An FDI Robust Filter Based-on LMI Control

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Abstract: - Starting from the narrow relationship between robust filtering and the design of robust filters to fault detection, an approach for fault detection and diagnosis robust filter design in linear systems is presented. The method consists on transforming the problem of robust detection of faults in a problem of robust control based on Linear Matrix Inequalities (LMI). The transformation is obtained by means of the design of a dynamic system (post-filter), which is obtained through the synthesis of robust controllers based on LMI. Thus, performance index in $\mathcal{H}_2$, $\mathcal{H}_\infty$, and multi-objective criteria ($\mathcal{H}_2/\mathcal{H}_\infty$) can be obtained. This formulation allows the application of any technique of robust control based on LMI's, and the robust fault detection and isolation is guaranteed.

Key-Words: - Fault Detection. Linear Matrix Inequalities (LMI) Control. $\mathcal{H}_2$-$\mathcal{H}_\infty$ Control. Robust Estimation.

1 Introduction

Some important connections exist between robust estimation or filtering and fault detection and isolation (FDI) robust filter design. In general, the problem consists in designing an asymptotic stable dynamical system (filter) able to cope with perturbations (rejection, for example) and uncertainties of the models, [13].

In the robust estimation problem, the goal is to derive an optimal estimate of the system state vector or a linear combination of the states being aware of the presence of perturbation and uncertainties (model). For the robust $\mathcal{H}_2$-$\mathcal{H}_\infty$ estimation problem, the $\mathcal{H}_2$ or $\mathcal{H}_\infty$ norm of the transfer matrix from the perturbations to the error estimation is lower than a prespecified level $\gamma > 0$: [11], [9].

Consider the following system:

$$\Sigma_1 \left\{ \begin{array}{rcl} \dot{x}(t) & = & Ax(t) + B_1\omega(t) \\ z(t) & = & C_1x(t) \\ y(t) & = & C_2x(t) + D\omega(t), \end{array} \right. \tag{1}$$

where $x \in \Re^n$ is the state, $z \in \Re^m$ is the signal to be estimated from the measured signal $y \in \Re^p$; $\omega \in \mathcal{L}_2$ is a perturbation. The matrices A, B_1, C_1, C_2 and D are matrices of appropriate dimensions. The pair (A, B_1) is stabilizable and pair (A, C_2) is detectable.

We are interested, in the first term, in to find a dynamical system denoted by $\mathcal{F}$, a filter, which allows to obtain an estimate $\hat{z}$ of z, where $\hat{z}$ is the output of the filter satisfying the following conditions: the filter is asymptotically stable and the effect of the perturbation on the estimation error is as small as possible, [6, 8, 9, 11, 18]. This is, $\mathcal{F}$ is such that

- $\hat{z}(t) = \mathcal{F}y(t)$.
- If the estimation error is $e_z(t) = z(t) - \hat{z}(t)$ and $\omega(t) = 0$, then $\lim_{t\to\infty} e_z(t) = 0$.
- If the transfer matrix from ω to e_z is denoted by $H_{e_z\omega}$ we want:
    1. $\mathcal{H}_2$ framework: $||H_{e_z\omega}(s)||_2 < \mu$, $\mu > 0$.
    2. In $\mathcal{H}_\infty$: $\sup_{0\neq\omega\in\mathcal{L}_2} \frac{||e_z||_2^2}{||\omega||_2^2} = ||H_{e_z\omega}||_\infty < \gamma$, $\gamma > 0$.

Under the considered assumptions, an admissible filter can be expressed as

$$\mathcal{F}_L \left\{ \begin{array}{rcl} \dot{\hat{x}}(t) & = & A\hat{x}(t) + L(y(t) - C_2\hat{x}(t)) \\ \hat{z}(t) & = & C_1\hat{x}(t), \end{array} \right. \tag{2}$$

where L is a gain matrix to be designed. Optimal solutions have been showed in [9, 11, 18].

On the another hand, for robust FDI filter design, the first step consists in the generation of residuals (fault detection), used in a second step for the diagnosis process (fault isolation). The residuals are generated by a dynamical system (filter) and are only significative when the system is affected by a fault. The input of this dynamical system is a measured output and it has to be able to distinguish if the qualitative changes on the system behavior are due to perturbations, uncertainties or to faults, [5, 15].

