IMM-UKF Algorithm and IMM-EKF Algorithm for Tracking Highly Maneuverable Target: A Comparison

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Abstract: - This paper aims to contribute in solving the problem of model-based body motion estimation by using data coming from visual sensors. We consider the case of state estimation in jump Markov nonlinear systems. The Interacting Multiple Model (IMM) algorithm is specially designed to track accurately targets whose state and/or measurement (assumed to be linear) models changes during motion transition. However, when these models are nonlinear, the IMM algorithm must be modified in order to guarantee an accurate track. In this paper we propose to compare the results given by an IMM algorithm Extended Kalman filter based (IMM-EKF) versus those given by an IMM algorithm Unscented Kalman filter based (IMM-UKF) in tracking target assumed to be highly maneuverable.

Keys-words: - Estimation, Kalman filtering, Target Tracking, Visual servoing.

1 Introduction

This paper hopes to be a contribution within the field of visual-based control of robots, especially in visual-based tracking [2]. To ensure a good track when the target switches abruptly from a motion model to another is not evident. Because of the complexity and difficulty of the problem, a simple case is considered. The study is restricted to 2-D motions of a point, whose position is given at sampling instants in terms of its Cartesian coordinates.

Several of maneuvring targets tracking algorithms are developed. Among them, the interacting multiple model (IMM) method based on the optimal Kalman filter, yields good performance when the measurement and state models are linear. However, if the latter are nonlinear, the Extended Kalman filter (EKF) generally substitutes the optimal Kalman filter, but some degradation should be expected according to the loss of optimality of the latter[4,5]. In this work, we present the Unscented Kalman filter (UKF) as a substitute of the EKF, then we give the results of the comparison between two nonlinear IMM algorithms based respectively on the EKF and on the UKF when considering Monte Carlo experiments with two scenarios where the target is supposed highly maneuverable.

The paper is organized as follows. In section 2 the mathematical formulation of 2-D motion is presented. In section 3 we briefly present the EKF and illustrate its expected limitations. In section 4 the Unscented Kalman filter is briefly described. In section 5 we present the nonlinear IMM algorithm with the two variants respectively EKF based and UKF based. In section 6 we present and discuss the results of simulations. Finally in section 7 we draw the conclusion.

2 Mathematical Formulation of 2-D Motion

The mathematical formulation of 2-D motion used is mainly inspired from Danes, Djouadi, and al in [3]. They make the hypothesis that the measurements are only the 2-D Cartesian coordinates of the moving point.

Let s(.) denote the curvilinear abscissa of M over time onto its trajectory, the origin of curvilinear abscissa is set arbitrarily. Functions x(.) and y(.), represent the Cartesian coordinates of M. The measurement equation may be written as:

$$\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \underline{h}\left(\underline{s}(t), \underline{p}(t)\right) \tag{1}$$

Where $\underline{p}(.)$ is a parameters vector function of minimal size. We can see that equation (1) is independent of the type of the motion of M onto its trajectory.

The state equation could be written as:

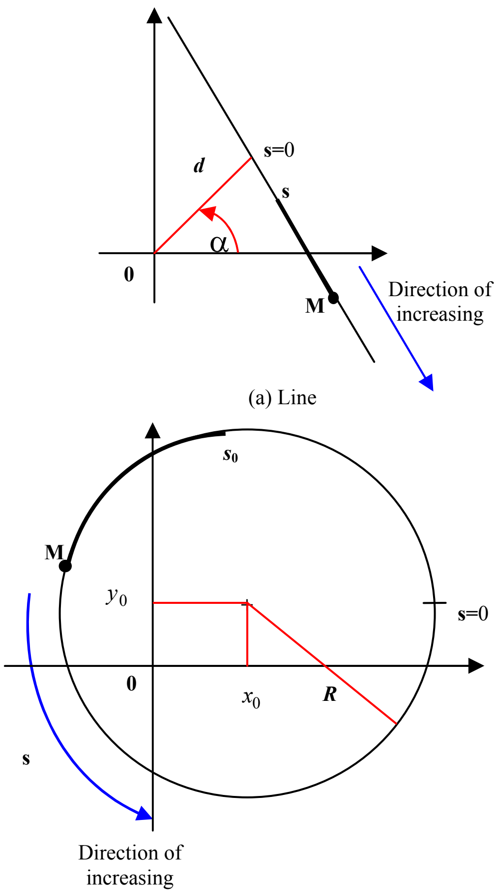
$$\dot{\underline{X}}(t) = A\underline{X}(t) \tag{2}$$

with $\underline{X}(t) = \begin{pmatrix} \underline{s}(t) \\ \underline{p}(t) \end{pmatrix}$

