Robust Anti-Windup Compensation for PID Controllers

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Abstract: - In this paper an approach for robust *anti-windup* compensation design for PID controllers based on the characterization of $\mathcal{H}_2$ and $\mathcal{H}_\infty$ norms as Linear Matrix Inequalities (LMI) is presented. The robustness is considered by assuring the closed loop performance, spite of unknown changes on the actuator saturation limits. The stability and performance margins are evaluated from $\mathcal{H}_2$ and $\mathcal{H}_\infty$ norms of the controller output with respect to the actuator output transfer function, which are LMI restrictions. The design of the robust compensation gain is assured by means of a parameter-dependent Lyapunov function obtained from a convex optimization procedure with LMIs, which can be solved in polynomial time by specialized algorithms.

Key-Words: - Anti-windup compensation. PID. Lyapunov stability. LMI. $\mathcal{H}_2$-$\mathcal{H}_\infty$ Control

1 Introduction

In general, the controlled industrial processes present actuator saturation problems. In control theory that restriction is denominated the bounded control problem, which leads to consider methods and technics that allow the practical installation of control systems.

On the other hand, the control systems can operate in multiple environments and with multiple objectives. Each specific situation defines the operation mode, which can require a controllercommutation. The modes commutation is the substitution in the plant inputs, considering that the controller output is replaced by another.

As a result of substitutions and limitations, the plant inputs will be different to the controller's output. When this happens, the controller outputs don't drive the plant appropriately and the controller's states will be strongly updated, [8, 9]. This effect is called Wind-Up. In global terms, the wind-up is one inconsistency among the control input given to the process and the internal states of the controller. The adverse effect of the wind-up is a significant performance deterioration, overshoots and even inestability, [9, 3].

The wind-up problem can be handled by means of compensation where, in a first stage, it is designed the control system without taking into account the restrictions; and in a second stage, some compensation scheme is found, with the purpose of minimizing the limitations and commutations effect. The last outlined focus has been denominated the *anti-windup bumpless transfer problem* (AWBT), [8].

A general framework for the AWBT problem has been presented in [8]. The development is based on the paradigm of designing a linear controller, which ignores the non linear inputs and incorporate AWBT compensation in order to minimize those adverse effects due to any non linearity in the control input.

The main advantage of this design methodology is that no restrictions are placed on the original linear controller design. The main disadvantage is that although the linear controller and anti-windup compensator both affect the closed-loop performance; so, the effect of the linear controller on the performance under saturation is completely ignored. In addition, the possible changes of the saturation limits is not considered, which can result very inconvenient.

In all solution cases for the AWBT problem by compensation, it is required of one residual signal obtained between the controller's output and the actuator's nonlinear output, [1, 3, 7, 8, 9, 11].

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2 Problem formulation

In order to introduce the robustness problem for the anti-windup compensation design; let us consider the following benchmark model [4, 5]:

$$\begin{aligned}
\dot{x}_1(t) &= -0.1x_1(t) + 0.5\sigma(u_1) + 0.4\sigma(u_2)\\
\dot{x}_2(t) &= -0.1x_2(t) + 0.4\sigma(u_1) + 0.3\sigma(u_2),\\
y_1(t) &= x_1,\\
y_2(t) &= x_2,
\end{aligned}$$

where the non-linear function $\sigma(\circ)$ denotes the actuator saturation, which is defined by

$$\sigma(u_i) = \begin{cases} u_{i_{min}} & \text{if} \quad u(t) < u_{i_{min}} \\ u(t) & \text{if} \quad u_{i_{min}} \leq u(t) \leq u_{i_{max}} \\ u_{i_{max}} & \text{if} \quad u(t) > u_{i_{max}}, \quad i = 1, 2, \ldots, p \end{cases} \tag{1}$$