The relations between the $\mathcal{H}_\infty$ robust estimation problem and FDI filter design has been investigated in [5]. In this case the addressed problem is the fault detection in presence of perturbations; the diagnosis problem is considered in a second level and the fault separation is obtained through a multiple filtering, [12]. In [15] this relations are reformulated and an FDI robust filter based on $\mathcal{H}_\infty$ optimization approach is obtained. In a same way, in [19] the robust fault detection problem has been formulated as a model matching problem in $\mathcal{H}_\infty$, which is solved through LMI's.

A way to consider robustness in the FDI filter design can be defining a sensitivity measure which characterizes the filter sensitivity regarding to the possible faults in comparison to the filter sensitivity regarding to perturbations.

Let us introduce

$$\mathcal{S}_{2i} = \frac{||H_{e_z\nu_i}||_2}{||H_{e_z\omega}||_2}; \quad \text{or} \quad \mathcal{S}_{\infty i} = \frac{||H_{e_z\nu_i}||_\infty}{||H_{e_z\omega}||_\infty};$$

where ν_i are the signals characterizing the faults, ω is the perturbation and e_z the estimation error. It is clear that if for a given i, $\mathcal{S}_{2i}$ or $\mathcal{S}_{\infty i}$ is significative, this means that for fault i, the filter is more sensitive to ν_i than to ω. In this context, it is necessary to generate novel approaches to improve the sensibility $\mathcal{S}_{2i}$ or $\mathcal{S}_{\infty i}$ of the filters. The problem consists in designing a filter which in some sense maximizes $\mathcal{S}_{2i}$ or $\mathcal{S}_{\infty i}$: [5, 12, 15, 19]. It is also clear that the problem can be formulated as a multi objective design problem and multiple filters are necessary in this case to solve the fault isolation problem.

In this paper, a method is proposed for the design of a robust FDI filter. The particularity of the approach is the use of a post-filter which in connection with the robust fault detection filter, obtained by solving an $\mathcal{H}_2$-$\mathcal{H}_\infty$ control design problem based on LMI's, which allows to solve simultaneously the fault detection and diagnosis problems and where multi-objective criteria can be formulated.

2 Robust filtering with post-filter

The post-filter is a dynamical system whose input is the innovation signal defined as the difference between the output and its estimate. We denote the post-filter by $\mathcal{F}_p$. We consider the filter $\mathcal{F}_L$ defined by:

$$\begin{aligned} \dot{\hat{x}}(t) &= A\hat{x}(t) + L(y(t) - C_2\hat{x}(t)) - \mathbf{B}_e u_e(t) \\ \hat{z}(t) &= C_1\hat{x}(t); \end{aligned} \qquad (3)$$

where $u_e(t)$ is the output of the post-filter $\mathcal{F}_p$, and $\mathbf{B}_e$ is its input matrix of appropriate dimension.

The error dynamic is obtained manipulating straightforwardly the filter equation (3). It is given by (see Fig. 1):

$$\begin{aligned} \dot{e}_x(t) &= (A - LC_2)e_x(t) + (B_1 - LD)\omega(t) + \mathbf{B}_e u_e(t) \\ e_z(t) &= C_1 e_x(t). \end{aligned} \qquad (4)$$

First, we can note that if $\mathbf{B}_e = -(B_1 - LD)$ and $u_e = \omega$, we can isolate completely the error from perturbation ω. The reconstruction of ω can be obtained from (1), considering $V = D^TD$ non singular, by the inverse system

$$\begin{aligned} \dot{\zeta}(t) &= (A + B_1V^{-1}D^TC_2)\zeta(t) - B_1V^{-1}D^Ty(t) \\ u_e(t) &= V^{-1}D^TC_2\zeta(t) - V^{-1}D^Ty(t); \end{aligned}$$

where $u_e(t)$ can be considered as an estimate of $\omega(t)$. In this way, a good rejection level can be attained and this is the pursued idea in the introduction of a post-filter.

2.1 The post-filter design

In this case, the robust filtering problem is transformed in a robust control problem. Introduce the innovation signal

$$e_y(t) = y(t) - C_2\hat{x}(t) = C_2e_x(t) + D\omega(t);$$

and define the dynamical post-filter equation

$$\mathcal{F}_p \left\{ \begin{aligned} \dot{\zeta}(t) &= \mathbf{A}_p\zeta(t) + \mathbf{B}_p e_y(t) \\ u_e(t) &= \mathbf{C}_p\zeta(t) + \mathbf{D}_p e_y(t), \end{aligned} \right. \qquad (5)$$

where the matrices $\mathbf{A}_p$, $\mathbf{B}_p$, $\mathbf{C}_p$, and $\mathbf{D}_p$ must be designed. In closed loop we obtain

$$\left\{ \begin{aligned} \dot{e}_x(t) &= (A - LC_2 + \mathbf{B}_e\mathbf{D}_pC_2)e_x(t) + \mathbf{B}_e\mathbf{C}_p\zeta(t) + \\ &\quad (B_1 - LD + \mathbf{B}_e\mathbf{D}_pD)\omega(t) \\ \dot{\zeta}(t) &= \mathbf{B}_pC_2e_x(t) + \mathbf{A}_p\zeta(t) + \mathbf{B}_pD\omega(t) \\ e_z(t) &= C_1e_x(t). \end{aligned} \right.$$

