Robust Adaptive Control of Robotic Systems using Additive Recurrent Neural Network

F.M. RAIMONDI, T. RAIMONDI AND P. BARRETTA
Dipartimento di Ingegneria dell'Automazione e dei Sistemi
University of Palermo
Viale delle scienze 90128, Palermo
ITALY

Abstract: - In this paper, an innovative robust adaptive tracking control method for robotic systems with unknown dynamics using a nonlinearly parameterized Additive Recurrent Neural Network (ARNN) is proposed. The ARNN uses the Gaussian Radial Basis Functions (GRBF) as activation functions. Through this method the training laws of all GRBF parameters are determined. Additionally, the system is augmented with sliding control to offset the higher-order terms in the Taylor series of RBF output. Such a development is necessary for the linearization of the GRBF with respect to the parameters and, therefore, to obtain the training laws of the ARNN. The study of the total system stability is based on the Lyapunov's theory. Finally, the effectiveness of the ARNN-based control approach is verified through simulations on a six-link robot manipulator.

Key-Words: - Recurrent neural networks, adaptive control, Gaussian radial basis functions, Lyapunov stability, robotic systems.

1 Introduction

In recent years there has been increasing interest in the use of neural networks for the identification and control of nonlinear systems. The neural network possesses powerful approximation capabilities and can therefore be used for identification of unknown, or rather partially unknown, nonlinear dynamic systems. In the last decade, the neural architectures used for identification and control are the MLP (MultiLayer Perceptron) networks and the RBF (Radial Basis Function) networks. Their application range goes from the indirect adaptive control [7], with heuristic approach, to the direct adaptive control [9], [12], and [13].

The RBF network is more suitable for on-line adaptation, being insensitive to the order of presentation of the signals used for adaptation. They also require less computation time for learning and have a more compact topology [8]. With the use of Gaussian activation functions, the RBF network forms a local representation (hyper-ellipsoids), as opposed to the sigmoidal MLP (hyper-planes), where each basis function responds only to inputs in the neighbourhood of a unit center and the spread is determined by the unit variance.

The first to have introduced the use of neural networks in dynamical systems identification and control were Narendra and Parthasarathy in [7]. They employed static MLP networks connected either in series or in parallel with linear dynamical systems, where the synaptic weights were updated through a gradient learning algorithm. However, the stability of the total system was verified only through results of simulations. Sanner and Slotine [13] incorporate Gaussian radial basis function neural networks with sliding mode control and linear feedback, to formulate a direct adaptive tracking control architecture. Besides, they developed a systematic procedure for determination off-line the variances and centers of the basis functions that censure the network approximation accuracy to be uniformly bounded everywhere within a relevant and finite region of state space. However, in the practice, the RBF networks require a very high number of hidden neurons to approximate functions in wide intervals. Polycarpou e Ioannou [9] employed Lyapunov stability theory to develop stable adaptive laws for identification and control of SISO dynamical systems with unknown nonlinearities, using various neural network architectures. Lewis and Fierro [3] applied the MLP networks with a robustifying control signal, to guarantee tracking performance in robotic systems. They have also shown that the backpropagation rule alone is insufficient to assure stability of the whole system. Rovithakis and Christodoulou [11] presented indirect and direct adaptive control schemes based on a recurrent neural network model of the unknown system. As activation functions were used the logistic functions. Also in this case the Lyapunov technique was used to provide answers to the problems of stability, convergence, and robustness.

Generally, the most of above works impose restrictions on the forms of allowable nonlinearities and, furthermore, the control laws need the a priori knowledge of the upper bound on the modelling error and on the norm of the optimal parameter values of the used neural network. However, in many practical cases such bounds may not be known. A recent result, obtain in [1], solves this type of problem.

On the other hand, the control of a robotic systems is currently of great interest when using neural network to approximate the unknown dynamics in the known model of robotic systems [6].

This paper deals with a recurrent neural network-based controller for motion dynamic control of robot manipulators. We present a feedback adaptive neurocontroller for robots which combines ARNN's with adaptive and robust control techniques. The ARNN uses the Gaussian radial basis functions (GRBF) as activation functions. We propose a new method in order to determine the training laws the parameters of the dilation (variance) and translation (center) of GRBF which allow to reduce the identification and control error for tracking tasks of time trajectories. Additionally, the system is augmented with sliding control to offset the higher-order terms in the Taylor series of RBF output. Such a development is necessary for the linearization of the GRBF with respect to the parameters and, therefore, to obtain the training laws of the ARNN. The study of the total system stability is based on the Lyapunov theory. In order to verify the effectiveness the ARNN-based controller, simulation studies were carried out using a PUMA-560 model robot. Simulation results showing the practical feasibility and performance of the proposed approach to robotics are given.

