REAL-TIME MANAGEMENT OF AIR TRAFFIC FLOW BASED ON THE CONTROLLED STOCHASTIC PETRI NET APPROACH.

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Abstract: - Several efforts have been focused on the improvement of the quality of service of *air transportation* in order to face the increasing of the demand. Consequently, in the latest decade, a growing body of advances concerning several aspects of this transportation mode has appeared in the operation research literature. At the best of our knowledge Stochastic Petri Nets (SPN) have not been yet used for the graphical description, the analysis of flows and the dynamic control of the ground holding problem. Due to the characteristics of the air transportation systems, this paper shows that the Controlled Stochastic Petri Net (CSPN) model of such a system is very efficient.

Indeed, the presented CSPN model aims at minimizing simultaneously the airborne and the ground delays, taking into consideration the probabilistic nature of the air traffic. As it is shown, this mathematical tool is very interesting for the real-time scheduling of the plane landing or flight at the airport.

The CSPN model of an elementary landing system is described and defined and this paper shows that it has interesting properties. It illustrates the interest of such a tool with a numerical application in which it analyses the optimal solution.

Keywords: System modeling, Air traffic flow management, Ground holding policy, Discrete event systems, Controlled Stochastic Petri nets.

1. Introduction

Several authors deal with issues of the air traffic planning in order to provide more efficient usage of the resources. Indeed, in the last decade, the competition becomes harder and the air traffic flow is very dense. Consequently, a growing body of advances concerning several aspects of the air transportation has appeared in the operation research literature. Most of the approaches aim at fulfilling the need for a better management of the air traffic that leads to minimize the traffic congestion at airports. The advantages of this are numerous, such as reducing airborne delays and the air pollution as well as increasing the safety level. These approaches are based on optimization techniques and on simulation approaches.

Due to the tremendous size and the complexity of air transportation systems, diverse models of these systems have been proposed in the literature [1] and [2]. Indeed, the air transportation industry is very rich in terms of problems, which can be modeled and solved using mathematical techniques. This paper focuses on the landing and on the flight-scheduling problem based on the well-known ground holding policy (GHP). In the literature, the modeling and the solutions of this problem are mainly based on a deterministic approach. However, the plane landing at the airport involves considerable uncertainty about weather conditions and plane safety. Thus, it is interesting to introduce a stochastic approach for modeling such a problem.

However, at the best of our knowledge, except the analysis of the number of the relief strips, stochastic Petri Nets (SPN) have not been yet used for the graphical description, the analysis and the real-time control of air traffic flows.

In this paper, the CSPN model aims at real-time optimizing delays, taking into consideration the airborne safety aspect and the quality of service. The use of this model allows to tackle the well-known problem of the GHP. The study is limited to one destination airport, which has to choose which flight to delay, in order to face the growing down of its capacity. As it is shown in the following, this mathematical tool is useful to take a decision at the airport when a conflict situation occurs about plane landing or flight. This paper illustrates the interest of such a tool with a numerical application.

This paper is organized as follows. In the second section, a brief description of the CSPN theory is given. The third section introduces the dealt traffic flow. Afterward, the CSPN model is shown and the simulation results are presented in the fourth section. Finally, a conclusion and prospects are given.

2. CSPN

The CSPN are widely used for the design of stochastic discrete event systems, especially for the study of manufacturing systems, telecommunication networks and computer systems [4]. They offer graphical and mathematical description of the system. This enables us to evaluate the performance and to compute the optimal control [5]. Thus they are very useful because they afford an appreciable aid to the designer, who attempts to predict the system behavior at the design stage rather than at the implementation one. This mathematical tool can be applied in the field of the transportation science, as it has been already shown in several recent works [6]. This paper introduces the CSPN-modeling of a particular problem of air traffic flow management.

More precisely the Petri Net (PN) allows modeling of sequential and concurrent actions including phenomena such as contention and synchronization. The additional structure in the Stochastic PN (SPN) [5] allows the extraction of additional performance information about the modeled system. Indeed, the reachability graph of the Continuous Time SPN (CTSPN) with exponentially distributed delays is isomorphic to homogeneous Continuous Time Markov Chain (CTMC) and this opens up an area of analysis for performance measures such as average delay and throughput. A Markov Reward Process (MRP) is obtained from the Stochastic Reward Nets (SRN) model, where a real-valued reward rate is associated to each marking of the reachability graph. This MRP allows an evaluation of the performances of the system. The SRN is a Controlled SPN (CSPN), when the isomorphic MRP is subject to control and a finite set of selections can be done.

In the following, we recall the basic Petri net terminology and notations.