As one can see, it is possible to select now L and after determine $\mathbf{A}_p$, $\mathbf{B}_p$, $\mathbf{C}_p$, $\mathbf{D}_p$, and $\mathbf{B}_e$ such that

$$\begin{pmatrix} A - LC_2 + \mathbf{B}_e\mathbf{D}_pC_2 & \mathbf{B}_e\mathbf{C}_p \\ \mathbf{B}_pC_2 & \mathbf{A}_p \end{pmatrix}$$

be asymptotically stable and $||H_{e_z\omega}||_2 < \mu$ or $||H_{e_z\omega}||_\infty < \gamma$. If $\mathbf{B}_e = 0$ we recover the previous case. We can note that in some cases $L = 0$ leads to a solution.

In summary, the problem to be solved is the design of a post-filter $\mathcal{F}_p$, whose output is u_e. In this case $\mathbf{B}_e$ is a design parameter. Thus, if $u_e \in \Re^q$, where $1 \leq q \leq n$, then $\mathbf{B}_e \in \Re^{n\times q}$.

Introduce the error equations:

$$\left\{ \begin{aligned} \dot{e}_x(t) &= \tilde{A}e_x(t) + \tilde{B}_1\omega(t) + \mathbf{B}_e u_e(t) \\ e_z(t) &= C_1e_x(t) \\ e_y(t) &= C_2e_x(t) + D\omega(t); \end{aligned} \right. \qquad (6)$$

where $\tilde{A} = A - LC_2$, and $\tilde{B}_1 = B_1 - LD$. The problem is to design a control u_e obtained, from the output e_y, such that the $\mathcal{H}_2$-$\mathcal{H}_\infty$ norm of the transfer matrix from the perturbation ω to the controlled output e_z be minimum. This is typically a well known $\mathcal{H}_2$-$\mathcal{H}_\infty$ optimal control design for which a solution is given in [4, 7, 17]. Then, we have translated the filtering problem to an $\mathcal{H}_2$ -$\mathcal{H}_\infty$ optimal control problem, where also the "control matrix" $\mathbf{B}_e$ is a design parameter. The problem solution is a dynamical controller, which defines the post-filter.

With this formulation, the transfer matrix $H_{e_z\omega}(s)$ is given by

$$H_{e_z\omega}(s) = \left[\begin{array}{c|c} \begin{pmatrix} \tilde{A} + B_e\mathbf{D}_pC_2 & \mathbf{B}_e\mathbf{C}_p \\ \mathbf{B}_pC_2 & \mathbf{A}_p \end{pmatrix} & \begin{pmatrix} \tilde{B}_1 + \mathbf{B}_e\mathbf{D}_pD \\ \mathbf{B}_pD \end{pmatrix} \\ \hline (C_1 \quad 0) & 0 \end{array}\right].$$

To design $\mathcal{F}_p$, the two steps procedure is the following:

- Design or choose L. Particular cases requires that the matrix $\tilde{A}$ has a particular structure, (for example in the fault isolation problem).

- To solve a $\mathcal{H}_2$-$\mathcal{H}_\infty$ optimal control problem for the system (6), where the obtained dynamical controller is the post-filter $\mathcal{F}_p$. Here the matrix $\mathbf{B}_e$ must also be designed.

The proposed diagram of blocks is shown in the Figure 1.

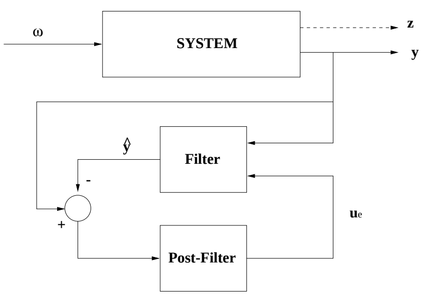

Figure 1: Post-filter scheme.