This work is organized as follows. Section II presents the modelling of the robot dynamics by using an ARNN-based identification scheme. In Section III, the problem of motion adaptive control of rigid robot manipulator is stated and formulated, and ARNN-based feedback robust adaptive controllers and stability results are given. Finally, Section IV shows the simulation results for the tracking adaptive control problem. Conclusions are given in Section V.

2 Modeling the Robot Manipulator Dynamics by using an ARNN

2.1 Robot manipulator dynamics

The dynamics of an *n*-link manipulator can be described in the Lagrange form:

$$M(q)\ddot{q} + V(q,\dot{q})\dot{q} + G(q) + F_d(\dot{q}) = \tau \,, \qquad (1)$$

where q consists of the joint variables, τ is generalized force vector, $M(q)$ is the symmetric inertia matrix, $V(q,\dot{q})$ is the matrix of Coriolis and centrifugal effects, the vector $G(q)$ denotes the gravity terms and $F_d(\dot{q})$ is the friction vector. The inertia matrix is positive-definite.

By choosing the following state variable:

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} q \\ \dot{q} \end{bmatrix}, \qquad (2)$$

we may represent the motion equation (1) in the state space form:

$$\dot{x} = f(x) + b(x)u \,, \qquad (3)$$

where:

$$f(x) = \begin{bmatrix} x_2 \\ -M^{-1}(x_1)[V(x_1,x_2)x_2 + G(x_1) + F(x_2)] \end{bmatrix},$$
$$b(x) = \begin{bmatrix} 0 & 0 \\ 0 & M^{-1}(x_1) \end{bmatrix}, \qquad u = \begin{bmatrix} 0 \\ \tau \end{bmatrix}. \qquad (4)$$

In order to design a suitable controller, we define following auxiliary variable:

$$q_a = \dot{q} + \Lambda q \,, \qquad (5)$$

where Λ is positive defined diagonal matrix. The system (3) can be rewritten as:

$$\dot{q}_a = F(q_a) + B(q_a)u \,, \qquad (6)$$

where F and B are *n*-dimension vector functions. Now we only assume the outputs q and $\dot{q}$ are available.

2.2 Additive recurrent neural network

We consider the dynamic model of the additive recurrent neural network as shown in Fig.1. Let n be the number of the state variables. The filter input of the Fig.1 is:

$$\hat{W}x + \hat{H}u \,, \qquad (7)$$

where x is the potential vector, u is the input vector, $\hat{W}$ is the matrix of synaptic weights, and $\hat{H}$ is the diagonal gain matrix. Let v be the input potential vector to the vector of nonlinear element $g(\cdot)$. Then, it is possible to express the dynamic of the order first filter through the following differential equation:

$$\dot{v} + Kv \,, \qquad (8)$$

where K is a diagonal positive define gain matrix. Given the activation potential vector v, we determined the ARNN output using the nonlinear relation:

$$x = g(v), \tag{9}$$

where $g(\cdot) = [g_1 \; g_2 \; \cdots \; g_n]^T$ is a vector of continuous nonlinear function and then differentiable, with elements $g_i(\cdot)$ so-called *activation functions*. It is possible to define the dynamic of the network through the following system of coupled differential equations of the first order:

$$\dot{v} = -Kv + \hat{W}g(v) + \hat{H}u, \tag{10}$$

The model described by the equation (10) is called *additive model*; this terminology is used to distinguish it from the multiplied models where $\hat{W}$ depends from x.