A equals $\begin{pmatrix} A_s & 0 \\ 0 & 0 \end{pmatrix}$, with A_s the n x n zero matrix with ones added on its first upper diagonal, and 0 the matrices of convenient sizes. The continuous time state equation (2) is linear time invariant and independent of M's trajectory, except on the sizes of $\underline{s}(.)$ and $\underline{p}(.)$. The dynamic and measurement noises are supposed to be stationary, white and Gaussian, non inter-correlated with known covariances.

The point M is supposed to move on straight or circular trajectories at constant or uniformly time-varying speed. Those motions belong to the set of the possible behaviors of a non-holonomic robot whose wheels are driven at constant velocities or accelerations.

One minimal description of a straight line is defined by the vector $\underline{p}=(\alpha,d)^T$ shown in figure 1(a), which is related to Plucker coordinates. Concerning a circular trajectory one minimal description is defined by the vector $\underline{p}=(R,x_0,y_0)^T$ shown in figure 1(b). The origin of curvilinear abscissa is uniquely defined from those parameterizations.



(b) Circle

Fig.1. Trajectory Parameterization

The output equations are as follows (trajectory parameter are considered time-invariant):

Straight Line: $$z(k) \overset{\Delta}{=} \begin{pmatrix} x(k) \\ y(k) \end{pmatrix} = \begin{pmatrix} d\cos\alpha + s(k)\sin\alpha \\ d\sin\alpha - s(k)\cos\alpha \end{pmatrix} + v(k) \quad (3)$$

Circle: $$z(k) \overset{\Delta}{=} \begin{pmatrix} x(k) \\ y(k) \end{pmatrix} = \begin{pmatrix} x_0 + R\cos\frac{s(k)}{R} \\ y_0 + R\sin\frac{s(k)}{R} \end{pmatrix} + v(k) \quad (4)$$

With $v(\cdot)$ Gaussian, and $E\{v(k)\} = 0$, $E\{v(k)v(k')^T\} = R\delta_{k,k'}$

3 Extended Kalman Filter

The Extended Kalman filter is a technique which allows the use of the standard Kalman filter on nonlinear process or measurement models. The process model and/or measurement model are linearized about the mean and covariance (the current operating point) at each iteration and the standard Kalman filter is then applied to the linearized models.

In the linear discrete Kalman filter, the state of the system can be updated with a straight forward matrix multiplication (F(k+1,k)). The same thing, converting from the state to measurement space is done with another matrix multiplication (H(k)). Both of these matrices are approximated in the EKF using a first order Taylor expansion.

To do this, the Jacobian matrix of both process model and measurement model are calculated. Since the process model is linear, the calculation is trivial. However, the Jacobian matrix of the measurement model is nontrivial:

1) Constant velocity on straight line

$$\frac{\partial H(k,X)}{\partial X} = \begin{bmatrix} \sin(\alpha) & 0 & (s(k)\cdot\cos(\alpha) - d\cdot\sin(\alpha)) & \cos(\alpha) \\ -\cos(\alpha) & 0 & (d\cdot\cos(\alpha) + s(k)\cdot\sin(\alpha)) & \sin(\alpha) \end{bmatrix}$$

2) Constant acceleration on straight line

$$\frac{\partial H(k,X)}{\partial X} = \begin{bmatrix} \sin(\alpha) & 0 & 0 & (s(k)\cdot\cos(\alpha) - d\cdot\sin(\alpha)) & \cos(\alpha) \\ -\cos(\alpha) & 0 & 0 & (d\cdot\cos(\alpha) + s(k)\cdot\sin(\alpha)) & \sin(\alpha) \end{bmatrix}$$

3) Constant velocity on circle

$$\frac{\partial H(k,X)}{\partial X} = \begin{bmatrix} -\sin\left(\frac{s(k)}{R}\right) & 0 & \cos\left(\frac{s(k)}{R}\right) & 1 & 0 \\ \cos\left(\frac{s(k)}{R}\right) & 0 & \sin\left(\frac{s(k)}{R}\right) & 0 & 1 \end{bmatrix}$$