In this example, the bounded controls $u_1 \in [-3, 3]$ and $u_2 \in [-10, 10]$ are considered. For satisfying control objectives, a PID controller with anti-windup compensation is designed, which is given by

$$\begin{aligned}
\dot{\zeta}_1(t) &= r_1 - x_1 + e_{11}[\sigma(u_1) - u_1]\\
&\quad + e_{12}[\sigma(u_2) - u_2]\\
\dot{\zeta}_2(t) &= r_2 - x_2 + e_{21}[\sigma(u_1) - u_1]\\
&\quad + e_{22}[\sigma(u_2) - u_2] \qquad (2)\\
u_1(t) &= 10(r_1 - x_1) + \zeta_1,\\
u_2(t) &= -10(r_2 - x_2) - \zeta_2,
\end{aligned}$$

where r_1, r_2 are the set-points, and $\mathbf{E_c} = \begin{pmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{pmatrix}$ is the compensation gain. According to [5], the calculated gain corresponds to

$$\mathbf{E_c} = \begin{pmatrix} 1.5 & 0.4 \\ 0.2 & 1.3 \end{pmatrix},$$

which satisfies the compensation requirements, just as is shown in Figure 1, where, for $r_1 = 0.6$ and $r_2 = 0.4$, the system outputs reach their references in spite of the actuator saturation.

On the other hand, if the saturation limits for the actuator change (for example, if $u_1 \in [-1, 1]$) then the closed loop system performance is not appropriate, just as is shown in Figure 2.

Thus, it can't be found robustness with respect to saturation limits changes, which is typical in an industrial processes control environment where the actuator elements: control valves, hydraulic actuators, etc., can be deteriorated for the intensive use, parts obsolescence, construction materials degradation, among other aspects. Therefore, it is necessary to design compensation mechanisms, in order to guarantee some robustness characteristics, which should consider changes in the actuator devices performance.

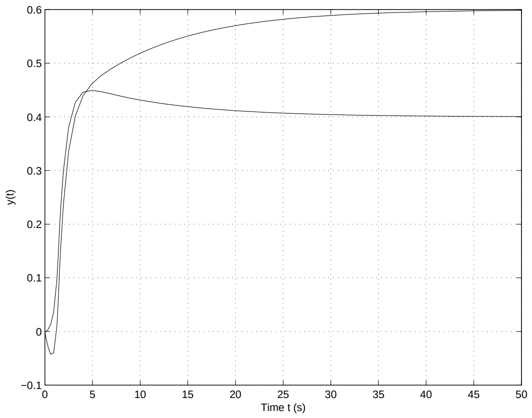

Figure 1: controlled Output using PI with compensation.

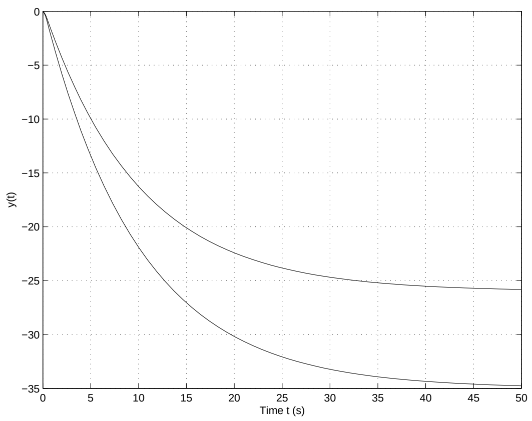

Figure 2: controlled system Output using PI with compensation and changed limits.

In this order of ideas, let us consider the linear system defined by

$$\begin{aligned}
\dot{x}(t) &= Ax(t) + B\sigma(u)\\
y(t) &= Cx(t),
\end{aligned} \tag{3}$$

where $x \in \mathbb{R}^n$ are the states, $u \in \mathbb{R}^p$ are the controls and $y \in \mathbb{R}^q$ are the outputs. The matrices A, B, C have appropriate dimensions.

The AWBT compensation problem is formulated from Figure 3, where, due to the limitations and/or substitutions, a non linearity appears among the controller's output and the effective process input.

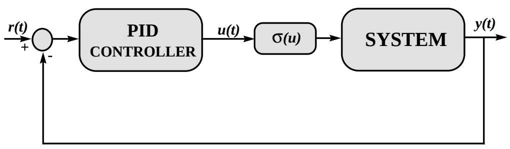

Figure 3: Control System with Actuators Saturation.

The effective control input $\sigma(u)$ is a non-linear function of the controller's output $u(t)$.

In order to satisfy the control requirements, a PID controller with compensation is considered, which is given by

$$
\begin{aligned}
\dot{\zeta}(t) &= e(t) + \mathbf{E_c}[\sigma(u) - u] \\
u(t) &= K_I\zeta(t) + K_P e(t),
\end{aligned}
\tag{4}
$$

where $e(t) = r(t) - y(t)$, K_I corresponds to the integral action gain, while K_P denotes the proportional action gain. $\mathbf{E_c}$, as it has already been mentioned, represents the compensation gain.