In this paper, we consider an ARNN that uses as activation potential the Gaussian radial basis function (GRBF). The ARNN model is described from (10), with:

$$g_i(v) = g_i(v, \hat{c}_i, \hat{a}_i) = \exp\left(-\left\|v - \hat{c}_i\right\|^2 / \hat{a}_i^2\right), \tag{11}$$

where $\hat{c}_i$ and $\hat{a}_i$ are, respectively, the vector representing the centers and the value representing the variance associated to the i-th element of the vector of activation functions. Then, the parameters characterizing the vector of activation functions are $\hat{c} = [\hat{c}_1 \; \hat{c}_2 \; \cdots \; \hat{c}_n]^T$ and $\hat{a} = [\hat{a}_1 \; \hat{a}_2 \; \cdots \; \hat{a}_n]^T$, that is $g(v) = g(v, \hat{c}, \hat{a})$. It has been proven [8] that the GRBF network satisfies the conditions of the Stone-Weierstrass theorem and is capable to uniformly approximate any real continuous nonlinear function on the *n*-dimensional compact set. This involves that GRBF networks are universal approximator. How it is possible to observe, the vector of the GRBF is nonlinear with respect to the parameters $\hat{c}_i$ and $\hat{a}_i$, we thus call it a *nonlinearly parameterized approximator*.

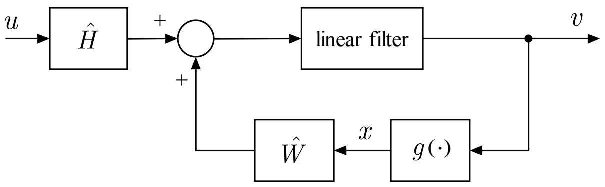

Fig. 1. Scheme of an ARNN.

2.3 ARNN-based identification of robot dynamics

In this section, we consider a series-parallel identification scheme based on the ARNN that uses as activation potential the GRBF described by (10) and (11). The problem of identification consists of choosing an opportune model and modifying the parameter values of the model itself according to some training laws such that his response of the identification model approximates the response of the real system, both forced to the same input signal u.

On the basis of the results found in the previous section, the real robotic system (6) can be expressed as:

$$\dot{q}_a = -Kq_a + Wg(q_a, c, a) + Hu + \Delta L, \tag{12}$$

where W, c, a, H are the *optimal estimation values*. At last, ΔL is the *modelling error*. We note that they are defined as the values of the parameters that correspond to minimum modelling error.

Assumption 1. The optimal estimation values and the modelling error are bounded in norm. □

Defined the *identification error* $e(t) = q_a(t) - v(t)$ and using the equations (10) and (12), the dynamic of the identification error can be written:

$$\dot{e} = -Ke + Wg(q_a, c, a) - \hat{W}g(v, \hat{c}, \hat{a}) + \tilde{H}u + \Delta L, \tag{13}$$

where $\tilde{H} = H - \hat{H}$. In order to obtain an adaptive law for the parameters, it is convenient to consider the first order approximation of the vector as activation functions. Using the Taylor series expansion of $g(q_a, c, a)$ around the point $(\hat{c}, \hat{a})$ we obtain:

$$g(q_a, c, a) = g(q_a, \hat{c}, \hat{a}) + J_g^c \tilde{c} + J_g^a \tilde{a} + O(q_a, \tilde{c}, \tilde{a}), \tag{14}$$

where $\tilde{c} = c - \hat{c}$, $\tilde{a} = a - \hat{a}$ are the estimates of the optimal centers and variances respectively, and $O(q_a, \tilde{c}, \tilde{a})$ represents the higher order terms of the expansion. The Jacobiane matrices present in (14) are given from:

$$J_g^c = diag\left\{\frac{\partial g_1}{\partial c_1^T}(q_a, \hat{c}_1, \hat{a}_1), \ldots, \frac{\partial g_n}{\partial c_n^T}(q_a, \hat{c}_n, \hat{a}_n)\right\}, \tag{15}$$

$$J_g^a = diag\left\{\frac{\partial g_1}{\partial a_1}(q_a, \hat{c}_1, \hat{a}_1), \ldots, \frac{\partial g_n}{\partial a_n}(q_a, \hat{c}_n, \hat{a}_n)\right\}, \tag{16}$$

with:

$$\begin{aligned} \frac{\partial g_i}{\partial c_i}(q_a, c_i, a_i) &= 2\frac{(q_a - c_i)}{a_i^2} g(q_a, c_i, a_i), \\ \frac{\partial g_i}{\partial a_i}(q_a, c_i, a_i) &= 2\frac{\left\|(q_a - c_i)\right\|^2}{a_i^3} g(q_a, c_i, a_i). \end{aligned} \tag{17}$$