2.2 Post-filter design via LMI control

We consider the case of optimal controller synthesis based on LMI, where the control matrix is a design parameter. Introduce the dynamical system

$$\begin{aligned} \dot{e}_x(t) &= \tilde{A}e_x(t) + \tilde{B}_1\omega(t) + \mathbf{B}_eu_e(t) \\ e_z(t) &= C_1e_x(t) + D_{11}\omega(t) + D_{12}u_e(t) \\ e_y(t) &= C_2e_x(t) + D_{21}\omega(t). \end{aligned} \quad (7)$$

A controller by output dynamic feedback is given by (5). Then, the closed loop system is given by

$$\dot{e}_x(t) = \mathbb{A}e_x(t) + \mathbb{B}\omega(t), \quad e_z(t) = \mathbb{C}e_x(t) + \mathbb{D}\omega(t), \quad (8)$$

where

$$\mathbb{A} = \begin{pmatrix} \tilde{A} + \hat{\mathbf{D}}C_2 & \hat{\mathbf{C}} \\ \mathbf{B}_pC_2 & \mathbf{A}_p \end{pmatrix}, \quad \mathbb{B} = \begin{pmatrix} \tilde{B}_1 + \hat{\mathbf{D}}D_{21} \\ \mathbf{B}_pD_{21} \end{pmatrix},$$

$$\mathbb{C} = (C_1 + D_{12}\mathbf{D}_pC_2 \qquad D_{12}\mathbf{C}_p),$$
$$\mathbb{D} = D_{12}\mathbf{D}_pD_{21} + D_{11};$$

and

$$\hat{\mathbf{C}} = \mathbf{B}_e\mathbf{C}_p, \quad \hat{\mathbf{D}} = \mathbf{B}_e\mathbf{D}_p.$$

Function transfer $H_{e_z\omega}(s)$ is given by: $H_{e_z\omega}(s) = \left[\begin{array}{c|c} \mathbb{A} & \mathbb{B} \\ \hline \mathbb{C} & \mathbb{D} \end{array}\right]$.

2.2.1 $\mathcal{H}_2$ Formulation

In this case we must design $\mathbf{B}_e$ and the controller given by (5) such that $||H_{e_z\omega}||_2 < \mu$, $\mu > 0$. It is well known that $||H_{e_z\omega}||_2^2 < \mu$ if and only if there exist symmetric $\mathbb{X} > 0$ such that, [2, 17]:

$$\begin{bmatrix} \mathbb{A}\mathbb{X} + \mathbb{X}\mathbb{A}^T & \mathbb{B} \\ \mathbb{B}^T & -\mathbb{I} \end{bmatrix} < 0, \quad \begin{bmatrix} \mathbb{W} & \mathbb{C}\mathbb{X} \\ \mathbb{X}\mathbb{C}^T & \mathbb{X} \end{bmatrix} > 0,$$

$$\text{tr}[\mathbb{W}] < \mu, \quad \mathbb{D} = 0.$$

Proposition 2.1 *We consider the system (7). That system is stabilizable by a dynamic controller given by (5), such that $||H_{e_z\omega}||_2^2 < \mu$ iff there exists symmetric matrices of n order $\mathbf{X} > 0$ and $\mathbf{Y} > 0$; matrices $\mathbf{Q}$, $\mathbf{L} \in \Re^{n\times n}$; matrices $\mathbf{F} \in \Re^{n\times p}$, $\mathbf{R} \in \Re^{q\times p}$, $\mathbf{M} \in \Re^{q\times n}$, $\mathbf{N} \in \Re^{n\times p}$; and a symmetric matrix $\mathbb{W} \in \Re^{m\times m}$ satisfying the following LMI's:*

$$\begin{bmatrix} \tilde{A}\mathbf{X} + \mathbf{X}\tilde{A}^T + \mathbf{L} + \mathbf{L}^T & \tilde{A} + \mathbf{N}C_2 + \mathbf{Q}^T & \tilde{B}_1 + \mathbf{N}D_{21} \\ (\star)^T & \mathbf{Y}\tilde{A} + \tilde{A}^T\mathbf{Y} + \mathbf{F}C_2 + C_2^T\mathbf{F}^T & \mathbf{Y}\tilde{B}_1 + \mathbf{F}D_{21} \\ (\star)^T & (\star)^T & -\mathbb{I} \end{bmatrix} < 0,$$

$$\begin{bmatrix} \mathbb{W} & C_1\mathbf{X} + D_{12}\mathbf{M} & C_1 + D_{12}\mathbf{R}C_2 \\ (\star)^T & \mathbf{X} & \mathbb{I} \\ (\star)^T & (\star)^T & \mathbf{Y} \end{bmatrix} > 0,$$

$$tr[\mathbb{W}] < \mu, \qquad \mathbb{D} = D_{12}\mathbf{R}D_{21} + D_{11} = 0.$$