Thus, the system in closed loop is given by

$$
\begin{aligned}
\dot{x}(t) &= Ax(t) + B\sigma(u) \\
\dot{\zeta}(t) &= -Cx(t) + r(t) + \mathbf{E_c}[\sigma(u) - u] \\
y(t) &= Cx(t), \\
u(t) &= -K_P Cx(t) + K_I\zeta(t) + K_P r(t),
\end{aligned}
\tag{5}
$$

which is equivalent to

$$
\begin{aligned}
\dot{x}(t) &= (A - BK_PC)x(t) + BK_I\zeta + BK_P r(t) \\
&\quad + B[\sigma(u) - u] \\
\dot{\zeta}(t) &= -Cx(t) + r(t) + \mathbf{E_c}[\sigma(u) - u] \\
y(t) &= Cx(t), \\
u(t) &= -K_P Cx(t) + K_I\zeta(t) + K_P r(t).
\end{aligned}
\tag{6}
$$

As it is well known, the gains K_P and K_I are designed without considering the actuator saturation, while the effect of the saturation on the performance in closed loop is minimized by means of the selection of the gain $\mathbf{E_c}$. The compensation have effect when the signal $\vartheta = \sigma(u) - u$ is not null, and the additional feedback, see the Figure 4, is incorporated in order to make that signal be null, again, through the upgrade of the control signal u, which will be inside of the actuator saturation limits. When the signal ϑ is different to zero, its effect on the performance of the closed loop system can be studied from the perspective of analysis of the subjected systems to external disturbances. In this case, the disturbance signal is, in fact, the presence of the actuator saturation. Such analysis corresponds to the minimization, in some sense, the effect of the disturbance signal on the control signal u. So, let us consider the transfer function of the controller output u with respect to the residual signal, or *perturbation* signal, ϑ. This is, if $\theta(t) = \begin{pmatrix} x(t) \\ \zeta(t) \end{pmatrix}$, then

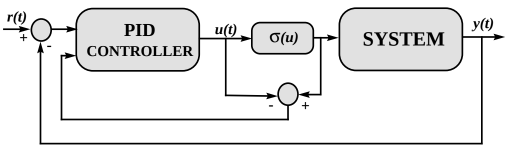

Figure 4: Control System with Saturation of Actuators.

$$
\begin{aligned}
\dot{\theta}(t) &= \mathbb{A}\theta(t) + \mathbb{B}\vartheta \\
u(t) &= \mathbb{C}\theta(t) + \mathbb{D}\vartheta,
\end{aligned}
\tag{7}
$$

where

$$
\mathbb{A} = \begin{pmatrix} A - BK_PC & BK_I \\ -C & 0 \end{pmatrix}, \quad \mathbb{B} = \begin{pmatrix} B \\ \mathbf{E_c} \end{pmatrix},
$$

$$
\mathbb{C} = \begin{pmatrix} -K_PC & K_I \end{pmatrix}, \quad \mathbb{D} = 0.
$$

Therefore, the transfer function $H_{u\vartheta}(s)$ is given by

$$
H_{u\vartheta}(s) = \left[\begin{array}{c|c} \mathbb{A} & \mathbb{B} \\ \hline \mathbb{C} & \mathbb{D} \end{array}\right] = \mathbb{C}(s\mathbb{I} - \mathbb{A})^{-1}\mathbb{B} + \mathbb{D}.
$$

The main condition to be satisfied is the closed loop stability, even under the saturation situation. It is guaranteed by means of the global stability of the bounded input system, that which, in case that this problem concerns, is guaranteed by means of the compensation gain design. This way, the synthesis problem consists in designing $\mathbf{E_c}$ that guarantees the effectiveness of the compensation under some robust stability condition in perturbed systems. Therefore, the compensation gain design problem can be focused starting from the norms $\mathcal{H}_2$ y $\mathcal{H}_\infty$.

Problem: Given the dynamic system (3), to design the *compensation gain* $\mathbf{E_c}$ for the controller (4), such that:

1. The closed loop system (7) be asymptotically stable.
2. The effect of signal ϑ on the control signal u be minimum, in some sense.