Now, by replacing (14) in (13) we obtain:

$$\begin{aligned} \dot{e} = -Ke + \tilde{W}[g(v, \hat{c}, \hat{a}) - J_g^c \hat{c} - J_g^a \hat{a}] + \\ + \hat{W}[J_g^c \tilde{c} + J_g^a \tilde{a}] + \tilde{H}u + \Delta E, \end{aligned} \tag{18}$$

where $\tilde{W} = W - \hat{W}$ represents the synaptic weight estimation error, and ΔE represents the *disturbance terms* expressed as:

$$\Delta E = WO(q_a, \tilde{c}, \tilde{a}) + W[g(q_a, \hat{c}, \hat{a}) - g(v, \hat{c}, \hat{a})] + \\ + \tilde{W}[J_g^c c + J_g^a a] + \Delta L. \tag{19}$$

Lemma 1. Given the disturbance terms ΔE, there exist a vector $l = [l_0, l_1, l_2, l_3]^T$, such that:

$$\|\Delta E\| \le l_0 + l_1 \|\hat{W}\| + l_2 \|\hat{c}\| + l_3 \|\hat{a}\| = l^T s(\hat{W}, \hat{c}, \hat{a}), \tag{20}$$

where $s(\hat{W}, \hat{c}, \hat{a}) = [1, \|\hat{W}\|, \|\hat{c}\|, \|\hat{a}\|]^T$.

Proof. Using (14), since the GRBF and partial derivatives are superiorly and lowerly bounded, the following inequality is verified:

$$O(x, \tilde{c}, \tilde{a}) \le d_0 + d_1 \|\tilde{c}\| + d_2 \|\tilde{a}\|, \tag{21}$$

where d_0, d_1 and d_2 are positive constants. Given the disturbance terms (19), using (21), we obtain:

$$\Delta E = \|W\| (d_0 + d_1 \|\tilde{c}\| + d_2 \|\tilde{a}\|) + \\ + \|W\| \|g(q_a, \hat{c}, \hat{a}) - g(v, \hat{c}, \hat{a})\| + \\ + \|\tilde{W}\| \|J_g^c c + J_g^a a\| + \|\Delta L\|. \tag{22}$$

Considering the assumption 1, opportunely picking up the constant terms present in (22), we obtain (20) . Q.E.D.

We consider the Lyapunov function candidate:

$$V = \frac{1}{2} e^T e + \frac{1}{2\alpha_0} tr\left\{\tilde{W}^T \tilde{W}\right\} + \frac{1}{2\alpha_1} \tilde{c}^T \tilde{c} + \\ + \frac{1}{2\alpha_2} \tilde{a}^T \tilde{a} + \frac{1}{2\alpha_3} tr\left\{\tilde{H}^T \tilde{H}\right\}, \tag{23}$$

where $\alpha_0, \alpha_1, \alpha_2, \alpha_3$ are positive constants. Differentiating (23) with respect to the time and using (18) we obtain:

$$\dot{V} = -e^T K e + \\ - \frac{1}{\alpha_0} tr\left\{\left[\dot{\hat{W}}^T - \alpha_0 e[g(v, \hat{c}, \hat{a}) - J_g^c \hat{c} - J_g^a \hat{a}]^T\right] \tilde{W}\right\} + \\ - \frac{1}{\alpha_1} [\dot{\hat{c}}^T - \alpha_1 e^T \hat{W} J_g^c] \tilde{c} - \frac{1}{\alpha_2} [\dot{\hat{a}}^T - \alpha_2 e^T \hat{W} J_g^a] \tilde{a} + \\ - \frac{1}{\alpha_3} tr\left\{[\dot{\hat{H}}^T - \alpha_3 e u^T] \tilde{H}\right\} + e^T \Delta E. \tag{24}$$

Using the following training laws of the parameters:

$$\dot{\hat{W}} = \alpha_0 [g(v, \hat{c}, \hat{a}) - J_g^c \hat{c} - J_g^a \hat{a}] e^T, \\ \dot{\hat{c}} = \alpha_1 (\hat{W} J_g^c)^T e, \\ \dot{\hat{a}} = \alpha_2 (\hat{W} J_g^a)^T e, \\ \dot{\hat{H}} = \alpha_3 u e^T, \tag{25}$$

the (24) is modified to:

$$\dot{V} = -e^T K e + e^T \Delta E. \tag{26}$$

3 ARNN-Based Motion Adaptive Control

In this section, we determine a control algorithm for tracking tasks of time trajectories. The objective is to determine the control law $u(t)$ such that the state of the first order system (6) can track a reference trajectory given by:

$$q_r = \dot{q}_d + \Lambda q_d, \tag{27}$$

where $q_d(t)$ is the desired trajectory of the system. If $\hat{H}$ is invertible, we consider the following input control:

$$u = \hat{H}^{-1} [\dot{q}_r + K q_r - \hat{W} g(v, \hat{c}, \hat{a})]. \tag{28}$$

