Progressive Methods in the Numerical Modeling by the Finite Elements

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Abstract: - This paper presents information about new directions in the modelling of lighting systems, and an overview of methods for the modelling of lighting systems. The new R-FEM method is described, which is a combination of the Radiosity method and the Finite Elements Method. The paper contains modelling results and their verification by experimental measurements and by the Matlab simulation for this R-FEM method.

Key-Words: - Light, lighting systems modelling methods, finite elements methods.

1 Introduction

This paper provides an overview of the Radiosity, Ray-tracing, and Elementary Projection methods, and a new wave of modelling methods used in illumination (the R-FEM method).

In the past, in Lighting Engineering, the elementary projection method and other geometrical optical methods were used for the modelling of illumination. Today, new directions in the modelling of lighting systems are emerging. These new directions start from the Computer Graphics modelling methods and from new methods of numerical modelling used for the solution of technical problems from different industrial sectors. The main applications of the Computer Graphics modelling methods are in architectural engineering and domestic architecture.

2 The Computer Graphics modelling methods

Computer Graphics allows us to use the method of Global illumination. The goal of global illumination is to generate realistic-looking images of objects and scenes, which may or may not exist in reality, by simulating the way light is transported (an example of the distribution of light in an imaginary room is given in Fig. 1).

The Global illumination method is based on the light transport mechanism in the real world. The light transport mechanism can be expressed in terms of BRDF (Bi-directional Reflectance Distribution Function, for more information see reference [2]) of each element, and the most common and practical way is dividing the BRDF into specular and diffuse components. Mathematically, Global illumination is a problem of solving numerical equations concerned with the convergence, convergence speed and whether the solution converges the right answer.

Fig. 1. The distribution of light in an imaginary room

Category	BRDF and its complexity	Solutions
Radiosity	Diffuse only	Hemi-cube Radiosity Progressive Radiosity Analytical form-factor method
Radiosity and ray tracing	Diffuse and specular (planar surface only) and single pass	Two-pass method
	Diffuse and specular (non-planar surface) multi pass	General Two-pass method
Ray tracing (for solving the difuse komponent, various techniques are suggested)	Forward statistic ray tracing enables caustic	Kajia´s Path tracing
	Diffuse and specular (non-planar surface) deterministic forward ray tracing	Radiance method
	Forward and backward combination	Bi-directional path tracing
	Path mutation in ray tracing	Metropolis light transfer method
	Bi-directional Photon tracing	Photon map method

Table 1. The capabilities of each method dealing with BRDF

The Radiosity and Ray-tracing methods are used to calculate energy propagation in each iteration step. The BRDF of the geometrical element is important as regards the efficiency of each method. If the BRDF is ideally diffused, the Radiosity method will converge and it will converge to right the answer. However, the Ray-tracing algorithm works more efficiently in calculating specular reflection, refraction and caustic surfaces. The Radiosity and Ray-tracing algorithms can be measured in two aspects, namely the accuracy and efficiency in BRDF simulation, and the rendering speed. We can classify the major solutions as given in Table 1 illustrating the capability of each method dealing with BRDF and its complexity.

2.1 The Ray Tracing

Ray tracing is a versatile technique that uses the same model to integrate aspects of light/object interaction that were previously handled by separate ad hoc algorithms - reflections, hidden surface removal and shadows. The idea of ray tracing is tracing the light for each pixel, from an eye or viewpoint through the pixel and into the scene. The method of tracing light from a light source and propagating it to an eye or viewpoint is called 'backward ray tracing'.

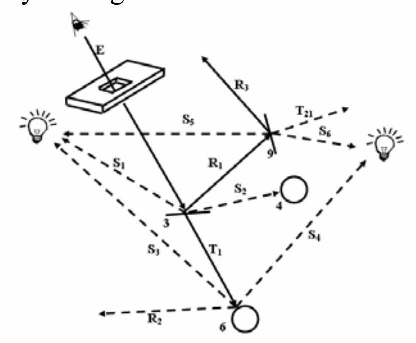

Fig. 2. Principle of the ray tracing method

A primary ray is shot through each pixel and tested for intersection with all objects in the scene. If there is an intersection with an object, then several other rays are generated. Shadow rays are sent towards all light sources to determine if any objects occlude the intersection spot.

2.1.1 The Ray Tracing computation

An appropriate local illumination model is applied at each level and the resultant intensity is passed up through the tree, until the primary ray is reached. The principle of the ray tracing method is described in Fig. 2. Thus we can modify the local illumination model (at each tree node) by

$$I = I_{local} + K_r \cdot R + K_t \cdot T \quad (1)$$

where R is the intensity of light from the reflected ray, and T is the intensity of light from the transmitted ray. Kr, and Kt are the reflection and transmission coefficients, and Ilocal is the local intensity. For a very specular surface, such as plastic, we sometimes do not compute using a local intensity, but only using the reflected/transmitted intensity values.

