Convergence-Speed Enhancement of Two Classes of (Ω,α)-Fair Rate Allocation Algorithms

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Abstract: - Fairness is an important aspect of any rate allocation strategy. Many fairness criteria have been proposed by different researchers. Mo and Walrand have introduced the notion of (Ω,α)-fairness in their pioneering work in 2000. Different fairness criteria such as *proportional, minimum potential delay* and *max-min* fairness are derived as special cases of (Ω,α)-fairness. In the current paper, a fast second-order rate allocation algorithm is proposed that can improve the convergence speed of conventional methods. Under certain (and almost) practical premises about the network topology, an end-to-end implementation of well-known *proportional* and *minimum potential delay* fair rate allocation algorithms is introduced. The proposed algorithm has been compared with the conventional method and TCP using simulation. The simulation results show that the proposed method, outperforms the conventional ones in convergence rate.

Key-Words: - Proportional fairness, Elastic traffic, (Ω,α)-fairness, Penalty function.

1 Introduction

There are plenty of fairness criteria such as *max-min*,[1] *proportional* [2] and *minimum potential delay* fairness [3]. Selecting a fairness criterion depends on the network's designer strategy. For example in the *max-min* criterion, the attention is strictly to users with lowest rates whereas in the Kelly's *proportional* criterion the objective is maximizing the overall throughput and we pay less attention to lower rate users and more penalize users who use long routes in network. In *minimum potential delay* criterion L. Massoulie et al. define a delay measure in terms of user throughputs and try to minimize that delay.

In the current paper we assume that the network traffic is of '*elastic*' type which was introduced by S. Shenker in [4]. As well-known examples of such traffic type we can mention FTP traffic in the current Internet and ABR traffic in the ATM networks.

The paper is organized as follows. In §2 we review some related works and specially the work of Walrand et al. in [5]. In §3 we introduce the algorithm. §4 is devoted to simulation results and finally we conclude in §5 our paper with conclusion.

2 Background

Consider a network with a set J of resources or links and a set $\Re$ of users and let C_j denotes the finite capacity of link $j\in J$. Let $\mathbf{\Omega=(}\omega_r \mathbf{,}\ r\in\Re)$ be the vector, representing users' ω parameters and assume that α is a positive number which is greater or equal to unity. Each user r has a fixed route R_r, which is a nonempty subset of J. Then Walrand's formulation of the **(Ω,α)**-fair rate allocation would be:

$$x_r[n+1] = \left\{x_r[n] + k_r \cdot \left(\omega_r - \left(x_r[n]\right)^\alpha \cdot \lambda_r[n]\right)\right\}^+ \quad (1)$$

Where:

$$\lambda_r[n] = \sum_{j\in R_r} p_j\left(\sum_{s\in j} x_s[n]\right),\ \{x\}^+ \triangleq \max(0,x) \quad (2)$$

Parameter k_r controls the speed of convergence in equation (1). Penalty function $p_j(y)$ is the amount that link 'j' penalizes its aggregate traffic 'y' and is a non-negative, continuous increasing function and tends to infinity as aggregate rate 'y' tends to link capacity C_j [2]. Given λ_r, user r selects an amount that is willing to pay per unit time, ω_r, and finally receives a rate $x_r = (\omega_r / \lambda_r)^{1/\alpha}$.

One of the interpretations is that using (1), the system tries to equalize ω_r with $(x_r[n])^{\alpha} \cdot \sum_{j\in R_r} p_j[n]$ by adjusting $x_r[n]$ value. System (1)-(2) show that the unique equilibrium x^*_r is the solution of the following equation:

$$\omega_r = \left(x_r^*\right)^{\alpha} \cdot \sum_{j\in R_r} p_j\left(\sum_{s:j\in R_s} x^*_s\right) \quad , r\in\Re \tag{3}$$

It must be noted that α=1,2 are related to *proportional* and *minimum potential delay* fairness respectively and as α tends to infinity system (1)-(2) approach to the *max-min* fairness criterion [5].

