider the notion of privacy introduced by Evfimievski *et al*. [4] in terms of an amplification factor γ . In [4], the amplification γ is proposed in a framework where every data record should be randomized with a factor greater than γ, before the data are sent to the data miner, to limit privacy breach. However, in this paper, we use amplification γ purely as a worst-case quantification of privacy for a designed post randomization scheme. It is proved in [4] that if the randomization operator is at most γ amplifying, revealing $\tilde{X}_i = k$ will cause neither an upward ρ_1-to- ρ_2 privacy breach nor a downward ρ_2-to- ρ_1 privacy breach if $\frac{\rho_2}{\rho_1}\frac{1-\rho_1}{1-\rho_2} > \gamma$. Clearly, the smaller the value of γ, the better is the worst case privacy. Ideally we would like to have $\gamma = 1$. The at most γ amplification provides a worst case quantification of privacy. However, it does not provide any information of privacy in general. Besides γ, we use $K = \min_{k'} \#\{k \mid P(\tilde{X}_i = k' \mid X_i = k) > 0\}$, which is the minimum number of possible categories that can be randomized to category k' in a designed post randomization, as another quantification of privacy. This K indicates the privacy preserved in general. It is similar to the K defined in K-anonymity in [8] but in probabilistic sense. If we group the categories of a variable into several groups, then K becomes smaller in general

4. Parameter Learning Framework

For parameter learning, we assume the structure G is fixed and known to every participating party. For Model I, we use the definition of cross variable and cross parents defined in [3]. N_{ijk} is the number of records such that X_i is in kth category while its parents are in jth category.

For each party a_i:

(1) Randomize cross parents belong to its own party according to their respective privacy requirements using post randomization described in Section 3. Randomizations are done independently for each (combined) variable and each record. (2) Send randomized cross parents of party a_i for party a_j to party a_j together with the probability transition matrix used. (3) Learn parameters for local variables in party a_i . This step does not involve randomized data. (4) Estimate the sufficient statistics N_{ijk} s for each cross variable belonging to its own party a_i using local data and randomized parent data from other parties. (5) Compute the parameters for cross variables using the estimated sufficient statistics $\hat{N}_{ijk}$ s. (6) Share the parameters with all other parties. Variables in one party are not randomized for its own calculations.

Steps of learning parameters for Model II:

For each party a_i:

(1) Randomize all its sensitive variables according to their respective privacy requirements using post randomization described in Section 3. Randomizations are done independently for each (combined) variable and each record. (2) Send randomized data and their corresponding probability transition matrices to the data miner.

For the data miner:

(1) Estimate the sufficient statistics N_{ijk} for each

node X_i using the randomized data from participated parties. (2) Estimate the parameters using the estimated sufficient statistics $\hat{N}_{ijk}$. (3) Broadcast the parameters to all parties.

The details of estimation of sufficient statistics and parameter (step 4 and 5 for Model I, Step 1 and 2 for data miner in Model II) from randomized data are described in Section 5.

5. Estimation of BN Parameters

The problem of privacy-preserving BN Parameter learning can be decomposed into a set of estimation of N_{ijk} s for each node X_i and a given fixed structure G from the randomized data $\tilde{D}$. Consider the following general case: Variable X_i with cardinality K_i has Q parent nodes $Pa_i(1),\Lambda\ ,Pa_i(Q)$. The cardinality of $Pa_i(q)$ is $K_{Pa_i(q)}$. These variables can be arbitrary vertically partitioned to different parties in both models. The randomization of each (combined) variable can also be done by grouping the categories of the variables into groups. We have the following different cases for estimating N_{ijk} s from the randomized data $\tilde{D}$ due to simultaneous randomization: (a) X_i and its parents are all randomized independently each other. (b) Some parents of X_i are randomized simultaneously. (c) X_i is randomized simultaneously with some of its parents. (d) X_i is randomized simultaneously with non-parent variables.