Thus, the control matrix $\mathbf{B}_e$ is given by

$$\mathbf{B}_e = \mathbf{L}\mathbf{M}^T\left(\mathbf{M}\mathbf{M}^T\right)^{-1}.$$

The controller is obtained from

$$\begin{pmatrix} \mathbf{A}_p & \mathbf{B}_p \\ \hat{\mathbf{C}} & \hat{\mathbf{D}} \end{pmatrix} = \begin{pmatrix} \mathbf{V}^{-1} & -\mathbf{V}^{-1}\mathbf{Y} \\ 0 & \mathbb{I} \end{pmatrix} \begin{pmatrix} \mathbf{Q} - \mathbf{Y}\tilde{A}\mathbf{X} & \mathbf{F} \\ \mathbf{L} & \mathbf{N} \end{pmatrix} \begin{pmatrix} \mathbf{U}^{-1} & 0 \\ -C_2\mathbf{X}\mathbf{U}^{-1} & \mathbb{I} \end{pmatrix},$$

$$\mathbf{C}_P = \left(\mathbf{B}_e^T\mathbf{B}_e\right)^{-1}\mathbf{B}_e^T\hat{\mathbf{C}}, \qquad \mathbf{D}_P = \mathbf{R},$$

where $\mathbf{V}$ and $\mathbf{U}$ are non singular matrices satisfying $\mathbf{Y}\mathbf{X} + \mathbf{V}\mathbf{U} = \mathbb{I}$.

Proof
The proof is based on the typical linearization procedure of the matrix inequalities through the congruence transformation and variable changes. ■

In the particular case where the system is given by (6), and $\mathbb{D} = 0$, the obtained dynamic controller represents the post-filter, which allows to attenuate the perturbation effects in an optimal way.

2.2.2 $\mathcal{H}_\infty$ Formulation

In this context we should design $\mathbf{B}_e$ and the controller (5) such that $||H_{e_z\omega}||_\infty < \gamma$, $\gamma > 0$.

From Bounded Real Lemma, it is well known that $||H_{e_z\omega}||_\infty < \gamma$ iff there exists $\mathbb{X} > 0$ such that, [2, 7]:

$$\begin{bmatrix} \mathbb{A}^T\mathbb{X} + \mathbb{X}\mathbb{A} & \mathbb{X}\mathbb{B} & \mathbb{C}^T \\ \mathbb{B}^T\mathbb{X} & -\gamma\mathbb{I} & \mathbb{D}^T \\ \mathbb{C} & \mathbb{D} & -\gamma\mathbb{I} \end{bmatrix} < 0.$$

Proposition 2.2 *Let be the system (7). The dynamic controller given by (5) stabilizes that system and $||H_{e_z\omega}||_\infty < \gamma$ iff there exists symmetric matrices of n order $\mathbf{X} > 0$ and $\mathbf{Y} > 0$; matrices $\mathbf{Q}, \mathbf{L} \in \Re^{n\times n}$; matrices $\mathbf{F} \in \Re^{n\times p}$, $\mathbf{R} \in \Re^{q\times p}$ and $\mathbf{M} \in \Re^{q\times n}$,$\mathbf{N} \in \Re^{n\times p}$; satisfying the following LMI's:*

$$\begin{bmatrix} \tilde{A}\mathbf{X} + \mathbf{X}\tilde{A}^T + \mathbf{L} + \mathbf{L}^T & \tilde{A} + \mathbf{N}C_2 + \mathbf{Q}^T & \tilde{B}_1 + \mathbf{N}D_{21} & \mathbf{X}C_1^T + \mathbf{M}^T D_{12}^T \\ (\star)^T & (\mathbb{YY}) & \mathbf{Y}\tilde{B}_1 + \mathbf{F}D_{21} & C_1^T + C_2^T\mathbf{R}^T D_{12}^T \\ (\star)^T & (\star)^T & -\gamma\mathbb{I} & D_{11}^T + D_{21}^T\mathbf{R}^T D_{12}^T \\ (\star)^T & (\star)^T & (\star)^T & -\gamma\mathbb{I} \end{bmatrix} < 0,$$

$$\begin{bmatrix} \mathbf{X} & \mathbb{I} \\ (\star)^T & \mathbf{Y} \end{bmatrix} > 0,$$

where

$$(\mathbb{YY}) = \mathbf{Y}\tilde{A} + \tilde{A}^T\mathbf{Y} + \mathbf{F}C_2 + C_2^T\mathbf{F}^T.$$