Note that the derivation operations on the desired trajectory don't present any problem, after this last one is analytically well known. The error between the estimated state $v(t)$ and $q_r(t)$ is given as:

$$e_r(t) = v(t) - q_r(t), \tag{29}$$

and is necessary obtaining the dynamic equation of the tracking error $\tilde{q}(t) = q(t) - q_d(t)$ to evaluate the property of convergence of the real system state on the desired state. We note that, the tracking error can be obtained as solution of:

$$\dot{\tilde{q}} + \Lambda \tilde{q} = e + e_r. \tag{30}$$

Finally, by differentiating (29) and substituting the (28) in (10) we obtain:

$$\dot{e}_r + K e_r = 0. \tag{31}$$

Theorem 1. Consider the nonlinear system of the first order (6). We assume that the adaptive control law is given by:

$$u_a = u + u_s, \tag{32}$$

with:

$$u_s = -\hat{l}^T s \frac{e}{\|e\|}. \tag{33}$$

Furthermore, we assume that the training laws of the parameters are given by:

$$\dot{\hat{W}} = \alpha_0 [g(v, \hat{c}, \hat{a}) - J_g^c \hat{c} - J_g^a \hat{a}] e^T, \\ \dot{\hat{c}} = \alpha_1 (\hat{W} J_g^c)^T e, \\ \dot{\hat{a}} = \alpha_2 (\hat{W} J_g^a)^T e, \\ \dot{\hat{H}} = \alpha_3 u e^T, \\ \dot{\hat{l}} = \alpha_4 s \|e\|, \tag{34}$$

where $\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4$ are positive constants. Then, the tracking error $\tilde{q}(t)$ asymptotically converges to zero and the estimate error of the parameters is uniformly bounded.

Proof. We consider the Lyapunov function candidate:

$$V_e = V + \frac{1}{2} e_r^T e_r + \frac{1}{2\alpha_4} \tilde{l}^T \tilde{l} \geq 0, \qquad (35)$$

where the scalar function V is defined by (23). Differentiating (35) with respect to time, using (31), (32), (34) and observing:

$$\dot{V} = -e^T K e + e^T (\Delta E + u_s), \qquad (36)$$

we obtain:

$$\dot{V}_e = -e^T K e - e_r^T K e_r + e^T (\Delta E + u_s) - s^T \tilde{l} \, \|e\|. \qquad (37)$$

Substituting the robustifying term (33) in (37) we get:

$$\dot{V}_e = -e^T K e - e_r^T K e_r + e^T \Delta E - \hat{l}^T s \|e\| - \tilde{l}^T s \|e\|, \qquad (38)$$

since:

$$\|e\| = \sqrt{e^T e}. \qquad (39)$$

The (38) is bounded by:

$$\begin{aligned} \dot{V}_e &\leq -e^T K e - e_r^T K e_r + l^T s \|e\| - \hat{l}^T s \|e\| - \tilde{l}^T s \|e\| \\ &\leq -e^T K e - e_r^T K e_r. \end{aligned} \qquad (40)$$

By (40) we obtain that all the error are bounded. Therefore by integrating (40) we have:

$$\int_0^\infty (\|e(\tau)\|^2 + \|e_r(\tau)\|^2) d\tau \leq \frac{1}{\lambda_{\min}(K)} [V_e(0) - V_e(\infty)] < \infty, \qquad (41)$$

which implies that $e, e_r \in L_2$. By (18) and (31), since the desired trajectory $q_d(t)$ and the time derivative they assume bounded, we obtain that $\dot{e}, \dot{e}_r \in L_\infty$. Then, using the Barbalat's Lemma, the identification error $e(t)$ and the reference error $e_r(t)$ asymptotically converge to zero, that is $\lim_{t\to\infty} e(t) = 0$ and $\lim_{t\to\infty} e_r(t) = 0$. From (30), we can conclude that $\lim_{t\to\infty} \tilde{q}(t) = 0$. Q.E.D.