3 Proposed Algorithm

The high-speed algorithm is composed of a two-level hierarchical structure. First look at an example. Consider the Fig.1. Let's assume that the network is consisted of 11 elastic sources that are included in four source virtual users. Dotted lines show the boundaries of the virtual users and thick lines show the aggregate flow of each virtual user that is traversing through the links that belong to backbone (these links are denoted by letters L6, L7 and L8). Each source (destination) of information is denoted by's' ('d') and as mentioned before, the rate associated with each (source, destination) pair is denoted by 'x'. Links are unidirectional and in Fig.1, links 6, 7 and 8 constitute the backbone.

As Walrand et al. have shown in [5], stabilized rates of users are:

$$x_r^* = \left(\omega_r / \lambda_r^*\right)^{1/\alpha} \quad , r\in\Re$$

Where:

$$\lambda_r^* = \sum_{j\in R_r} p_j\left(\sum_{u:j\in R_u} x_u^*\right)$$

Since it is assumed that the congestion may only occur in the links which belong to backbone, we may consider that λ^*_r is only affected by backbone links and is approximated by:

$$\lambda_r^* \cong \sum_{\substack{j\in R_r \\ \& j\in \text{Backbone}}} p_j\left(\sum_{u:j\in R_u} x_u^*\right) \tag{4}$$

For example, for users 's_1' and 's_2' in Fig.1, we would have:

$$x_1^* = \left(\frac{\omega_1}{\lambda_1^*}\right)^{1/\alpha}, \; x_2^* = \left(\frac{\omega_2}{\lambda_2^*}\right)^{1/\alpha} \tag{5}$$

Define:

$$\Lambda_1^* \overset{\Delta}{=} p_6\left(\sum_{u:L_6\in R_u} x_u^*\right)$$

where Λ_1^* is the aggregate penalty of users 's_1' and 's_2' (λ^*_1 and λ^*_2) in backbone of the network (link '6' in this case).

Then, at the equilibrium point, the aggregate rate of virtual user 1 is:

$$x_1^* + x_2^* = \left(\frac{\omega_1}{\lambda_1^*}\right)^{1/\alpha} + \left(\frac{\omega_2}{\lambda_2^*}\right)^{1/\alpha} \cong \frac{(\omega_1)^{1/\alpha} + (\omega_2)^{1/\alpha}}{(\Lambda_1^*)^{1/\alpha}} \tag{6}$$

If we denote the aggregate rate of virtual user 1 with χ_1, at the equilibrium point we have:

$$\chi_1^* = \frac{(\omega_1)^{1/\alpha} + (\omega_2)^{1/\alpha}}{(\Lambda_1^*)^{1/\alpha}} \tag{7}$$

By considering equations (5) and (7) and the assumption that $\lambda_1^* \cong \Lambda_1^*$, then:

$$x_1^* \cong \frac{(\omega_1)^{1/\alpha}}{(\omega_1)^{1/\alpha} + (\omega_2)^{1/\alpha}} \cdot \chi_1^* \tag{8}$$

Now, in the mathematical terms [6]. let $\mathcal{S} \triangleq \{\mathcal{S}_i \mid i=1,2,\ldots,Q\}$ and $\mathcal{D} \triangleq \{\mathcal{D}_i \mid i=1,2,\ldots,Q\}$ be the sets that represent the virtual sources and virtual destinations . Where, Q represents the number of virtual sources (destinations). For example, in Fig.1 we have Q=4 and $\mathcal{S}_3=\{s_6,s_7\}$, $\mathcal{D}_3=\{d_6,d_7\}$.

If the rate associated with virtual user 'i' at iteration 'n' is denoted by '$\chi_i[n]$', and the rate of end users (as mentioned before) are denoted by small 'x' letter, algorithm behaves in the following manner:

At the beginning, algorithm works in the first level of hierarchy and allocates rates to the virtual sources using some high-speed algorithm (such as Jacobi method). Then, each virtual user assigns some proportions of its rate to each end-user within the virtual user.