A Petri net (cf. Fig. 1) is a five-tuple $N=(P,T,I,O,M_0)$ where :

- $P=\{p_1,p_2,\ldots,p_m\}$ is a finite set of places(represented with circles),
- $T=\{t_1,t_2,\ldots,t_n\}$ is a finite set of transitions (represented with line segments),
- I: is an input function such that $I(p_i,t_j)$ is the weight of the arc directed from place p_i to transition t_j,
- O: is an output function such that $O(p_i,t_j)$ is the weight of the arc directed from transition t_j to place p_i,
- M_0; is an initial marking that associates zero or more tokens to each place.

The state of a Petri net is defined by the number of tokens in each place and is represented by a vector $M=[M(p_1),\ldots,M(p_m)]^t$, called the marking vector of the Petri net, where $M(p_i)$ is the number of tokens in place p_i. A transition $t_j \in T$ is said to be enabled if and only if $M(p_i) \geq I(p_i,t_j)$, with $p_i \in P$. An enabled transition may fire. When transition t_j fires, $I(p_i,t_j)$ tokens are removed from each input place p_i of t_j, and $O(p_i,t_j)$ tokens are added to each output place p_i of t_j. The dynamic behavior of the modeled system is described by the transitions firing mechanism. If the transition t_j is fired then the marking $M_0(p_i)$ results in a new marking $M(p_i)$ such that $M(p_i) = M_0(p_i) + O(p_i,t_j) - I(p_i,t_j)$.

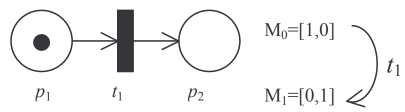

Fig. 1 Petri net

To obtain the performance of the dealt air transport system, the SPN model required to be solved analytically. The general procedure to solve an SPN model includes [??]:

1- Generating the reachabilitry graph from SPN model,
2- Transforming the reachability graph into Markov model
3- Solving the Markov model

Let t designate the time. We use the following theoretical definitions to model the air transportation system. Let $Z=\{Z(t),t\geq 0\}$denote a CTMC with finite state space Ω. Define the infinitesimal generator matrix $Q=\lfloor q_{ij}\rfloor$ consisting of the direct transition rates from state i to j and the diagonal entries defined as: $q_{ii}=-\sum_{j,j\neq i} q_{ij}$. A homogeneous CTMC can be completely described by its infinitesimal generator matrix Q and its initial probability vector $p(0)$. The transient probability vector at time $t>0$ for a CTMC, $p(t)$, is obtained by solving the equation

$$\frac{dp(t)}{d\tau} = p(t)\cdot Q \qquad (1)$$

whose solution can be formally written as $p(t) = p(0)\cdot H(t)$ with $H(t) = e^{Qt}$.

Let a real-valued reward rate r_i for each state $i\in\Omega$ and, if the Markov chain stays in state i for duration t, a reward $r_i \cdot t_i$ is gained. Let $\Re(t)$ represents the random variable corresponding to the instantaneous reward rate. The expected value of the reward rate as a function of time can be computed as $X(t) = E[\Re(t)] = \sum_{i\in\Omega} r_i \cdot p_i(t)$. In the time interval $[0,\ldots t)$ the accumulated reward is defined by $Y(t) = \int_0^t \Re(u)du$. The expected value of the accumulated reward $E[Y(t)]$ can be computed as:

$$Y(t) = \int_0^t X(u)du = \sum_{i\in\Omega} r_i \int_0^1 p_i(u)du \qquad (2).$$

Let the MDP be an MRP subject to control. We define the decision arcs that denote the option to select the transition rate from one state to another. At every point of time a set of transition rates is possible for each decision arc. A strategy $S(T)$ comprises a set of done selection for all options of the model at particular points of time $t_0, t_1 \ldots t_n$ with $0 \le t_0 < t_1 \ldots < t_n \le T$. A strategy $\hat{S}(T)$ is considered optimal if the performability measure under strategy $\hat{S}(T)$ is greater or equal than the performability measure under any other strategy $S(T)$.

3. Traffic description

The GHP deals with the airport congestion that is due to the probabilistic nature of the air traffic. Actually, for example, even if the maximum airport capacity were adequate to meet the scheduled demand, it does happen that bad weather conditions drop this capacity by half or more. Consequently, several flights can be delayed. The planners have to take an option either to give preference to the airborne delay or to the ground delay. The GHP is based on the principle that ground delay is safer, less pollutant and less expensive than the airborne delay. The GHP consist in giving preference to ground delays over airborne delays.

However, even under optimal assignment of ground-holds, there will be instances in which airport landing capacity is lost where planes sit waiting on the ground. These occurrences could be more frequent than necessary when bad weather is expected. Indeed, airports tend to provide conservative capacity forecasts to protect their airspace from saturation. The trade-off in establishing ground-hold delays is between conservative policies that may at times assign excessive ground-holds and optimistic ones that may result in more expensive airborne delays.