Remark 1. We note that, the tracking error $\tilde{q}(t)$ asymptotically converges to zero also in presence of disturb torques on the control input, that is:

$$M(q)\ddot{q} + V(q, \dot{q})\dot{q} + G(q) + F_d(\dot{q}) + \tau_d = \tau, \qquad (42)$$

where τ_d is the disturb torque vector which act on the mechanical systems. The disturb torque vector is bounded in norm.

4 Simulation Experiments

To show the performance of the proposed ARNN-based adaptive controller, as well as the stability properties obtained in the preceding theoretical development, a simulation study has been carried out for the PUMA-560 robot system. This was possible using the SIMULINK, a simulation toolbox put at disposal from MATLAB. The mathematical model of the robot manipulator used is given in [2]. Our algorithm can be separated into two phases: the first takes into consideration only the identification of the system, and the second, in case of success of the first, determines the control laws that allow the system to tracking a desired trajectory.

In the identification phase, we consider the ARNN described by the equation (10) with $n = 6$, and the input is given by sinusoidal functions. The parameters of the ARNN are trained according to the equations (25) without requiring a preliminary offline learning phase. The sampling time is fixed to the value of 0.1 ms. The initial values of all the tunable variables are small random numbers, and set of the design constants are $K = diag(50, 100, 50, 40, 50, 40)$, and $\alpha_i = 10$ with $i = 0, \ldots, 4$. Note that high gain is required on joint 2 in order to counter the significant disturbance torque due to gravity.

In the control phase the problem is to develop a control law such that the state of the system (1) can track a desired trajectory given by:

$$\begin{aligned} q_d(t) = [&0.5\sin(\frac{t}{4}), 0.5\sin(\frac{t}{4}), 0.5\sin(\frac{t}{4}), \\ &0.2\cos(\frac{t}{4}), 0.5\sin(\frac{t}{4}), \cos(\frac{t}{4})]^T, \end{aligned} \qquad (43)$$

and the initial conditions are:

$$q(0) = [\frac{\pi}{4}, 0, 0, 0, 0, 0]^T, \text{ and } \dot{q}(0) = 0. \qquad (44)$$

Applying the results found in the previous section, we obtain the feedback linearising control based on the ARNN obtained in the phase of identification. To ensure the convergence of the tracking error to zero, the control law (28) is augmented by a sliding mode control term (33). In practical robotic systems, the load may vary while different tasks are performed and some neglected nonlinearities may appear as disturbances at the control inputs. Therefore, the robustness of the proposed control methodology for robots has practical value. The initial values of the variables present in (34) are small random numbers and the design constants are the same as in identification phase.

This simulation examines the approximation capability, tracking performance and the high dimension problem of the ARNN-based controller. The simulation results show that the proposed controller with adaptive update law can overcome