If the assumption in equation (4) is true, when the system is in the vicinity of equilibrium point, users' rates are close to the optimal values. The rate assignment by virtual user 'i' to a user 'u' located within virtual user 'i' is:

$$x_u[n+1] = \chi_i[n].\frac{(\omega_u)^{1/\alpha}}{W_i} \quad ,n = 0,1,2,... \qquad i = 1,2,...,Q\,,u \in i \tag{9}$$

Where notation '$u \in i$', means that user 'u' is located within virtual user 'i' and:

$$W_i \overset{\Delta}{=} \sum_{u \in i} (\omega_u)^{1/\alpha} \tag{10}$$

By the notion of (Ω,α)-fairness, the Ω_i associated with the virtual user 'i' would be:

$$\Omega_i = (W_i)^{\alpha} \quad ,i = 1,2,...,Q$$

Evolving $\chi_i[n]$ in the equation (9) is as Jacobi iteration [1] (i=1,2,…,Q):

$$\chi_i[n+1] = \left\{\chi_i[n] + K_i \cdot \left[\Omega_i - (\chi_i[n])^{\alpha} \cdot \Lambda_i[n]\right] \Big/ \left[\alpha(\chi_i[n])^{\alpha-1} \cdot \Lambda_i[n] + (\chi_i[n])^{\alpha} \cdot \frac{\partial}{\partial \chi_i(t)} \Lambda_i(t)\Big|_{t=n}\right]\right\}^{+} \tag{11}$$

Where $\chi_i[0] = \varepsilon \cong 0 \quad ,\forall i$ and also:

$$\Lambda_i[n] \overset{\Delta}{=} \sum_{\substack{j \in R_{S_i} \\ \& j \in \text{Backbone}}} p_j\Big(\sum_{u: j \in R_u} \chi_u[n]\Big)$$

Equation (11) is in fact a form of the projected Jacobi method, as Bertsekas et al. have defined in[1].

4 Simulation Results

Consider the network topology of Fig.2 which is composed of 87 elastic users and 94 links. Gray nodes are the network's backbone boundary.

We have adopted a similar approach as that of Walrand[5] and Başar[7] for simulating the rates allocated to the users with different propagation delays. We have used the OPNET discrete-event simulator. The bottleneck links are in the backbone, and all of the backbone links have the capacity equal to 150kBps, other link capacities are selected 100MBps. All backbone links' propagation delays is set to 5 ms and we assume that propagation delay of the other links to be negligible. We have assumed that sources have data for sending at all times (greedy sources). All backbone links' buffer sizes are set to 100 packets.

We have used *go back* n method [8] for re-sending the packets that are double acknowledged. Links' scheduling discipline is FIFO. As in TCP, Slow-Start method is used for initializing the rate allocation.

Receivers' window sizes are set to unity. Sender window size in Walrand method is updated according to the following relations:

$$cwnd_r[n+1] = \left\{cwnd_r[n] - k_r \cdot \frac{cwnd_r[n]}{RTT_r[n]} \cdot s_r[n] \cdot u_r[n]\right\}^{+} \tag{12}$$

Where:

$$s_r[n] = cwnd_r[n] - \frac{cwnd_r[n]}{RTT_r[n]} \cdot \bar{d}_r - \frac{\omega_r}{\left(\frac{cwnd_r[n]}{RTT_r[n]} + 1\right)^{\alpha-1}} \tag{13}$$

And also:

$$u_r[n] = \bar{d}_r - (\alpha - 1) \cdot \frac{\omega_r}{\left(\frac{cwnd_r[n]}{RTT_r[n]} + 1\right)^{\alpha}} \tag{14}$$

In Jacobi hierarchical method, sender congestion window evolves as follows:

$$CWND_i[n+1] = \left\{CWND_i[n] - K_i \cdot \left(\frac{CWND_i[n]}{RTT_i[n]} \cdot S_i[n] \cdot U_i[n]\right) \Big/ \left(\alpha \cdot d_i[n] \cdot \left(\frac{CWND_i[n]}{RTT_i[n]}\right)^{\alpha-1} + \left(\frac{d_i[n] - d_i[n-1]}{\Delta_n^{\chi_i}}\right) \cdot \left(\frac{CWND_i[n]}{RTT_i[n]}\right)^{\alpha}\right)\right\}^{+} \tag{15}$$