The closed loop stability is satisfied, in first place, for the appropriate selection of the PID controller gains. The second condition, lied to the bounded input systems stability, will allow the compensation under global stability. In this case, it is necessary to design the compensation gain in order to guarantee the closed loop stability and to minimize the effect of the *disturbance* signal ϑ on the control signal u when the saturation becomes present.

2.1 A framework in $\mathcal{H}_2$

In this case, we should design $\mathbf{E_c}$ such that $||H_{u\vartheta}||_2^2 < \mu$, for all $\mu > 0$. The following lemma is a well known result, which completely characterizes the $\mathcal{H}_2$ norm constraint through LMI [2, 10].

Lemma 2.1 *The inequality $||H_{u\vartheta}||_2^2 < \mu$ holds if, and only if, $\mathbb{D} = 0$ and there exists symmetric matrices $\mathbb{X} > 0$, and $\mathbb{W}$ such that*

$$
\begin{bmatrix} \mathbb{A}\mathbb{X} + \mathbb{X}\mathbb{A}^T & \mathbb{B} \\ (\circ)^T & -\mathbb{I} \end{bmatrix} < 0, \quad \begin{bmatrix} \mathbb{W} & \mathbb{C}\mathbb{X} \\ (\circ)^T & \mathbb{X} \end{bmatrix} > 0,
\tag{8}
$$
$$
tr(\mathbb{W}) < \mu.
$$

is feasible.

Proposition 2.1 *Consider the system defined by (3) and the PID controller with compensation given by (4). The controlled system is asymptotically stable with robust compensation, because $||H_{u\vartheta}||_2^2 < \mu$ if, and only if, there exist symmetrical matrices $\mathbb{X} > 0$ and $\mathbb{W} > 0$, and the matrix $\mathbf{Q}$, such that the following LMIs are satisfied:*

$$\begin{bmatrix} \mathbb{A}\mathbb{X} + \mathbb{X}\mathbb{A}^T & M_1 B + M_2 \mathbf{Q} \\ (\circ)^T & -\mathbb{I} \end{bmatrix} < 0, \quad \begin{bmatrix} \mathbb{W} & \mathbb{C}\mathbb{X} \\ (\circ)^T & \mathbb{X} \end{bmatrix} > 0, \quad tr(\mathbb{W}) < \mu; \tag{9}$$

where $M_1 = \begin{pmatrix} \mathbb{I} \\ 0 \end{pmatrix}$ and $M_2 = \begin{pmatrix} 0 \\ \mathbb{I} \end{pmatrix}$, have appropriate dimensions.

The compensation gain matrix $\mathbf{E_c}$ is given by

$$\mathbf{E_c} = \mathbf{Q}. \tag{10}$$

Proof

The proof is direct. The first inequality in (8) is re-written starting from the design variable $\mathbf{E_c}$, using the variables change $\mathbf{Q} = \mathbf{E_c}$. ■

This formulation guarantees the stability in the case of saturation with a minimum effect on the controller's output signal.

2.1.1 Extended $\mathcal{H}_2$

Proposition 2.2 *Consider the system defined by (3) and the PID controller with compensation given by (4). The controlled system is asymptotically stable with robust compensation, because $||H_{u\vartheta}||_2^2 < \mu$ if, and only if, there exist symmetrical matrices $\mathbb{X} > 0$ and $\mathbb{W} > 0$, and the matrices $\mathbb{F}$, $\mathbb{G}$, and $\mathbf{Q}$, satisfy the following LMIs are satisfied:*

$$\begin{bmatrix} \mathbb{A}\mathbb{F} + \mathbb{F}^T\mathbb{A}^T & \mathbb{X} - \mathbb{F}^T + \mathbb{A}\mathbb{G}^T & M_1 B + M_2 \mathbf{Q} \\ (\circ)^T & -(\mathbb{G}^T + \mathbb{G}) & 0 \\ (\circ)^T & (\circ)^T & -\mathbb{I} \end{bmatrix} < 0,$$
$$\begin{bmatrix} \mathbb{W} & \mathbb{C}\mathbb{G} \\ (\circ)^T & \mathbb{G}^T + \mathbb{G} - \mathbb{X} \end{bmatrix} > 0, \quad tr(\mathbb{W}) < \mu; \tag{11}$$

where $M_1 = \begin{pmatrix} \mathbb{I} \\ 0 \end{pmatrix}$ and $M_2 = \begin{pmatrix} 0 \\ \mathbb{I} \end{pmatrix}$, have appropriate dimensions.