2.1.2 The Ray Tracing conclusion

The design of an interior by the ray tracing method is described in Fig. 3, with electric lighting on the scene of the theatre. With a simple implementation of ray tracing, the effects of shadow, reflection and transparency are presented in the final image without any further extension of algorithm. Potentially, we can extend ray tracing to support CSG solid modelling by checking the relationship between intersection points for the same light. What we have to do is based on the Boolean operation with each object, namely to compare all intersection points of the same light to find out the existing point which is nearest to the light.

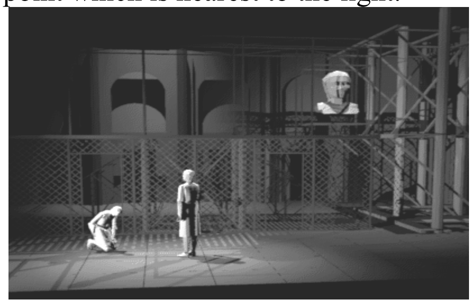

Fig. 3. Design of an interior by ray tracing method

A major disadvantage of ray tracing is that its computation complexity is about the number of objects squared. For more objects on the scene, much more time is needed for rendering. Some optimal methods are available to enhance the efficiency of ray tracing. The bounding volume, Ray coherence and 3-directional subdivision are some of the well-known methods. These methods are used, for example, in the Radiance software system, which is well suited for the analysis and visualization of lighting in design. More specifications are in the literature [11].

2.2 The Radiosity

The description of the radiosity model that follows is based on the original radiosity system developed by Goral (for more information, see reference [5]). The radiosity method is based on a simple model of energy transfer. On each surface in a model the amount of energy that is given off is comprised of the energy that the surface emits internally, plus the amount of energy that is reflected off the surface.

The amount of energy that is reflected off the surface can further be characterized by the product of the amount of energy incident on the surface and the reflectivity constant of the surface.

$$B_j = \rho_j H_j + E_j \quad (2)$$

where B_j is the radiosity of surface *j*, ρ_j is the reflectivity of surface *j*, H_j is the energy incident on surface *j*, E_j is the energy emitted by surface *j*. The principle of the radiosity method is described in Fig.4.

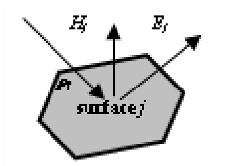

Fig. 4. Principle of the radiosity method

The radiosity of a surface is the energy that is given off. This is what is used to determine the intensity of the surface and what is being solved. The amount of light emitted from a surface must be specified as a parameter in the model, just as in the conventional lighting methods, where the location and intensity of light sources must be specified. The reflectivity of the surface must also be specified in the model, just as in the conventional lighting methods. The only unknown in the equation is the amount of incident light hitting the surface. This can be found by summing for all other surfaces the amount of energy that they contribute to this surface.

$$H_j = \sum_{i,j=1}^{N} B_i F_{i,j} \quad (3)$$

where F_{ij} is form factor i,j. The form factor in the above equation is defined to be the fraction of energy that leaves surface i and lands on surface j, and is therefore a number in the range (0...1). This form factor can be computed via analytical means, or through a geometric analog. See [6] for more information on form factor computation. The radiosity equation now looks like this:

$$B_j = E_j + \rho_j \sum_{i,j=1}^{N} B_i F_{i,j} \quad (4)$$

2.2.1 The Radiosity Matrix

The derived radiosity equations (4) form a set of N linear equations in N unknowns. This leads nicely to a matrix solution (5).

$$\begin{bmatrix} 1-\rho_1 F_{11} & -\rho_1 F_{12} & \cdots & -\rho_1 F_{1N} \\ -\rho_2 F_{21} & 1-\rho_2 F_{22} & \cdots & -\rho_2 F_{2N} \\ \vdots & & & \vdots \\ -\rho_N F_{N1} & -\rho_N F_{N2} & \cdots & 1-\rho_N F_{NN} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_N \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_N \end{bmatrix} \quad (5)$$

This matrix has two interesting properties: it is diagonally dominant and is therefore guaranteed to converge when using the Gauss-Seidel iteration, and the upper right of the matrix is computable from the lower left [5]. Alternative methods for computing the solution of this matrix have been proposed by Cohen et al. [6].

2.2.2 The Radiosity Pipeline

The design of an interior by the radiosity method is shown in Fig.5 and an example of the radiosity method modelling is in Fig. 6. Follow-up information about radiosity solutions can be found in [4] and [7]. Each of the steps in a radiosity render has been described.

Fig. 5. Design of an interior by radiosity method

The following is a list of procedures that a radiosity render would take in computing a scene:

1 - Generate Model
2 - Compute Form Factors
3 - Solve Radiosity Matrix
4 – Render

Fig. 6. Example of radiosity method modelling

It is important to note that when rendering a radiosity scene, changing the parameters does not require a restart from step 1. In fact, only if the geometry of the model is changed must the system start over from step 1. If the lighting or reflectance parameters of the scene are modified, the system may start over from step 3. If the view parameters are changed, the system must merely re-render the scene.

