Modified Input Shaping for Circular Trajectory Following

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Abstract: - Input shaping for circular trajectory following through a second-order system usually results in a smaller than desired radius. The reasons of this smaller radius by using input shaping are investigated in this paper. A coefficient with two parts is found to correct this effect. Using this coefficient to modify the input shaper, better performance of applying input shaping to circular trajectory following is achieved. Simulation is made to show the effect of input shaping and the use of the developed coefficient.
Key-words: Flexible structures, Input shaping, Trajectory following, Vibration control

1. INTRODUCTION

Vibration is a serious problem in mechanical system especially for those requiring both precise motion and structural flexibility. Mostly used examples of such systems are the positioning of a disk driver's head, container cranes, large space structures, and coordinate measuring machines. There is active research interest in finding methods that will eliminate vibration for a variety of mechanical and structural systems. Making the system stiffer or making the load lighter are the most obvious suggestions. But it is usually difficult to substantially increase the stiffness or decrease the weight due to cost or physical limitations. Lowering the acceleration or using jerk-limited curve can help to reduce the vibration. But in the final analysis, they only lower the system's performance. Using a close loop control is traditional method, but the result is rarely time optimal.

Input shaping was first introduced to control residual vibration by Singer and Seering in 1989[2]. It origins from 'posicast' method that was developed by Smith in 1957[1]. After Singer and Seeing, a lot of papers appear to address the different aspects of this technology, such as sensitivity, robustness[3][4][5], frequency domain designing [13][6][7][8], trajectory following[9][10][11], and so on. All those work of using input shaping to suppress residual vibrations are well done and demonstrated for point to point motions. Singhose[9][10][11] then showed us that input shaping can also be applied to spatial trajectories for trajectory following where only the shape of the movement is important. But, there is a 'smaller-than-desired radius around most of the circle' problem for the circular trajectories as stated in [11]. A solution was also given as by 'using an unshaped circle command that has a radius larger than desired'. But the reason of this 'smaller-than-desired-radius' by input shaping is not analysed or given, what's more, no feasible solution to determine the 'unshaped circle command that has a radius larger than desired' is given.

This paper will first analyse the reasons that cause the smaller radius of circular trajectory by input shaping in section 2. Then based on this, a coefficient expression to correct this effect is deduced. Either the parameter of the input shaper or the unshaped circle command can then be changed according to this coefficient expression to get a more precise trajectory following for circular trajectory. Simulations is then made to show the effect of this modified input shaping in section 3. Conclusions are made in section 4.

2. REASONING

A circular trajectory is produced by two orthogonal sine curve. The radius of the circle reflected on the sine curve is the amplitude of the sine curve. So the smaller than desired circular radius effect that is found through using input shaper could be analysed with the amplitude of a sine curve. Without exception, the plant is modelled as a second-order system(mass-spring-mass).

There are two factors that influence the amplitude of a sine curve through a second-order system with input shaper. The first factor is due to the addition of two sine curve with a phase difference which is produced by the input shaper. This factor makes the radius smaller then designed; The second factor is due to the response of a second order system to a sine curve. Normally, this factor makes the radius bigger than designed. To some extent, the second factor can compensate the smaller radius effect that is caused by the first factor.

2.1 Introduction of input shaping

Input shaping is a feedforward control technique for reducing residual vibrations in flexible systems. It convolves the ordinary input signal with a sequence of impulses that is deduced from the system parameters to result in a shaped input. The energy, that is near the system's natural frequency, is removed by the impulses, thus residual vibration is greatly eliminated.

The input shaping method used by [11] is designed for step changed velocity input. Zero Vibration(ZV) input shaping produces two impulses to convolve with system's input signal. The amplitude and time location of the two impulses in ZV are:

$$A_0 = \frac{1}{1+K},\ T_0 = 0;\ A_1 = \frac{K}{1+K},\ T_1 = T_d/2 \quad (1)$$

with $K = e^{-\xi\pi/\sqrt{1-\xi^2}}$;

$T_d = \frac{2\pi}{\omega_d} = \frac{2\pi}{\omega\sqrt{1-\xi^2}}$: damped period of vibration;

ω : the system's natural frequency;

$\omega_d = \omega\sqrt{1-\xi^2}$: the damped frequency;

ξ : the system's damping ratio.