4) Constant acceleration on circle

$$\frac{\partial H(k,X)}{\partial X} = \begin{bmatrix} -\sin\left(\frac{s(k)}{R}\right) & 0 & 0 & \cos\left(\frac{s(k)}{R}\right) & 1 & 0 \\ \cos\left(\frac{s(k)}{R}\right) & 0 & 0 & \sin\left(\frac{s(k)}{R}\right) & 0 & 1 \end{bmatrix}$$

These linearized matrices are incorporated into the EKF using the following equations:

The Kalman gain is:

$$K(k+1)=P(k+1|k)H^T(k+1)\left(R(k+1)+H(k+1)P(k+1|k)H^T(k+1)\right)^{-1} \quad (5)$$

The estimate is updated with the measurement by:

$$\hat{X}(k+1|k+1)=\hat{X}(k+1|k)+K(k+1)\left(z(k+1)-h\left(\hat{X}(k+1|k)\right)\right) \quad (6)$$

The error covariance of this estimate is:

$$P(k+1|k+1)=\left(I-K(k+1)H(k+1)\right)P(k+1|k) \quad (7)$$

For the next iteration the estimate is predicted by:

$$\hat{X}(k+1|k)=F(k)\hat{X}(k|k) \quad (8)$$

The predicted error covariance for the next iteration is:

$$P(k+1|k)=F(k)P(k|k)F^T(k)+Q(k) \quad (9)$$

In spite of being widely used over several decades, a number of drawbacks are usually expected on the EKF, among of them [4,6], the fact that linearization can lead to unstable filters if the local linearity assumption is violated.

Many substitutes to the EKF have been proposed in the literature, among them, the Unscented Kalman filter (UKF) introduced by Julier and Uhlmann in [5].

4 The Unscented Kalman Filter

The basis of the UKF is the unscented transform, where a distribution is approximated using a number of vectors which are passed through the nonlinear function to determine the probability distribution of the output from the function in spite of linearizing the function itself.

4.1 The Unscented Transform

A set of vectors, selected to be representative of the probability distribution, are chosen so that their mean and covariance are respectively $\bar{x}$ and P_{xx} [4,5].

The nonlinear function is applied to each point; the result is a set of transformed points with the statistics $\bar{y}$ and P_{yy}. The n-dimensional random variable x with mean and covariance respectively $\bar{x}$ and P_{xx} is approximated by $2n+1$ weighted points given by:

$$\chi_0=\bar{x} \qquad W_0=\frac{\kappa}{(n+\kappa)}$$
$$\chi_i=\bar{x}+\left(\sqrt{(n+\kappa)P_{xx}}\right)_i \qquad W_i=\frac{1}{2(n+\kappa)} \quad (10)$$
$$\chi_{i+n}=\bar{x}-\left(\sqrt{(n+\kappa)P_{xx}}\right)_i \qquad W_{i+n}=\frac{1}{2(n+\kappa)}$$

Where $\kappa\in R$, $\left(\sqrt{(n+\kappa)P_{xx}}\right)_i$ is the i th row or column of the matrix square root of $(n+\kappa)P_{xx}$ and W_i is the weight which is associated with the i th point, note that $\sum_{i=0}^{2n}W_i=1$. The transformation procedure occurs in three steps:

1. The transformed set of vectors are: $y_i=f(\chi_i)$ (11)
2. The mean is given by: $\bar{y}=\sum_{i=0}^{2n}W_i y_i$ (12)
3. The covariance is given by:

$$P_{yy}=\sum_{i=0}^{2n}W_i\{y_i-\bar{y}\}\{y_i-\bar{y}\}^T \quad (13)$$

4.2 The Unscented Kalman Filter

The unscented Kalman filter is obtained by occurring little modifications on standard one [4,5,6]. At first the state vector is augmented with the process and noise terms this leads to an $n^a=n+q$ dimensional vector.

$$x^a(k)=\begin{bmatrix}x(k)\\v(k)\end{bmatrix} \quad (14)$$

The process model is now a function of $x^a(k)$,

$$x(k+1)=f\left(x^a(k),u(k),k\right) \quad (15)$$

The unscented transform needs to $2n^a+1$ vectors which are selected from:

$$\hat{x}^a(k/k)=\begin{pmatrix}\hat{x}(k/k)\\0_q\end{pmatrix}$$

and (16)

$$P^a(k/k)=\begin{bmatrix}P(k/k) & P_{xv}(k/k)\\P_{xv}(k/k) & Q(k)\end{bmatrix}.$$