For (b) and (c) above, we can consider the simultaneously randomized variables as combined variables in estimating the sufficient statistics. For example, if variable X_i is randomized simultaneously with one of its parents $Pa_i(1)$, N_{ijk} is equal to the number of records such that $(X_i; Pa_i(1)) = (k, j^1)$, $Pa_i(2) = j^2$, …, $Pa_i(Q) = j^Q$, where $(X_i; Pa_i(1))$ is a combined variable. Thus, we can estimate the N_{ijk} s from the randomized data by considering $(X_i; Pa_i(1))$ as a single variable with cardinality $|K_i| \times |K_{Pa_i}(1)|$. For case (d), since the current N_{ijk} does not involve the variable randomized simultaneously with X_i, the learner can get the marginal transition probability matrix from the given transition matrix of the combined variable.

From the above arguments, we conclude that the cases (b), (c), and (d) above can effectively be considered to be equivalent to case (a). Hence, without loss of generality, we can discuss case (a) only. We denote by $Pa(X_i)$ as a compound variable for all the parents of Variable X_i. Hence $Pa(X_i)$ takes $J_i = \prod_{q=1}^{Q} K_{Pa_i(q)}$ different values. $N_{ij} = \sum_{k=1}^{K_i} N_{ijk}$ and N_i is $J_i K_i$ dimensional vector of N_{ijk} values, that is $N_i = (N_{i11}, N_{i12}, \Lambda\ , N_{i1K_i}, N_{i21}, \Lambda\ N_{iJ_iK_i})^t$, where superscript t denotes matrix transpose. $N_i(l)$ is an element of N_i. $\tilde{N}_{ijk}$, $\tilde{N}_{ij}$, and $\tilde{N}_i$ are defined similarly as N_{ijk}, N_{ij}, and N_i, respectively but for the randomized data $\tilde{D}$. $\hat{N}_{ijk}$, $\hat{N}_{ij}$, and $\hat{N}_i$ are estimators of N_{ijk}, N_{ij}, and N_i respectively. Given the training data D with N records of variables X_i and its Q parents in the above general case, if they are post-randomized with probability transition matrices P^i, $P^{Pa^i(1)}$, …, $P^{Pa^i(Q)}$ respectively, we have the following theorem.

Theorem 1: $E[\tilde{N}_i \mid D] = P^t N_i$, where

$P = P^i \otimes P^{pa^i}$ and $P^{pa^i} = P^{pa^i(1)} \otimes \Lambda \otimes P^{pa^i(Q)}$, $\otimes$ denotes Kronecker matrix product. Moreover,

$Cov[\tilde{N}_i \mid D] = \sum_{l=1}^{J_i K_i} N_i(l) V_l$, where V_l is a $K_i J_i \times K_i J_i$ matrix with (l_1, l_2) th element

$$V_l(l_1, l_2) = \begin{cases} P(l,l_1)(1-P(l,l_1)) & l_1 = l_2 \\ -P(l,l_1)P(l,l_2) & l_1 \neq l_2 \end{cases}.$$

Proofs are omitted here due to the page limitations. Interested readers can refer to a longer version of this paper for details [6]. The following theorem establishes the bias and variance of the estimator $\hat{N}_i = (P^t)^{-1} \tilde{N}_i$. Its proof is straight-forward and is omitted.

Theorem 2: $\hat{N}_i = (P^t)^{-1} \tilde{N}_i$ is an unbiased estimator for N_i and $Cov\{\hat{N}_i \mid D\} = (P^{-1})^t Cov\{\tilde{N}_i \mid D\}(P^{-1})$, where P and $Cov\{\tilde{N}_i \mid D\}$ are as defined in Theorem 1.

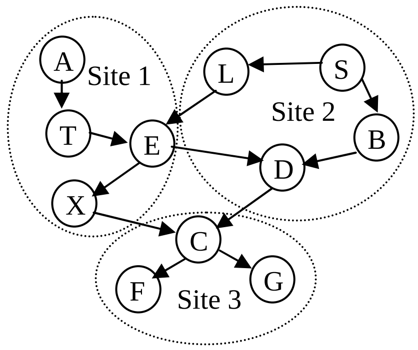
We use the estimated sufficient statistics to get ML estimate of the parameter as $\hat{\theta}^{ML}_{ijk} = \frac{\hat{N}_{ijk}}{\hat{N}_{ij}} = \frac{\hat{N}_{ijk}}{\sum_{k=1}^{K_i} \hat{N}_{ijk}}$

and the MAP estimate of the parameter as $\hat{\theta}_{ijk} = \frac{\alpha_{ijk} + \hat{N}_{ijk}}{\alpha_{ij} + \hat{N}_{ij}}$, where we assume the prior distribution of θ_{ij} is Dirichlet with parameter $\{\alpha_{ij1}, \alpha_{ij2}, \Lambda, \alpha_{ijr_i}\}$.