From eq.1, it is clear that the parameters of the input shaping are dependent on the system's parameters ξ and ω. The change of the system's parameters will cause the ZV input shaping ineffective. Therefore, other input shapings that are more robust to the change of system's parameters were developed. Among them are Zero Vibration Derivative(ZVD) input shaping. The ZVD produces three impulses to convolve with the input signal of the system. The amplitude and time location of the three impulses are:

$$A_0 = \frac{1}{1+2K+K^2},\ T_0 = 0;$$
$$A_1 = \frac{2K}{1+2K+K^2}, T_1 = T_d/2;$$
$$A_2 = \frac{K^2}{1+2K+K^2}, T_2 = T_d; \quad (2)$$

The amplitudes of the impulses in both ZV and ZVD should be designed to have no influence on the amplitude of the input signal after the overall delayed time, that is the amplitude of the impulses should be: $\sum_i A_i = 1$ for step input signal. This is because the result signal is simply the summation of step signals with the amplitudes that are scaled by those amplitude of impulses. But if an input signal has other form, for example, a sine curve, the amplitude of result signal is not any more a simple summation of amplitude of individual sine curves, as those sine curves have a time difference as $T_d/2$.

2.2 Addition of two sine curve with phase difference

Generally, we assume that a sine signal has the form

$$B\sin\Omega t \quad (3)$$

and the system parameters ξ, ω_n are known. Applying the ZV method of input shaping, we could express the shaped input in this way:

$$BA_0\sin(\Omega t) + BA_1\sin(\Omega(t-T_0)) \quad (4)$$

resulting in

$$B_1\sin(\Omega t - \theta) \quad (5)$$

with $B_1 = B\sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\Omega T_0)}$,

$\theta = arctg\left(\frac{A_2\sin(\Omega T_0)}{A_1 + A_2\cos(\Omega T_0)}\right)$,

A_1, A_2: the amplitudes of the two impulses of ZV,

Ω : the frequency of the sine signal.

Obviously, the shaped amplitude B_1 is smaller than the original amplitude B. For a circular trajectory, when velocity amplitudes of the two orthogonal axis are smaller than desired, then smaller radius circle will appear.

Fig. 1 shows an example of a shaped sine curve. The dotted sine curve in the figure is the original curve with unit amplitude. The solid curve is the shaped sine curve. As can be seen from the figure, the shaped curve has a smaller amplitude than the original curve.

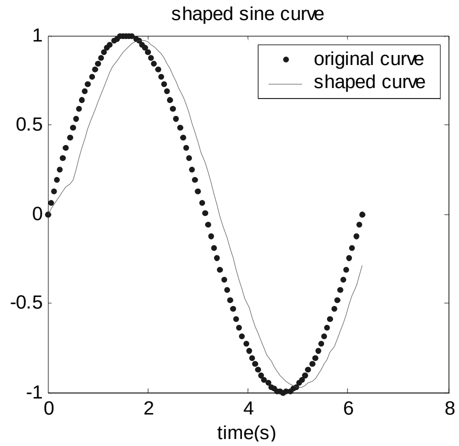

Fig. 1. Addition of sine curves with phase difference

In order to make the shaped sine curve to have the same amplitude as the unshaped sine curve, a coefficient can be used to modify the amplitude of the impulses of the input shaper. The ratio between unshaped amplitude and shaped amplitude of the sine curve is the coefficient. Thus the coefficient is according to (5):

$$coef1 = 1\Big/\sqrt{A_0^2 + A_1^2 + 2 * A_0 A_1 \cos(\Omega T_1)} \qquad (6)$$

Using this ratio, we can modify the amplitude of impulses in ZV method, so that the shaped sine curve has no change in amplitude compared with unshaped input signal. The amplitude of the two impulse in the ZV method is then:

$$A_{0\_new} = A_0 \times coef1;\ A_{1\_new} = A_1 \times coef1 \quad (7)$$

according to (1) and (6).

Through this coef1 we can easily remove the effect that is caused by the addition of two sine curves with phase difference.

2.3 Response to a sine curve

The transfer function of a second order system can be expressed by :

$$F = \frac{\omega^2}{s^2 + 2\omega\xi s + \omega^2} \qquad (8)$$

The response of such a second order system to a sine curve $B\sin\Omega t$ is:

$$y(t) = C\sin(\Omega t - \vartheta) \qquad (9)$$

with $C = \frac{B}{\sqrt{(1-\mu^2)^2 + (2\xi u)^2}}$, $\vartheta = \tan^{-1}(\frac{2\xi\mu}{1-\mu^2})$, $\mu = \frac{\Omega}{\omega_n}$.

The factor $\sqrt{(1-\mu^2)^2 + (2\xi u)^2}$ will then change amplitude of the sine signal after going through the second order system. In order to get the same amplitude of the responded signal as the initial input signal, we can use another coefficient to modify the amplitude of the impulses of the input shaping:

$$coef2 = \sqrt{(1-\mu^2)^2 + (2\xi u)^2} \qquad (10)$$

So, the amplitude of the impulse should be :

$$A_{0\_new} = A_0 \times coef2;\ A_{1\_new} = A_1 \times coef2 \quad (11)$$

by combining (1) and (10).