The prediction phase in the KF using the unscented transform is as follow:

1. The set of vectors are created by applying equation (10) to the augmented system given by equation (16)
2. The transformed set of vectors are given by
$$\chi_i(k+1)=f\left(\chi_i^a(k/k),u(k),k\right) \quad (17)$$
3. The predicted mean is calculated by
$$\hat{x}(k+1/k)=\sum_{i=0}^{2n^a}W_i\chi_i(k+1/k) \quad (18)$$
4. The predicted covariance is calculated by
$$P((k+1/k)=\sum_{i=0}^{2n^a}W_i\{\chi_i(k+1/k)-\hat{x}(k+1/k)\}\cdot\{\chi_i(k+1/k)-\hat{x}(k+1/k)\}^T \quad (19)$$
5. Each prediction vector is instantiate through the measurement model
$$z_i(k+1/k)=h\left(\chi_i(k+1/k),u(k),k\right) \quad (20)$$
6. The predicted observation is calculated by
$$\hat{z}(k+1/k)=\sum_{i=0}^{2n^a}W_i z_i(k+1/k) \quad (21)$$
7. Assuming the fact the measurement noise is additive and independent, the innovation covariance is
$$P_{vv}(k+1/k)=R(k+1/k)+\sum_{i=0}^{2n^a}W_i\{z_i(k+1/k)-\hat{z}(k+1/k)\}\{z_i(k+1/k)-\hat{z}(k+1/k)\}^T \quad (22)$$

8. The cross correlation matrix is calculated by

$$P_{xz}(k+1/k) = \sum_{i=0}^{2n^a} W_i\{\chi_i(k+1/k)-\hat{x}(k+1/k)\}\{z_i(k+1/k)-\hat{z}(k+1/k)\}^{T} \quad (23)$$

Given these predicted values the state and covariance estimates are computed according to the equations:

$$\hat{x}(k+1/k+1)=\hat{x}(k+1/k)+W(k+1)v(k+1)$$

$$P(k+1/k+1)=P(k+1/k)-W(k+1)P_{vv}(k+1/k)W^{T}(k+1) \quad (24)$$

With $v(k+1)$ the innovation and $W(k+1)$ the weight chosen to minimise the mean squared error of the estimate.

$$v(k+1)=z(k+1)-\hat{z}(k+1/k)$$

$$W(k+1)=P_{xz}(k+1/k)P_{vv}^{-1}(k+1/k) \quad (25)$$

5 The Nonlinear Interacting Multiple Model Algorithm

The Interacting Multiple Model (IMM) [1] is one of the rare algorithms which are able to overcome the problem of model changes during motion transition and is able of switching from one model to another accurately.

5.1 The IMM Algorithm

Let a system be described by the equations:

$$x(k)=F[M(k)]x(k-1)+v[k-1,M(k)]$$
$$z(k)=H[M(k)]x(k)+w[k,M(k)] \quad (26)$$

Where $M(k)$ denotes the model at time k. It's a finite state Markov process tacking values in $\{M_j\}_{j=1}^{r}$, according to a Markov transition probability matrix p assumed to be known, v and w represent white Gaussian processes and are assumed to be mutually independent.

A cycle of the IMM algorithm could be summarized in four steps:

5.1.1 The mixed initial condition for filters

Starting with $\hat{x}^i(k-1/k-1)$, we compute the mixed initial condition for the filter matched to M_j(k)

$$\hat{x}^{0j}(k-1/k-1)=\sum_{i=1}^{r}\hat{x}^{i}(k-1/k-1)\mu_{i|j}(k-1/k-1) \quad i,j=1,\ldots,r \quad (27)$$

the covariance corresponding to the above is

$$P^{0j}(k-1/k-1)=\sum_{i=1}^{r}\mu_{i|j}(k-1/k-1)\cdot \quad (28)$$
$$\Big\{ P^{i}(k-1/k-1)+\left[\hat{x}^{i}(k-1/k-1)-x^{0j}(k-1/k-1)\right]\cdot$$
$$\left[\hat{x}^{i}(k-1/k-1)-x^{0j}(k-1/k-1)\right]^{T} \Big\} \quad i,j=1\ldots r$$

Where:

$$\mu_{i|j}(k-1/k-1)=\frac{1}{\bar{c}_j}p_{ij}\mu_i(k-1) \quad i,j=1,\ldots,r \quad (29)$$

is the probability that model M_i was in effect at k-1 given that M_j is in effect at time k conditioned on Z^{k-1}.