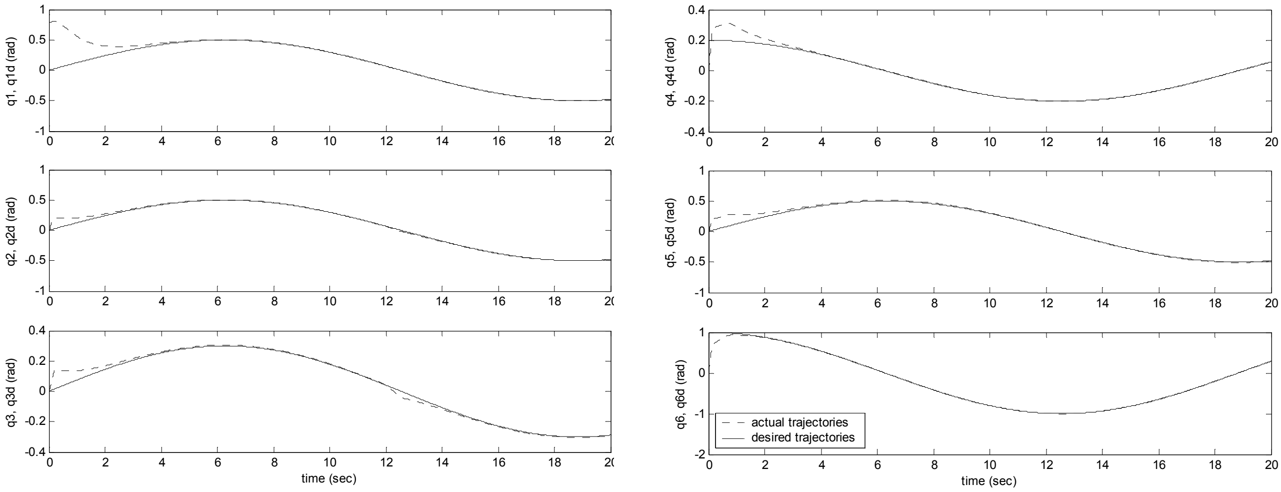

Fig. 2. Tracking performance of ARNN-based controller for PUMA-560 robot.

such problems. The results obtained during the control phase are shows in Fig. 2. The moving of the real trajectory of the system from the respective reference shows that the characteristics of tracking of the ARNN scheme are very good, as it is predicted by the theoretical analysis.

5 Conclusion

In this paper, a new robust adaptive tracking control design for robotic systems characterized by unknown dynamics has been presented. The proposed additive recurrent neural network nonlinear with respect to the parameters can achieve the desired tracking performance, as shown in the simulation results. The considered activation functions for the ARNN are the Gaussian radial basis functions with the update parameters (centers and variances). In particular, the control algorithm does not need an off-line learning or training phase. It is showed that the ARNN uses a minimum hidden neuron number to approximate the function taken into consideration. A robustifying control term is also needed to overcome higher-order modelling error terms. Finally, the simulation results of a six-link robot manipulator confirm that the ARNN can handle the high-dimension problems and show good performance of the proposed approach to robotics.

References:

[1] F. M. Raimondi, T. Raimondi and P. Barretta, "Identification and control of nonlinear systems using nonlinearly parameterized ARNN", *Proc. IEE Int. Conf. on Control 2004*, Bath, UK, 2004.

[2] J. Burdick, O. Khatib and B. Armstrong, "The Explicit dynamic model and inertial parameters of the PUMA 560 arm", *IEEE*, pp. 510-518, 1986.

[3] R. Fierro and F. L. Lewis, "Control of a nonholonomic mobile robot using neural networks", *IEEE Trans. Neural Networks*, vol. 9, no. 4, pp. 589-600, 1998.

[4] T. Fukao, H. Nakagawa and N. Adachi, "Adaptive tracking control of a nonholonomic mobile robot", *IEEE Trans. Robot. Autom.*, Kyoto, Japan, vol. 16, no. 5, pp. 609-615, 2000.

[5] R. I. W. Lang, "A future for dynamic neural networks", *Dept. Cybernetics*, University of Reading, UK, pp. 1-26, 2000.

[6] F. L. Lewis, A. Yesildirek and K.Liu, "Multilayer neural-net robot controller with guaranteed tracking performance", *IEEE Trans. Neural Network*, vol. 7, no. 2, pp. 388-399, 1996.

[7] K. S. Narendra and K. Parthasarathy, "Identification and control of dynamical systems using neural networks", *IEEE Trans. Neural Networks*, vol. 1, no. 1, pp. 4-27, 1990.

[8] T. Poggio and F. Girosi, "Networks for approximation and learning", *Proc. IEEE*, vol. 78, pp. 1481-1497, 1990.

[9] M. M. Polycarpou and P. O. Ioannou, "Neural networks as on-line approximators of nonlinear systems", *IEEE Conf. Decision Contr.*, Tucson, AZ, pp. 7-12, 1992.

[10] M. M. Polycarpou, "Stable adaptive neural control scheme for nonlinear systems", *IEEE Trans. Automat. Contr.*, vol. 41, no. 3, pp. 447-451, 1996.

[11] G. A. Rovithakis and M. A. Christodoulou, "Adaptive control of unknown plants using dynamical neural networks", *IEEE Trans. Syst. Man. Cybern.*, vol. 24, pp. 400-412, 1994.

[12] N. Sadegh, "A perceptron network for functional identification and control of nonlinear systems", *IEEE Trans. Neural Networks*, vol. 4, pp. 982-988, 1993.

[13] R. M. Sanner and J. J. Slotine, "Gaussian networks for direct adaptive control", *IEEE Trans. Neural Networks*, vol. 3, no. 6, pp. 837-864, 1992.