Thus to overcome the drawback of both type of policies, the authors of [7] define the requirements of an effective ground delay program, as follows: t_{Gj}

- Considering relative costs of ground and air delays
- Taking into account uncertainty regarding airport capacities
- Being able to respond to a constantly changing system.

The considered air traffic can be described in reference to the single-destination network shown in Fig.2. The proposed CSPN model (cf. Fig.3) of this system attempts to address the key issues involved by the authors of []. The semantic of the model is presented on Table 1. This model

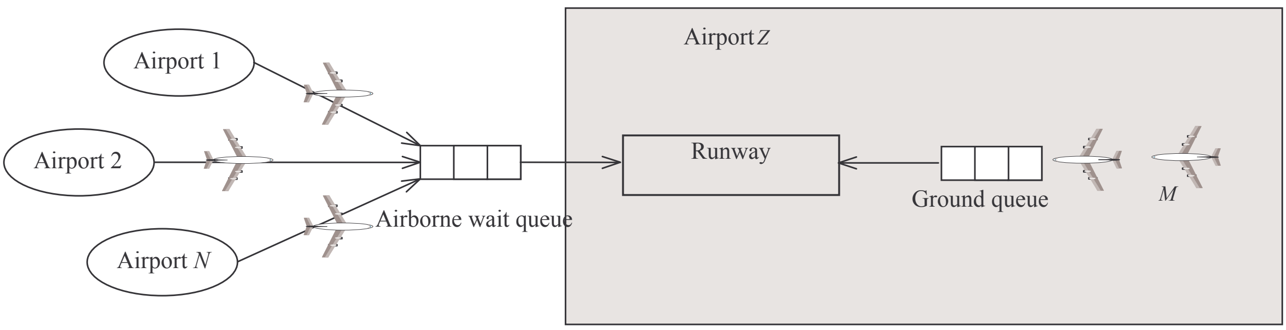

Fig.2 Traffic at the airport

captures the essential elements needed to solve the GHP problem:

- Airport Z is the only capacitated element of the network.
- N planes (flight F_{A1}, ..., F_{AN}) are scheduled to arrive at the "arrival" airport Z from the "departure" airports.
- M planes (flight F_{G1}, ..., F_{GN}) are scheduled to fly form airport Z.
- The departure and the arrival times (θ_G and θ_A, respectively) of each plane are deterministic and known in advance.
- The capacity of airport Z is expressed by means of the minimum admitted time interval, which separates two successive plane actions, i.e. landing or flight.
- Ground and air delay cost functions for each flight are known.

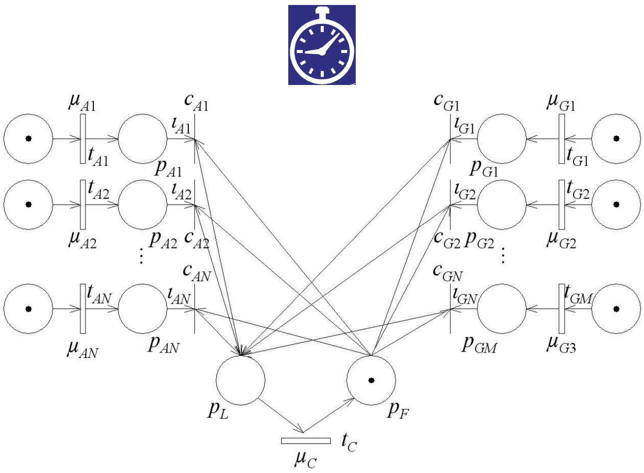

Fig.3 CSPN model

The semantic of the model is as follows:

- The marking of places:
  - p_{Ai} with i=1...N represents the plane i flying and waiting for an order to land.
  - p_{Gi} with i=1...M represents the plane i on the ground and waiting for an order to fly.
  - P_F represents the fact that the runway is free.
  - P_L represents the fact that the runway is not free.

- There are two classes of transitions: the first one represents the *timed transitions* whose delays are exponentially distributed random variables. Each timed transition is associated with a constant firing rate. This rate is the inverse of the average firing time of the transition. The one represents the *controlled transitions*, c_{Ai} and c_{Gj}, whose delays are deterministically ι_{Ai} and ι_{Gj} respectively. These times represent the moments when the planes are allowed to land or to fly. This means that there is a clock the time ι. Thus, when $\iota<\iota_{Ai}$ or $\iota<\iota_{Gj}$, planes F_{Ai} or F_{Gi} are not allowed to land or to fly, even if transitions c_{Ai} or c_{Gj} are enabled respectively. The meanings of the firing rates whose are depicted in the figure 2 are explained in the table 1.