And where: - $\mu_i(k-1)$ the probability that the mode M_i is in effect at time k-1.

- $\bar{c}_j=\sum_{i=1}^{r}p_{ij}\mu_i(k-1)$ the normalizing constants.

5.1.2 Model-matched filtering

The above estimate and covariance are used as input to the filter matched to $M_j(k)$, which uses z(k) to obtain $x^j(k/k)$ and $P^j(k/k)$.

5.1.3 Model probability update

The r model probabilities are updated from the innovation of the r Kalman filters.

The likelihood functions corresponding to the r filters are given by:

$$\Lambda_j(k)=N\{x(k);\hat{z}^j\{k/k-1;\hat{x}^{0j}(k-1/k-1)\}, S_j\{k;P^{0j}(k-1/k-1)\}\} \quad j=1\ldots r \quad (30)$$

$$\mu_j(k)=\frac{1}{c}\Lambda_j(k)\bar{c}_j \quad j=1,\ldots,r \quad (31)$$

Where $c=\sum_{j=1}^{r}\Lambda_j(k)\bar{c}_j$ is the normalization constant.

5.1.4 Estimation and covariance combination.

The output estimates and covariances are computed according to the mixture equations

$$\hat{x}(k/k)=\sum_{j=1}^{r}x^j(k/k)\mu_j(k)$$

$$P(k/k)=\sum_{j=0}^{r}\mu_j(k) \quad (32)$$
$$\left\{P^j(k/k)+\left[\hat{x}^j(k/k)-\hat{x}(k/k)\right]\left[\hat{x}^j(k/k)-\hat{x}(k/k)\right]^{T}\right\}$$

The problem here is that this algorithm is designed with the assumption that the target motion models are linear, so, for its optimality, the use of the Kalman filter is recommended. However, in our case the motion models considered are nonlinear. As a solution we propose to test the EKF and the UKF and to compare between theirs performances. The modification to operate on the IMM algorithm is to use as state and covariance estimator once the UKF then the EKF, the resulting algorithm is called the Nonlinear Interacting Multiple Model (NIMM).

6 Simulation and Results

In this section, we perform some simulations to evaluate our algorithm (NIMM). The motion models considered are: - constant velocity on straight line (M_1), -constant acceleration on straight line (M_2), - constant velocity on circle (M_3), - constant acceleration on circle (M_4).

To explore the capability of our NIMM algorithm to track a highly maneuvering target, Monte Carlo simulation is done on two scenarios.

We assume that the target is in a 2-D space and its position is sampled every T=1s. For all cases we assume:

- The measurement noise is zero mean, white, independent of the process noise, and with variance σ^2=0.01.
- The process noise Q_k=0.001, for all motion models.
- The initial condition with the state

$$X=[s,\dot{s},\ddot{s},\alpha,d,R,x_0,y_0]$$

$$X(0)=[1m \quad 1m/s \quad 0.1m/s^2 \quad 1\ rad \quad 50m \quad 90m \quad 30m \quad 40m]$$

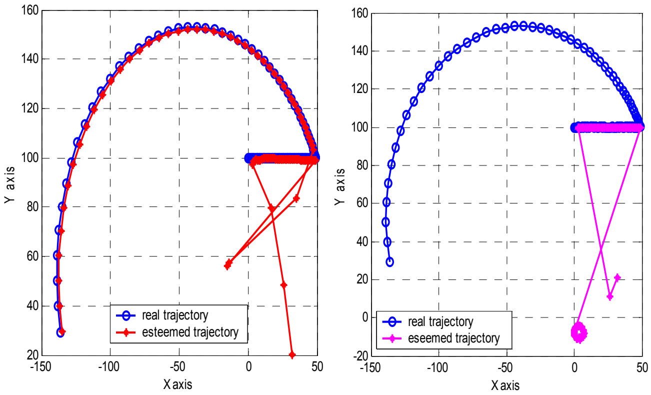
- The probability transition matrix of four models is

$$p=\begin{bmatrix} 0.97 & 0.01 & 0.01 & 0.01 \\ 0.01 & 0.97 & 0.01 & 0.01 \\ 0.01 & 0.01 & 0.97 & 0.01 \\ 0.01 & 0.01 & 0.01 & 0.97 \end{bmatrix}$$

- The initial probability of selecting a model is 0.25, that's to say, at the start all models have the same chance to be selected.
- The curvilinear abscissa s (.) remains continuous even if a trajectory jump occurs.