The compensation gain matrix $\mathbf{E_c}$ is given by

$$\mathbf{E_c} = \mathbf{Q}. \tag{12}$$

Proof

Consider $\mathbf{Q} = \mathbf{E_c}$. As $\mathbb{B} = M_1 B + M_2 \mathbf{Q}$, then the first inequality in (8) is obtained multiplying the first inequality in (11) on the left by $\mathcal{G}$, and on the right by $\mathcal{G}^T$, where

$$\mathcal{G} := \begin{pmatrix} \mathbb{I} & \mathbb{A} & 0 \\ 0 & 0 & \mathbb{I} \end{pmatrix}.$$

On the other hand, assume that the inequalities (11) are feasible. Therefore $\mathbb{G}^T + \mathbb{G} > \mathbb{X} > 0$. Hence, this implies that $\mathbb{G}$ is non-singular. Since $\mathbb{X}$ is positive definite the inequality $(\mathbb{X} - \mathbb{G})^T \mathbb{X}^{-1} (\mathbb{X} - \mathbb{G}) \geq 0$ holds. Then, $\mathbb{G}^T \mathbb{X}^{-1} \mathbb{G} \geq \mathbb{G} + \mathbb{G}^T - \mathbb{X}$ can be established, which yields

$$\begin{bmatrix} \mathbb{W} & \mathbb{C}\mathbb{G} \\ (\circ)^T & \mathbb{G}^T\mathbb{X}^{-1}\mathbb{G} \end{bmatrix} \tag{13}$$

Recalling that $\mathbb{G}$ is non-singular, then multiplying the inequality (13) on the right by $\mathcal{M} := \mathrm{diag}[\mathbb{I}, \mathbb{G}^{-1}\mathbb{X}]$ and on the left by $\mathcal{M}^T$, the second inequality in (8) is hold. ■

2.2 A framework in $\mathcal{H}_\infty$

In this environment, we want to design $\mathbf{E_c}$ such that $||H_{u\vartheta}||_\infty < \gamma$, for all $\gamma > 0$. It is well known that the $\mathcal{H}_\infty$ norm has a characterization as LMI constraints according to the Bounded Real Lemma [2, 10]:

Lemma 2.2 *The inequality $||H_{u\vartheta}||_\infty < \gamma$ holds if, and only if, there exist a symmetric matrix $\mathbb{X}$, such that*

$$\begin{bmatrix} \mathbb{A}^T\mathbb{X} + \mathbb{X}\mathbb{A} & \mathbb{X}\mathbb{B} & \mathbb{C}^T \\ (\circ)^T & -\gamma\mathbb{I} & \mathbb{D}^T \\ (\circ)^T & (\circ)^T & -\gamma\mathbb{I} \end{bmatrix} < 0. \tag{14}$$

Starting from 2.2, it is possible to extend that characterization of the ∞-norm for systems by means of using parameters dependent Lyapunov's functions. This formulation has the advantage that is less conservative [6].

Lemma 2.3 *The inequality $||H_{u\vartheta}||_\infty < \gamma$ holds if, and only if, there exist symmetric positive $\mathbb{Y}$, and appropiate dimension matrices $\mathbb{F}$ and $\mathbb{G}$ such that*

$$\begin{bmatrix} \mathbb{A}\mathbb{F} + \mathbb{F}^T\mathbb{A}^T & \mathbb{Y} - \mathbb{F}^T + \mathbb{A}\mathbb{G}^T & \mathbb{B} & \mathbb{F}^T\mathbb{C}^T \\ (\circ)^T & -(\mathbb{G} + \mathbb{G}^T) & 0 & \mathbb{G}\mathbb{C} \\ (\circ)^T & (\circ)^T & -\gamma\mathbb{I} & \mathbb{D}^T \\ (\circ)^T & (\circ)^T & (\circ)^T & -\gamma\mathbb{I} \end{bmatrix} < 0. \tag{15}$$

Proof

Consider $\mathbb{Y} = \mathbb{X}^{-1}$. The inequality (14) is re-written using the congruent transformation $\mathcal{T} := \mathrm{diag}[\mathbb{Y}, \mathbb{I}, \mathbb{I}]$; this is, the inequality is multiplied on the right by $\mathcal{T}$, and on the left by $\mathcal{T}^T$. Thus, the obtained result is equivalent to the one that is obtained of the multiplication, on the left, the inequality (15) by $\mathcal{F}$, and on the right by $\mathcal{F}^T$, where

$$\mathcal{F} := \begin{pmatrix} \mathbb{I} & \mathbb{A} & 0 & 0 \\ 0 & 0 & \mathbb{I} & 0 \\ 0 & \mathbb{C} & 0 & \mathbb{I} \end{pmatrix}$$