6. Experimental Results

In this experiment, we use the Bayesian Network shown in Fig. 2, where the variables are distributed over three sites. All variables are binary except variables *L* and *B* which are ternary. The conditional probabilities of the different nodes are also shown. 20000 samples were generated from this Bayesian Network to form the dataset *D*.

This data was then randomized according to the scheme described in Table 1, where variables *T*, *S*, and *G* were considered not sensitive and hence not randomized. The corresponding at most γ amplification is also shown in Table 1. *K*=2 for Binary randomization while *K*=3 for ternary randomization. Table 2 shows a part of parameters learnt from the randomized data using the algorithm described in Section 4 for model II.[1] The remaining part can be calculated by one minus the given part. All the values in the Table are average over 5 runs, with the corresponding standard deviation indicated in parenthesis. It is clear from the Table that the proposed algorithms can accurately learn the BN parameters for both scenarios, even for moderate levels of randomization.



A	0.7,0.3	T	0.1,0.9,0.9,0.1
S	0.5,0.5	L	0.3,0.7,0.4,0.15,0.3,0.15
X	0.2,0.6,0.8 ,0.4	F	0.25,0.9,0.75,0.1
E	0.25,0.8,0.15,0.5,0.3,0.4,0.75,0.2,0.85,0.5,0.7,0.6		
D	0.7,0.65,0.1,0.4,0.8,0.35,0.3,0.35,0.9,0.6,0.2 ,0.65		
C	0.9,0.4,0.6,0.25,0.1,0.6,0.4,0.75		
B	0.8 ,0.15,0.1,0.5,0.1,0.35		
G	0.2,0.4,0.8,0.6		

Fig. 2 A Bayesian Network

[1] Less randomization occurs in Model I, so the results for Model I were better than those for model II. We present only the results for model II here.

A,D	Binary symmetric $p_1 = p_2 = 0.25$, $\gamma = 3$
L,B	Ternary symmetric $p_1 = p_2 = 0.15$ $\gamma = 4.67$
E	Binary symmetric $p_1 = p_2 = 0.2$ $\gamma = 4$
X	Binary symmetric $p_1 = p_2 = 0.2$ $\gamma = 4$
C,F	Binary $p_1 = 0.1$ $p_2 = 0.25$ $\gamma = 9$

Table 1: Randomization performed

A	0.70(0.70)	*T*	0.10(0.50) 0.90(0.77)
S	0.50(0.00)	*X*	0.20(0.80)0.60(1.1)
L	0.30(0.49)0.71(0.57)0.39(0.64)0.14(0.55)		
B	0.80(0.77)0.16(0.41)0.094(0.71)0.49(0.73)		
E	0.25(0.20)0.81(0.90)0.14(2.7)0.51(1.2) 0.31(2.6)0.41(2.34)		
D	0.69(2.2)0.65(1.3)0.11(3.3)0.38(0.77)0.79(1.7) 0.39(5.65)		
C	0.90(2.0)0.38(1.6)0.61(2.6)0.25(2.1)		
F	0.24(0.73)0.91(1.1)		
G	0.20(0.30) 0.40(0.29)		

Table 2: Mean and standard deviation ($\times 10^{-2}$) over 5 runs of parameters learnt from the randomized data**.**

7. Conclusion

We have proposed a post randomization technique to learn parameters of a Bayesian network from distributed heterogeneous data. Our method estimates the sufficient statistics from the randomized data, which are subsequently used to learn the parameters. Our experiments show that the post randomization is an efficient, flexible and easy-to-use method to learn Bayesian network from privacy sensitive data. Post Randomization method can be easily upgraded to learn BN Structures from sensitive data. It is clear that it can also be applied to Privacy-Preserving decision tree learning.

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