Then, the control matrix $\mathbf{B}_e$ is given by

$$\mathbf{B}_e = \mathbf{L}\mathbf{M}^T\left(\mathbf{M}\mathbf{M}^T\right)^{-1}.$$

The dynamic controller is obtained from

$$\begin{pmatrix} \mathbf{A}_p & \mathbf{B}_p \\ \hat{\mathbf{C}} & \hat{\mathbf{D}} \end{pmatrix} = \begin{pmatrix} \mathbf{V}^{-1} & -\mathbf{V}^{-1}\mathbf{Y} \\ 0 & \mathbb{I} \end{pmatrix} \begin{pmatrix} \mathbf{Q} - \mathbf{Y}\tilde{A}\mathbf{X} & \mathbf{F} \\ \mathbf{L} & \mathbf{N} \end{pmatrix} \begin{pmatrix} \mathbf{U}^{-1} & 0 \\ -C_2\mathbf{X}\mathbf{U}^{-1} & \mathbb{I} \end{pmatrix},$$

$$\mathbf{C}_P = \left(\mathbf{B}_e^T\mathbf{B}_e\right)^{-1}\mathbf{B}_e^T\hat{\mathbf{C}}, \qquad \mathbf{D}_P = \mathbf{R},$$

where $\mathbf{V}$ and $\mathbf{U}$ are non singular matrices satisfying $\mathbf{YX} + \mathbf{VU} = \mathbb{I}$.

Proof

In a similar way that in the previous case, the proof is constructed from appropriate transformation and changes of variables on the LMI's. ■

Thus, the post-filter dynamical with $\mathcal{H}_\infty$ performance criterions can be obtained. Similar procedures allows to design the post-filter from multi-objective criteria.

2.3 FDI Robust Filter Design based on a post-filter

It will be considered the design of robust filtering based on a post-filter to derive a robust detector of faults.

The FDI robust filter design problem can be formulated in the post-filter framework, [15]. Under this formulation a dynamic system (post-filter) is obtained applying the previous results on robust filtering based on LMI's, in order to generate the residues and to separate the faults.

Some results on FDI filter design using LMI's are shown in [3, 12, 19], in which the problem of fault separation is not systematically solved.

The robust filters synthesis based on post-filters are obtained from previous results. In this case, the system (1) is used. Then, for $\mathcal{H}_2$ and $\mathcal{H}_\infty$ robust filtering, the dynamic controller is given by

$$\mathbf{A}_p = \mathbf{V}^{-1}(\mathbf{Q} - \mathbf{F}C_2\mathbf{X})\mathbf{U}^{-1} + \left(\tilde{A}\mathbf{X} + \mathbf{L} - \mathbf{N}C_2\mathbf{X}\right)\mathbf{U}^{-1} \quad (9)$$

$$\mathbf{B}_p = \mathbf{V}^{-1}\mathbf{F} + \mathbf{N} \quad (10)$$

$$\hat{\mathbf{C}} = (\mathbf{L} - \mathbf{N}C_2\mathbf{X})\mathbf{U}^{-1}, \qquad \hat{\mathbf{D}} = \mathbf{N} \quad (11)$$

where $\mathbf{V} = -\mathbf{Y}$ and $\mathbf{U} = \mathbf{X} - \mathbf{Y}^{-1}$.

The post-filter dynamic matrices $\mathbf{C}_p$ and $\mathbf{D}_p$ are obtained choosing $\mathbf{B}_e$ such that $\mathbf{C}_P = \left(\mathbf{B}_e^T\mathbf{B}_e\right)^{-1}\mathbf{B}_e^T\hat{\mathbf{C}}$ and $\mathbf{D}_P = \left(\mathbf{B}_e^T\mathbf{B}_e\right)^{-1}\mathbf{B}_e^T\hat{\mathbf{D}}$. In the particular case where $\mathbf{B}_e = \mathbb{I}_{n\times n}$, then $\mathbf{C}_p = \hat{\mathbf{C}}$, $\mathbf{D}_p = \hat{\mathbf{D}}$.

Based on post-filter formulation, we consider FDI robust filter design. Let us consider the dynamic system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B_1\omega(t) + B_2u(t) + \sum_{i=1}^{f} L_i\nu_i(t), \\ y(t) &= C_2x(t) + D\omega(t) + \sum_{i=1}^{f} M_i\nu_i(t), \end{aligned} \quad (12)$$

where L_i, M_i are the *fault directions* in the actuators and sensors, respectively; $\nu_i(t)$ is a signal characterizing the *fault mode*.