Table 1. Timed Transitions Description of the CSRN Model

Parameter	Meaning
$\mu_{Ai} = \frac{1}{\theta_{Ai}}$	θ_{Ai} the scheduled arrival times of flight F_{Ai} with i=1...N
$\mu_{Gi} = \frac{1}{\theta_{Gi}}$	θ_{Gi} the scheduled arrival times of flight F_{Gi} with i=1...M
$\mu_{Gi} = \frac{1}{\theta_{C}}$	θ_C the minimum admitted time interval, which separates two successive plane actions, i.e. landing or flight.
ι_{Ai}	The new scheduling landing time
ι_{Gi}	The new scheduled flight time

One can note that a negative reward must be associated to each state where places p_{Ai} and p_{Gj} are marked. Indeed, such a marking means that plane landings or flights are delayed. The value of the negative reward depends on the nature of the expected plane action i.e. landing or flight. In other words, it is necessary to distinguish the ground delay cost and the air delay cost, which is more expensive. Hence, solving equations (1) and (3), it is possible to define the objective function to optimize.

4. Real-time management of an elementary traffic flow

The analyzed system is the one depicted in fig.4. This assumes that a plane F_{G1} is ready to fly and the arrival time of another plane F_{A1} is expected for θ_{Ai}. Thus we have to decide ι_{Gj} which minimizes both delay costs. The CSPN model of the system is presented in fig.5. The MDP has nine states. Five of them represents airborne-delay or ground-delay.

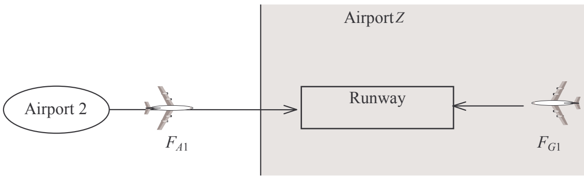

Fig.4 Elementary traffic flow

We assume that the airborne-delay is ten times more expensive than the ground delay. Thus, fig.6 shows the variation of the optimal ι_{Gj} against θ_{Ai}.

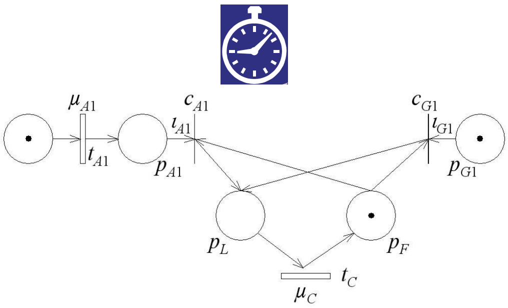

Fig.5 Elementary CSPN model

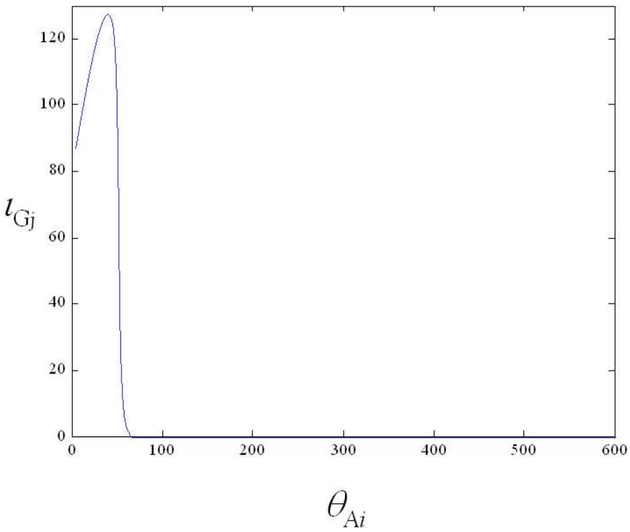

Fig.6 Variation of θ_{Ai} against ι_{Gj}

From fig.6 one can note the interest of such an approach where:

- An interval of safety is taken into consideration. Indeed, when $\iota_{G1}>0$, $\iota_{G1}-\theta_{A1}>0$.
- The model ovoid an unnecessarily ground delay. Indeed, when $\theta_{A1}>81{,}2$ sec $\iota_{G1}=0$.
- The ground holding policy is kept. Indeed, plane F_{A1} still priority.

5. CONCLUSION AND PROSPECTS

We have proposed in this paper an approach to model air transportation systems using the CSPN and MDP theories. This allows improving the real-time management of the air traffic flow. Indeed, we have shown through an elementary example that the proposed model takes into consideration the probabilistic nature of the air traffic flow, by avoiding an unnecessarily ground-delay and by introducing a safety interval. Besides, the GHP is kept.

Several issues deserve further investigation. We expect to compare the results obtained by CSPN model of GHP problem with the ones abstained by a deterministic approach.

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