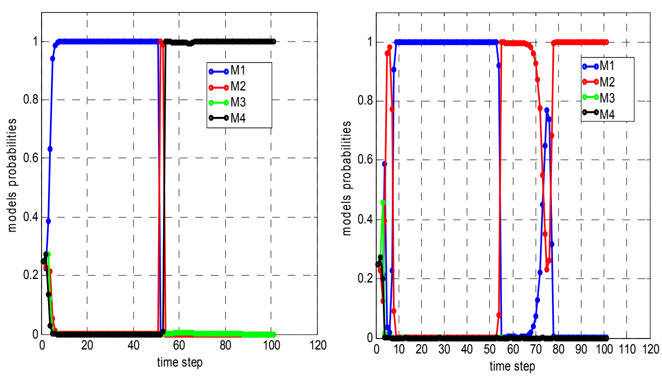
6.1 First scenario

The target starts moving according to model M_1 until the 50th sample when an abrupt acceleration about 0.2 m/s^2 and trajectory change occur and still moving according to this during 50 samples (switching from model M_1 to M_4).



a) IMM-UKF *b)* IMM-EKF

Fig.1. Real and Esteemed Trajectory



a) IMM-UKF *b)* IMM-EKF

Fig.2. Models Probabilities

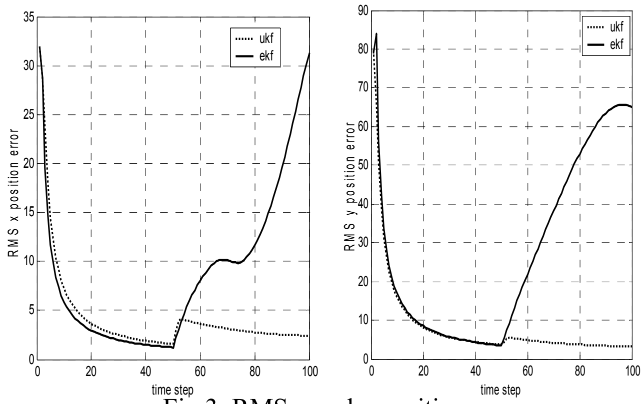


Fig.3. RMS x and y position error

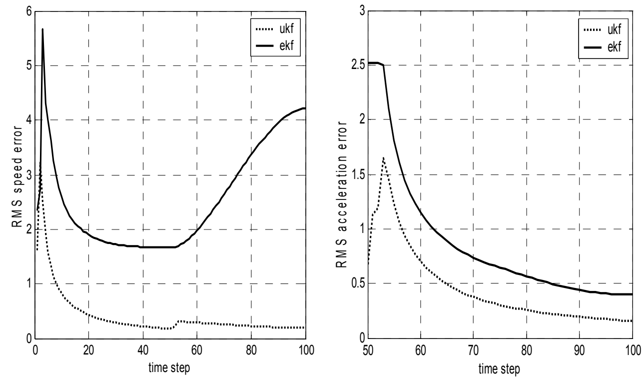
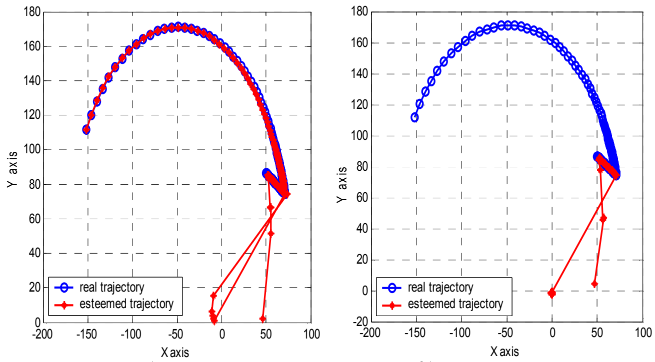