2.4 Combination

Of the two coefficients that are developed above, one is aimed to modify the smaller amplitude that is caused by the two sine curve addition; the other is aimed to modify the bigger amplitude that is caused by the response of a second order system. In order to get the same amplitude of the responded sine curve as the input sine curve, the two coefficients could be combined. According to (6) and (10), the combined coefficient to modify the amplitude of the impulses of ZV input shaper can be expressed by:

$$coef = coef1 \times coef2 = \frac{\sqrt{(1-\mu^2)^2 + (2\xi u)^2}}{\sqrt{A_0^2 + A_1^2 + 2 * A_0 A_1 \cos(\Omega T_1)}} \qquad (12)$$

and now the amplitude of the impulses should be:

$$A_{0\_new} = A_0 \times coef;\ A_{1\_new} = A_1 \times coef \qquad (13)$$

with $A_0 = \frac{1}{1+K}$, $A_1 = \frac{K}{1+K}$, $\mu = \frac{\Omega}{\omega_n}$, $T_1 = T_d / 2$, $K = e^{-\xi\pi/\sqrt{1-\xi^2}}$.

Fig.2. gives out a simulation result of the effect of this modified input shaper. Input signal is a sine curve with unit amplitude. The solid curve in Fig.2 is the response signal to the input sine curve through a second order system without any controller or input shaper. The dotted curve is the response to the same input through the same system but with the modified input shaping. It is clear from the figure that the dotted curve reserves the amplitude and shape of the input sine curve by using the modified input shaping.

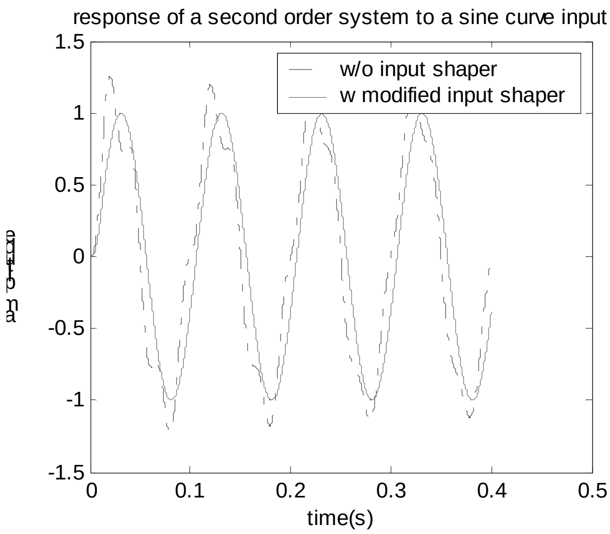

Fig. 2. Response of a second-order system to sine input

As ZV method is sensitive to the modelling error of the system parameter ω, the more robust method, ZVD, is mostly used. For the two parts of the coefficient, the second part(*coef2*) remains the same for those two methods, but the first parts(*coef1*) are different. The difference is caused by the different number of impulses in ZV, ZVD. In ZVD, three sine curves with another phase delay should be added together to form a shaped sine curve, which has a even smaller amplitude.

3 SIMULATION RESULT

In this section, simulation results are presented to show the effect of the coefficients developed above for the circular trajectory. The system model is described by two mass-spring-mass system which represent a two-orthogonal-mode model as shown in Fig. 3.

We assume the same frequency and same damping ratio for both vibration modes ($f_x = f_y = 1Hz, \xi_x = \xi_y = 0.01$). The inputs to the system are the planned x and y direction velocity of the load. The position outputs of the load from both direction form the trajectory. The radius of the circular is 0.5, and the velocity input command has a frequency of 0.15Hz. The circle is initiated to the +y direction at position (0,0)..

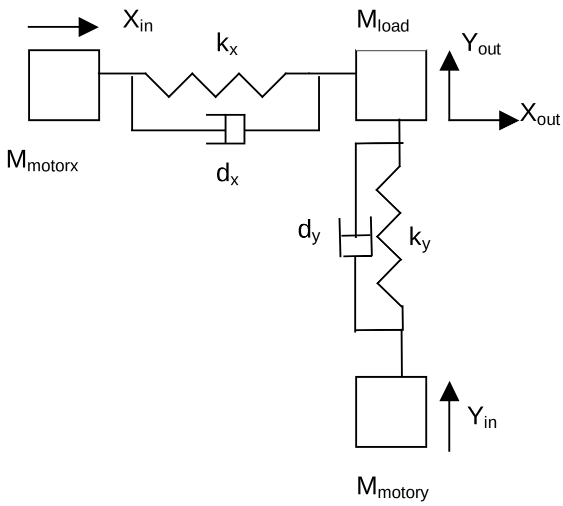

Fig. 3.[11]: Two-mode system model

The following Fig.4, Fig.5 and Fig.6 are the simulation results. Fig.4 shows the expected circle and the responded circle from the system mentioned above without any controller. Fig.5 gives out the responded circle from the same system with ZVD input shaping. Fig.6 demonstrates the effect of the modified input shaping.