To detect the faults, it is necessary to generate residuals obtained from the estimation of the following signal:

$$z(t) = C_1x(t),$$

using a state estimator. We want to minimize $||H_{e_z\omega}||_2$ or $||H_{e_z\omega}||_\infty$, and to maximize $||H_{e_z\omega}||_2$ or $||H_{e_z\nu_i}||_\infty$, for $i = 1, \ldots, f$, where $H_{e_z\nu_i}$ are transfer matrices from ν_i to e_z. It is difficult to take simultaneously these requirements, [3, 13, 15, 16, 19]. The conditions ensuring that faults are detectable and separable are given in [10, 14]. The FDI filter synthesis is supported by that conditions.

For FDI robust filter design, the strategy of the post-filter is applied, using the previous results for $\mathcal{H}_2$-$\mathcal{H}_\infty$ robust filtering . Thus, the following dynamical for the estimation error is obtained:

$$\begin{cases} \dot{e}_x(t) = \tilde{A}e_x(t) + \tilde{B}_1\omega(t) + \mathbf{B}_e u_e(t) + \\ \qquad \sum_{i=1}^{f}(L_i - LM_i)\nu_i(t) \\ e_z(t) = C_1e_x(t) \\ e_y(t) = C_2e_x(t) + D\omega(t) + \sum_{i=1}^{f} M_i\nu_i(t). \end{cases} \quad (13)$$

The control matrix $\mathbf{B}_e$ also is a design parameter in order to derive a admissible control. Control signal u_e is obtained from the Proposition 2.1 or the Proposition 2.2. So, the post-filter allows to guarantee a performance level appropriate for the fault robust detection.

The first step consists in selecting L in order to guarantee the fault separability. The matrix B_e is a parameter of design. We must choose a structure for B_e in order to guarantee the solvability of the $\mathcal{H}_2$-$\mathcal{H}_\infty$ optimal control problem. Results can be extended to systems where the dynamic matrix is given by $A(t) = A_0 + \sum_{i=1}^{g} a_i(t)A_i$. There, A_0 is a stable matrix, A_i are non destabilizing terms, and $a_i(t) \in \mathcal{L}_2$, [3, 6].

3 Numerical example

Consider the following state equation

$$\begin{aligned} \dot{x} &= \begin{pmatrix} 0 & -102 & 0 \\ 181 & -171 & 0 \\ 0 & -1.12 \times 10^{-2} & 0 \end{pmatrix} x + \begin{pmatrix} 0 \\ 4.44 \\ 0 \end{pmatrix} \omega + \\ & \begin{pmatrix} 102 \\ 163 \\ 0 \end{pmatrix} u + \begin{pmatrix} 0 \\ 181 \\ 0 \end{pmatrix} \nu_1 \\ y &= \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0.978 \end{pmatrix} x + \begin{pmatrix} 0 \\ 0.978 \end{pmatrix} \nu_2; \end{aligned}$$

which is the model of a diesel engine actuator, [1]. Two faults are considered: fault on the actuator or fault on the sensor. We suppose that $z = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0.978 \end{pmatrix} x$.

The first step consists in selecting L, we choose, in order to obtain $A - LC_2$ decoupled in relation to the faults:

$$L = \begin{pmatrix} -102 & 0 \\ 0 & 0 \\ 1.12 \times 10^{-2} & 10 \end{pmatrix};$$

and the dynamic of the estimation error is described by:

$$\begin{aligned} \dot{e}_x &= \begin{pmatrix} 0 & 0 & 0 \\ 181 & -171 & 0 \\ 0 & 0 & -9.78 \end{pmatrix} e_x + \begin{pmatrix} 0 \\ 4.44 \\ 0 \end{pmatrix} \omega + \\ & \begin{pmatrix} 0 \\ 181 \\ 0 \end{pmatrix} \nu_1 + \begin{pmatrix} 0 \\ 0 \\ -9.78 \end{pmatrix} \nu_2 + B_e u_e \\ e_z &= \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0.978 \end{pmatrix} e_x, \\ e_y &= \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0.978 \end{pmatrix} e_x + \begin{pmatrix} 0 \\ 0.978 \end{pmatrix} \nu_2 \end{aligned}$$