■

Proposition 2.3 *Consider the system defined by (3) and the PID controller with compensation given by (4). The closed loop system is asymptotically stable with robust compensation, because* $||H_{u\vartheta}||_\infty < \gamma$ *if, and only if, there exist a symmetrical matrix* $\mathbb{Y} > 0$*, and* $\mathbb{F}$*,* $\mathbb{G}$*, and* $\mathbf{Q}$ *matrices , such that the following LMI is satisfied:*

$$\begin{bmatrix} \mathbb{A}\mathbb{F} + \mathbb{F}^T\mathbb{A}^T & \mathbb{Y} - \mathbb{F}^T + \mathbb{A}\mathbb{G}^T & M_1 B + M_2 \mathbf{Q} & \mathbb{F}^T \mathbb{C}^T \\ (\circ)^T & -(\mathbb{G} + \mathbb{G}^T) & 0 & \mathbb{G}\mathbb{C} \\ (\circ)^T & (\circ)^T & -\gamma\mathbb{I} & \mathbb{D}^T \\ (\circ)^T & (\circ)^T & (\circ)^T & -\gamma\mathbb{I} \end{bmatrix} < 0. \quad (16)$$

where $M_1 = \begin{pmatrix} \mathbb{I} \\ 0 \end{pmatrix}$ *and* $M_2 = \begin{pmatrix} 0 \\ \mathbb{I} \end{pmatrix}$ *are matrices with appropriate dimensions.*

The compensation gain matrix $\mathbf{E_c}$ *is given by*

$$\mathbf{E_c} = \mathbf{Q}. \quad (17)$$

Proof

The demonstration is direct. Consider $\mathbf{Q} = \mathbf{E_c}$. Immediately, the matrix $\mathbb{B}$ is expressed from its components using the variables change. ∎

The performance index based-on the $\mathcal{H}_\infty$ norm corresponds to the $\mathcal{L}_2$ gain of the controller output signal with respect to the actuator output signal.

3 Numerical Example

Let us consider the example presented in the first part. We want to calculate the compensation gain matrix, in both cases $\mathcal{H}_2$ and $\mathcal{H}_\infty$. At this time, we use the LMI toolbox from Matlab® in order to formulate the LMIs and to obtain their numeric solution.

1. In the $\mathcal{H}_2$ case, the following results were obtained: performance level $\mu = 5.9560e(-008)$, and
$$\mathbf{E_c} = \begin{pmatrix} 15.0000 & 40.0000 \\ 12.0000 & 30.0000 \end{pmatrix}$$

2. For the $\mathcal{H}_\infty$ case, the results are: $\gamma = 1.3639e(-005)$, and
$$\mathbf{E_c} = \begin{pmatrix} 15.0000 & 40.0000 \\ 12.0000 & 30.0000 \end{pmatrix}$$

For the time analysis, a simulation was made for the $\mathcal{H}_\infty$ case, where the $u_1 \in [-1, 1]$ situation was considered. Figure 5 shows the results in the different cases: without saturation, with saturation and without compensation, and with compensation.

Just as it can be observed, the compensation guarantees the system closed loop performance, in spite of the change in the saturation limits. The temporary performance for the case without saturation and with compensation are similar, varying only the time response. On the other hand, the situation where compensation is not given the design objectives are not reached due to the saturation, such as is observed in the figure (c) where the signal remains in the saturation limit. This is the same situation that is presented for the case where doesn't have robustness in the synthesis of the compensation gain, as can be seen in Figure 2. Thus, it is important to consider some robustness characteristics for designing the gain compensation matrix.