The dotted circle in Fig.4 is the desired circle of the load. The solid curve is the simulated load response without any input shaping. Obviously, the responded circle has unexpected vibrations along the circular trajectory.

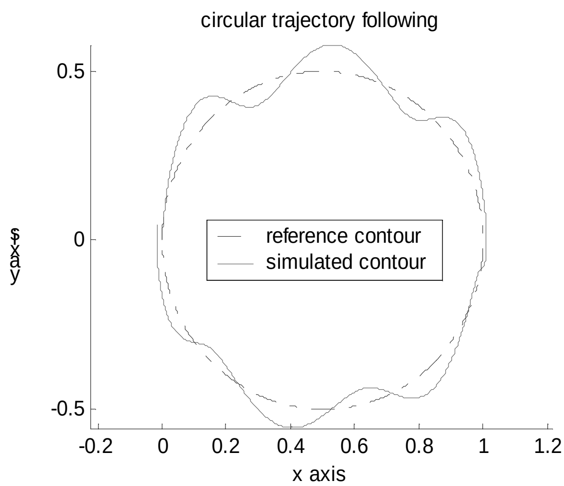

Fig. 4. Reference circle and unshaped response

The solid curve in the Fig.5 is the responded trajectory after ZVD input shaper is used. There is no obvious vibrations around the responded trajectory. Comparing the solid curve in Fig.4 and Fig.5, it is clear that the input shaper provides a better shape of trajectory following. But the radius of the responded circle in Fig.5 is smaller than desired. This smaller radius effect is caused by the addition of three(for ZVD method) sine curves with a phase difference which are produced by the input shaper, compensated a little by the response of a second order system to a sine curve input as analysed in section 2.

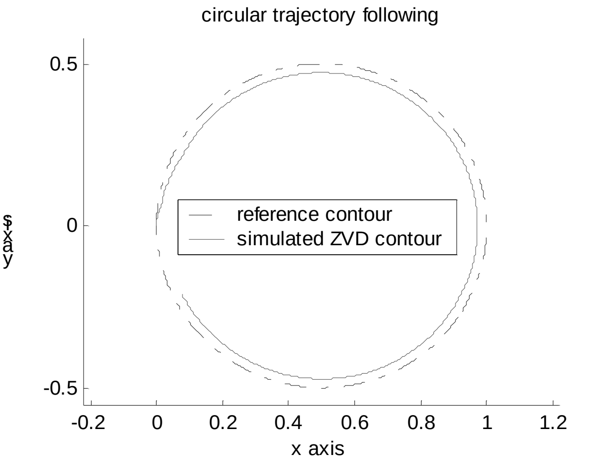

Fig. 5. Reference circle and ZVD-Shaped response

Now, the coefficient to modify the input shaper is applied to the ZVD input shaper. The simulated result is shown in Fig. 6. The dotted circle is again the designed load trajectory, the solid circle is the simulated load response. We could see that in most trajectory area, the solid curve and the dotted curve overlap. Thus, the performance of mean value, that is mentioned in [11], is greatly improved.

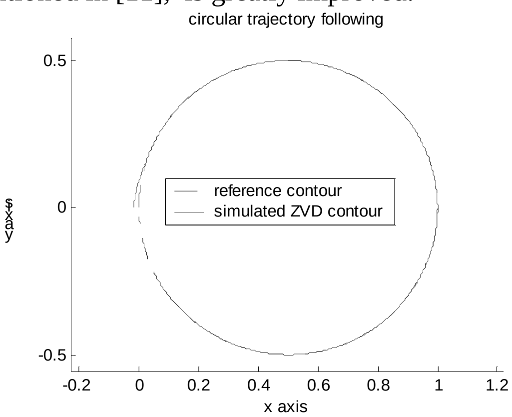

Fig. 6: Reference circle and modified ZVD-Shaped response

4 CONCLUSION

Input shaping helps to eliminate the residual vibration of a flexible system. But for a circular trajectory, using input shaping usually results in a smaller radius circle. This fact is caused by the addition of sine curves with phase difference that are caused by the input shaper. In a second order system, this smaller than desired radius effect can be partly compensated by the response of the system to a sine curve. Nevertheless, this effect cannot totally be compensated. A coefficient is than found to modify the amplitude of the impulses of the input shaper, so that the response of the load could have the same radius as designed. Simulations show that the modified input shaper gives better load response than ordinary input shaper.

But for an arbitrary trajectory, this coefficient is impractical to be applied.

As input shaping is a feedforward method, it has no resistance to disturbance. But it could be combined with the industrial cascaded control loop. Thus the input shaper deals with the residual vibration of the flexible system, and leaves the control loop the task of getting rid of the disturbance. For very flexible system, which cannot be stabilized by the cascaded control loop alone, the use of input shaper is every effective.

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