Where:

$$\Delta_n^{\chi_i} \overset{def}{=} \frac{CWND_i[n]}{RTT_i[n]} - \frac{CWND_i[n-1]}{RTT_i[n-1]} \tag{16}$$

$$S_i[n] = CWND_i[n] - \frac{CWND_i[n]}{RTT_i[n]} \cdot \bar{d}_i - \frac{\Omega_i}{\left(\frac{CWND_i[n]}{RTT_i[n]} + 1\right)^{\alpha-1}} \tag{17}$$

$$U_i[n] = \bar{d}_i - (\alpha - 1) \cdot \frac{\Omega_i}{\left(\frac{CWND_i[n]}{RTT_i[n]} + 1\right)^{\alpha}} \tag{18}$$

$$d_i[n] = RTT_i[n] - \bar{d}_i \qquad ,\forall i \tag{19}$$

Where, $\overline{d}_r\left(\overline{d}_i\right)$ is the user (virtual user) 'r'('i') propagation delay and its round trip time is $RTT_r(RTT_i)$.
The final rate that is allocated to each end user in the hierarchical method is based on the equation (9).

$$x_u[n+1] = \frac{CWND_i[n]}{RTT_i[n]} \cdot \frac{(\omega_u)^{1/\alpha}}{W_i} \quad , n = 0,1,2,... \qquad (20)$$
$$i = 1,2,...,Q \,, u \in i$$

We have used $k_r = K_i = 0.0003$ in Walrand and hierarchical method**.**
It is important that as congestion occurs only in the bottleneck links located in the backbone, the rate allocation algorithm is only consisted of equations (9) and (11). This relations reach the users' rates to the optimal ones as in (3).
The simulation results for 4 users in Fig. 2 are depicted in Figs.3 to 6. We have compared in these Figures, the proposed second order method with the Walrand's method and TCP for two values of α=1,2 which correspond to *proportional* and *minimum potential delay* fairness respectively. It can be verified that the proposed method, outperforms that of Walrand in convergence speed. In the simulated figures, MPDF, PF, TCP, HMPDF and HPF symbols stand for *minimum potential delay* fairness, *proportional* fairness, TCP, hierarchical *minimum potential delay* fairness and hierarchical *proportional* fairness respectively.
On the other hand, another outstanding feature of our rate allocation strategy is that the user rates in the proposed method and that of Walrand, have less fluctuations with respect to TCP.
As equations (12) to (19) use only the RTT and propagation delay of the connection, they can be implemented in an end-to-end manner.

5 Conclusion

In the current paper, we have compared the performance of a high-speed second-order algorithm with the conventional Walrand's algorithm in the *proportional* and *minimum potential delay* fairness criteria. Simulation results show that the proposed method, outperforms that of Walrand in the convergence speed.

As an outline for future work, the interested readers can focus on other fairness criteria such as *max-min* fairness and compare their performance against that of Walrand.

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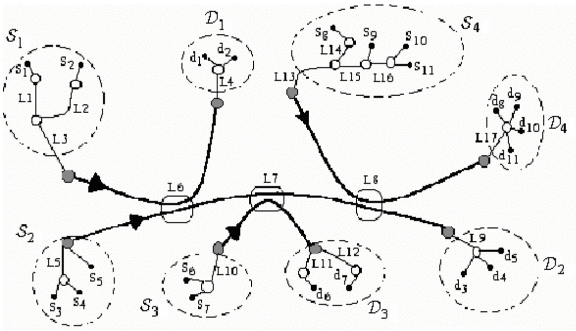