Fig.4. RMS Acceleration and Speed Error

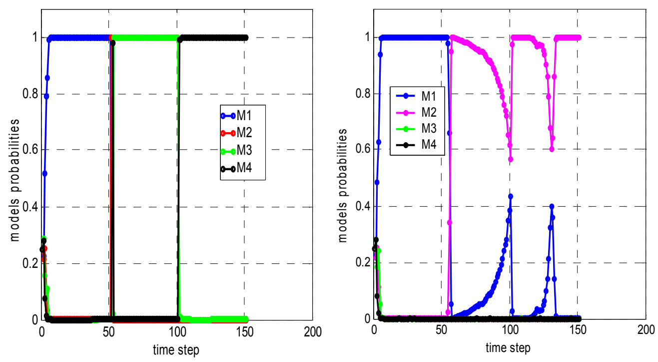
6.2 Second scenario

The target starts moving according to model M_1 until the 50th sample when it changes abruptly trajectory from straight line one to circle one and continue moving according to it until the 100th sample when an acceleration about 0.2 m/s^2 occurs and still maintained during 50 samples (switching from model M_1 to M_3 and then to M_4).



a) IMM-UKF *b)* IMM-EKF

Fig.5. Real and Esteemed Trajectory



a) IMM-UKF *b)* IMM-EKF

Fig.6. Models Probabilities

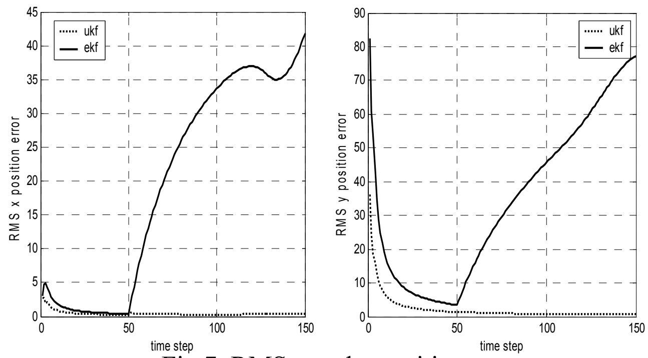


Fig.7. RMS x and y position error

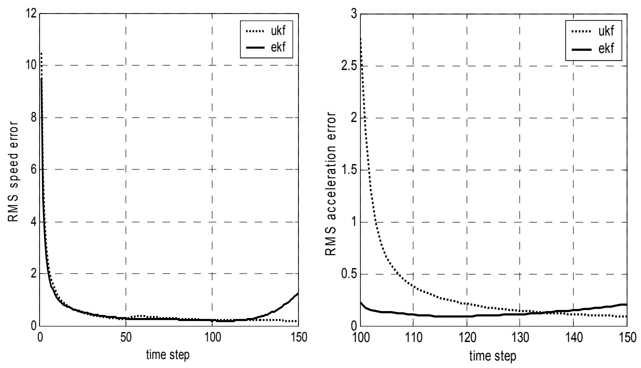


Fig.8. RMS Acceleration and Speed Error

6.3 Results interpretation:

Fig.1 and fig.5 show that before the abrupt change on the trajectory and dynamics occur (from straight line to circle and from constant velocity to constant acceleration), the two algorithms converge to the correct trajectory, but after the trajectory change only the IMM-UKF algorithm detects and converges to the correct new trajectory. In the second scenario, the second abrupt change is also detected by the IMM-UKF algorithm and converges to the correct trajectory, but the IMM-EKF algorithm is completely divergent. This is confirmed by fig.2 and fig.6 which show the correct transition of the IMM-UKF algorithm from model M_1 to model M_4 in the first scenario and from model M_1 to model M_3 and then to M_4 in the second scenario; and the incapability of the IMM-EKF to converge to the correct model after the abrupt change. Figures 3, 4, 7 and 8 also confirm this result, as it could be seen, only the IMM-UKF algorithm converges to the correct position and dynamics after the maneuver; the IMM-EKF completely diverges.

From this, we can say that when we have to track a highly manoeuvrable target whose trajectory and/or dynamics could change abruptly over time, and when the state and/or measurement models present strong nonlinearities, the IMM-UKF algorithm is a very interesting solution to be envisaged. In this case the IMM-EKF is completely rejected.

7 Conclusion

The model-based body motion estimation by using data coming from visual sensors still an open problem on which we try to provide a contribution. In this paper we presented, the IMM-EKF and the IMM-UKF, two nonlinear algorithms which attempt to track efficiently a highly maneuvering target whose trajectory and/or dynamic could change abruptly.

Simulations show that the IMM-UKF is a good investment while we are asked to track a highly manoeuvrable target whose measurement and/or state models present a strong nonlinearities. However, as it was expected the IMM-EKF (according to the several drawbacks of the EKF) is unable to track this kind of targets.

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