We can note that, in order to ensure the fault separability, one fault is associated to each output error. To solve the "control problem" presented in the previous section, we assume $\nu_1 = \nu_2 = 0$. For the $\mathcal{H}_\infty$ case, we consider that $\mathbf{B}_e \in \Re^{3\times3}$, then the following results are obtained:

$$\gamma = 3.8280e - 07$$

$$\mathbf{A}_p = 1.0e+08 \begin{pmatrix} -0.0000 & -0.1342 & 0 \\ 0.0000 & -2.1035 & 0 \\ 0 & 0 & -0.3866 \end{pmatrix},$$

$$\mathbf{B}_p = 1.0e+08 \begin{pmatrix} -0.0406 & 0 \\ 1.1883 & 0 \\ 0 & -0.0084 \end{pmatrix}.$$

If $\mathbf{B}_e = \mathbb{I}_{3\times3}$, then

$$\mathbf{C}_p = 1.0e+07 \begin{pmatrix} -0.0000 & -0.5093 & 0 \\ -0.0000 & 0.3455 & 0 \\ 0 & 0 & -3.7019 \end{pmatrix},$$

$$\mathbf{D}_p = 1.0e+07 \begin{pmatrix} -0.4055 & 0 \\ -3.0056 & 0 \\ 0 & -0.0840 \end{pmatrix}.$$

The Figure 2 represents the frequency responses of the transfer matrices from the fault to the estimation error ($H_{e_z\nu_i}$) and from the perturbation to the estimation error ($H_{e_z\omega}$). We can appreciate the sensitivity of the filtering with respect to the faults in relation to the perturbation. The perturbation is rejected.

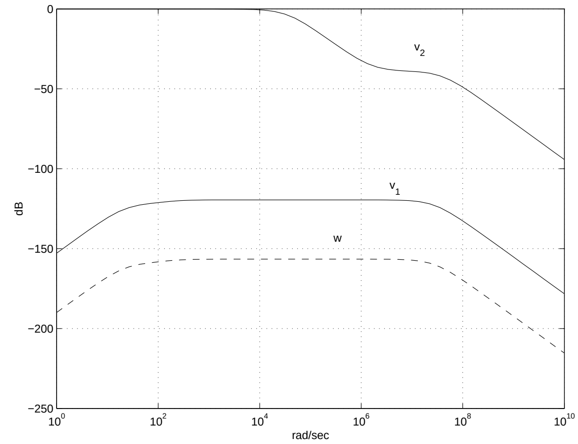

Figure 2: Singular values diagram: $H_{e_z\nu_2}$, $H_{e_z\nu_1}$, $H_{e_z\omega}$.

Figure 3 represents the estimation errors with the perturbation and in presence of faults. Fig. 3 shows the perturbation and a fault affecting the actuator at $t = 10s$. The perturbation magnitude is important. However, the fault is detected and isolated.

The fault sensor is present from $t = 20s$. Figure 3 shows that residues are significatives when the faults occur at $t = 10s$ and at $t = 20s$, and that each residual is associated with a fault guaranteeing faults separability. We can also note that the perturbation is rejected.

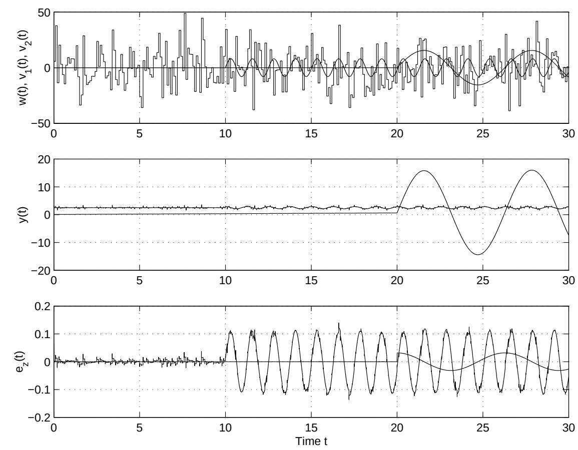

Figure 3: Filter responses in presence of faults and perturbation.

4 Conclusion

An approach for robust filter design of fault detection and isolation based on the optimal control problem has been presented. The method is based on the construction of a dynamical system called post-filter, which is obtained solving an optimal control problem through of LMI's, where the control matrix is considered a design parameter.

The method is based on two steps. The first step consists in designing a full state observer in a way ensuring faults separability. In a second step, the post-filter is designed to ensure asymptotic stability of the error dynamic and rejection of the perturbation. The fault detection and isolation problem thus is translated in an $\mathcal{H}_2$-$\mathcal{H}_\infty$ optimal control problem which is solved using LMI's machinery and multi-objective criteria can be applied.

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