Fig.1 A sample network with two levels of hierarchy

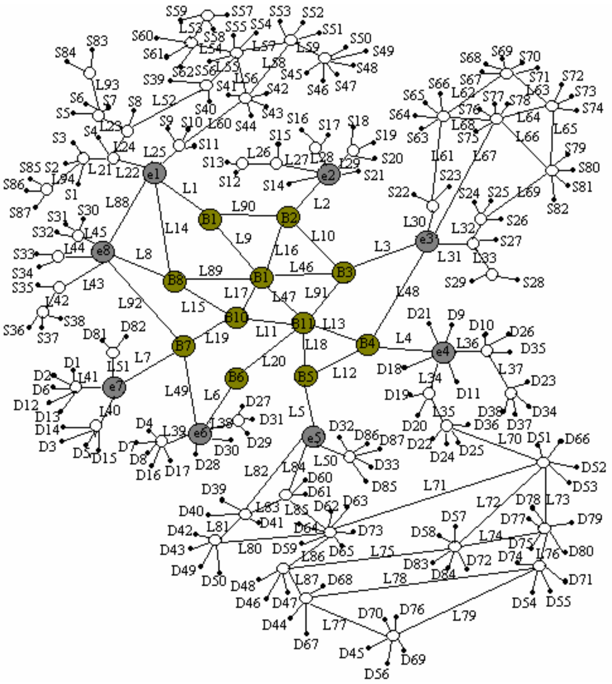

Fig.2 Simulated network topology

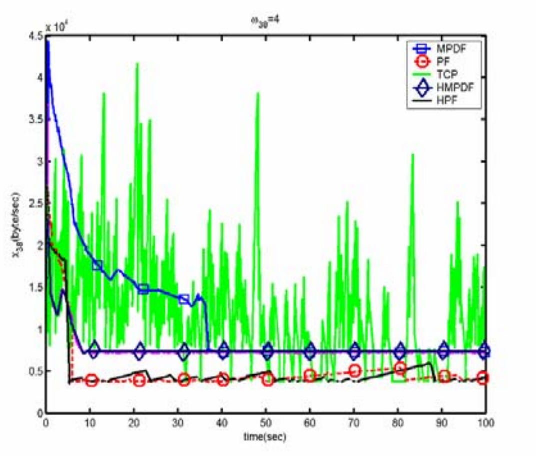

Fig.3 Rate allocated to user 38

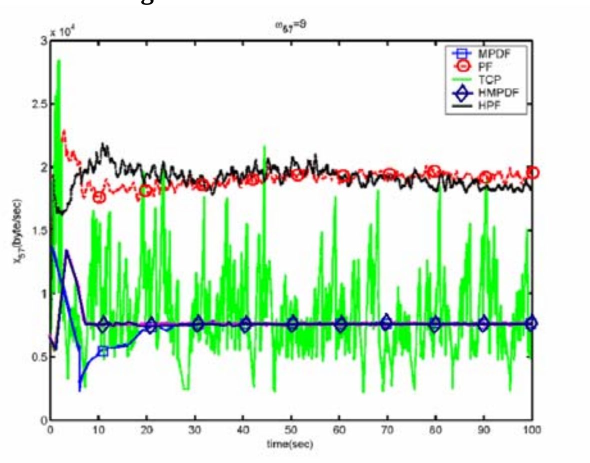

Fig.4 Rate allocated to user 57

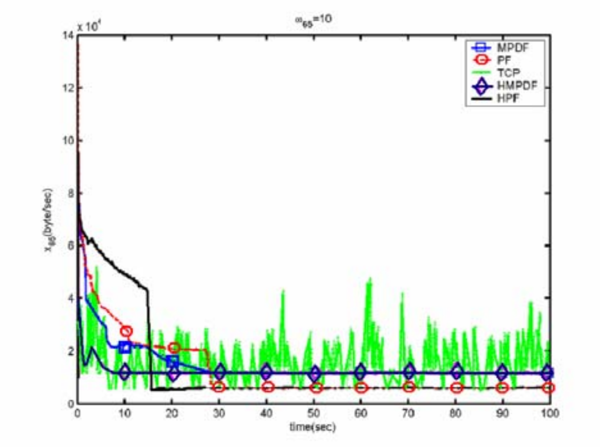

Fig.5 Rate allocated to user 65

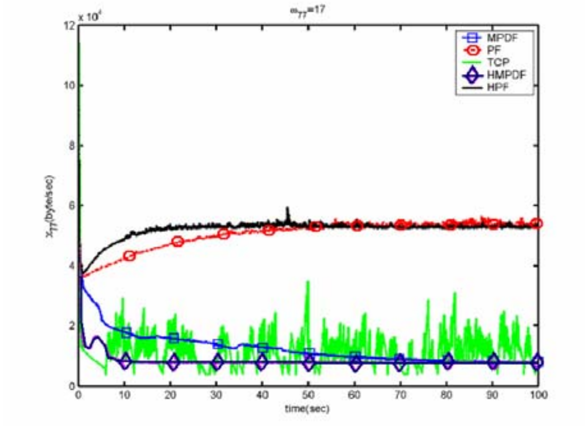

Fig.6 Rate allocated to user 77