4 Conclusions

An approach for robust *anti-windup* compensation design for PID controllers has been presented. The technique is based on $\mathcal{H}_2$ and $\mathcal{H}_\infty$ norms characterization as Linear Matrix Inequalities (LMI). The robustness analysis is considered on the closed loop transfer matrix, of the controlled output respect to the difference between the actuator output and controller output. The performance is assured in spite of unknown changes on the actuator saturation limits. The stability and performance margins are evaluated from $\mathcal{H}_2$ and $\mathcal{H}_\infty$ norms of the transfer matrix, which are LMI restrictions. The robust compensation gain design is obtained by means of a parameter-dependent Lyapunov function (less conservative), which allows to describe a convex optimization problem. This problem is presented from LMIs, which can be solved in polynomial time by specialized algorithms.

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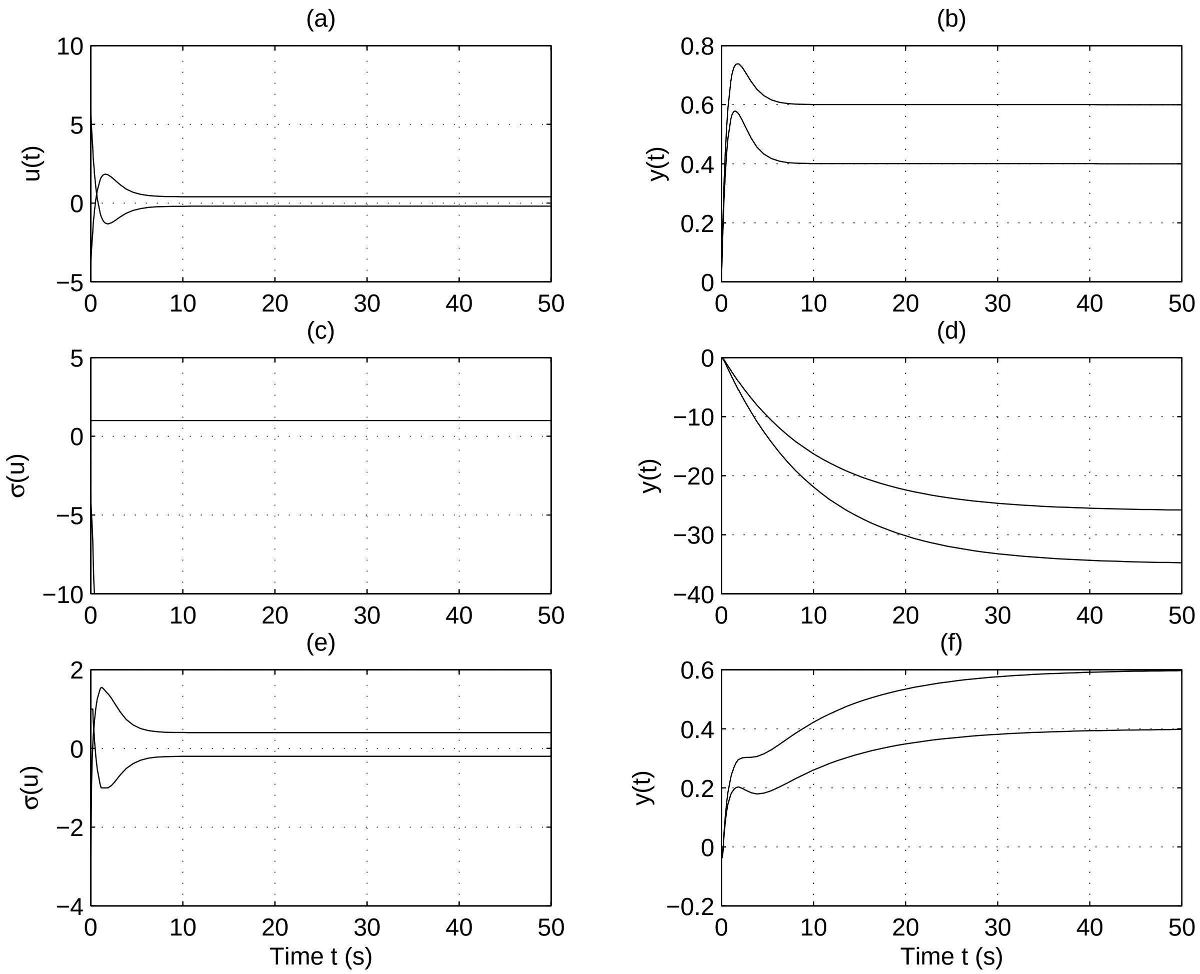

Figure 5. Actuator output and System output: (a)-(b) Without saturation. (c)-(d) With saturation and without compensation. (e)-(f) With compensation.