3 The lighting engineering modelling methods

3.1 The EPM method

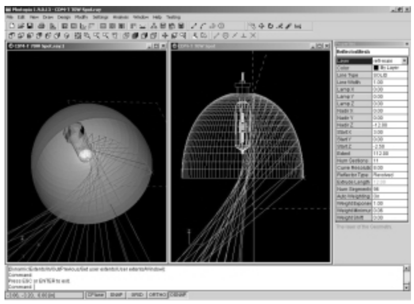

Fig. 7. The Photopia 2.0 design by the EPM method

EPM is stands for the Elementary projection modelling method. The EPM method is used most often from all the methods for designing lighting systems in Lighting

Engineering. There are, for example, two programs of the PHOTOPIA 2.0 design, (Fig. 7. describes the program execution, for more information see [12]) or LUX JUNIOR (more is in the literature [13]) and HIGH CURVES ST (more is in the literature [14]).

3.1.1 The principle of the EPM method

The principle of the EPM method and the design of an interior by EPM are given in Fig. 8. The beam of light from the reflector area is considered to be the sum of all beams of light, which went out from the source of light and from the reflection area of the reflector (for more information see [15], [16] and [17]).

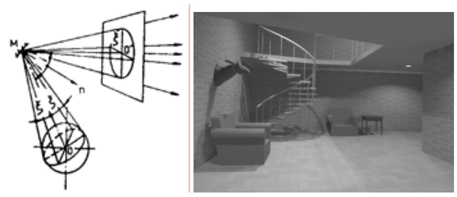

Fig. 8. The principle of EPM and the design of an interior

3.2 Matlab simulation

Matlab simulation is the modelling of lighting systems which combines the EPM and ray tracing methods and then approximation by the principle of radiation follows. The algorithm is created by combining the ray tracing method and the EPM method as a geometrical simulation in the Matlab and then the approximation of the light intensity by the principle of radiation is used. It gives the right behaviour of the light intensity distribution.

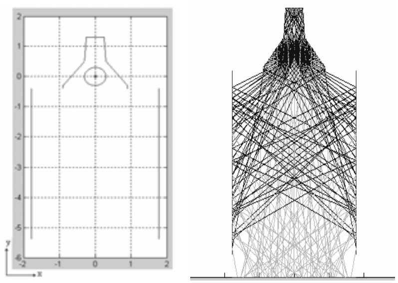

Fig. 9. The geometrical modelling by the Matlab simulation

This method was designed for second verification of the lighting systems modeling, which is described in chapter 3.3 of this article. Experimental measurements were used as first verification.

The main advantages of the Matlab simulation are the simplicity of the submission for the initial figures and the high speed of the calculations for the simple assignments. This modeling is suitable for the verification of the results, which are received from other type of the modeling method.

3.3 The R-FEM method

The R-FEM method is a new direction in the modelling of lighting systems. It utilizes the similarity between physical models. This paragraph demonstrates the usage of analogy between different physical models for the modelling of light problems. The R-FEM method is able to solve tasks that fulfill the condition λ_S << max (D) $\wedge$ λ_S < 10 . max (D), where λ_S is the source of light wavelength and *D* is one of the geometrical dimensions of the modelling task. It can be used for to model more complicated physical problems than the methods mentioned up to now. An example of a more complicated physical problem, which we can solve by the R-FEM method, is the modelling of light intensity distribution in interior or exterior spaces with non-homogeneous environment, where the light has passed through some impure air (e.g. filled with smoke, fog, mist, vapour, dust, etc.).

3.3.1 The design by R-FEM method

In technical praxis we often encounter conjugate problems. A necessary part of the design process during the development and measurement of light sources is the modelling and experimental verification of results. The most accurate mathematical models of the sources of light include models based on the radiation principle.

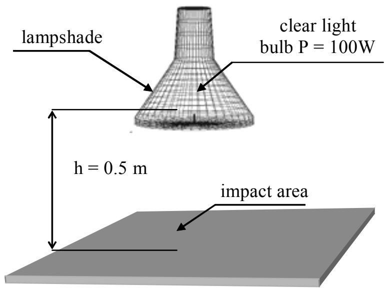

Fig. 10. Geometrical illustration of a simple source of light

One possibility is to use standard one–purpose programs while another possibility offers the usage of sophisticated numerical methods, among them the finite element method, for example the ANSYS program. The ANSYS program uses standard program tools such as modelling, discretization into a net of elements, solvers, evaluation, and interpretation of the results. The crux of the whole problem lies in the transformation of thermal field quantities into optical quantities. This can be done using the general rules described in [19].

In the following text the basics of modelling the primitive light problem are described. The verification of the model of light source is done via experiment and then it continues to the hollow light guide problems and it was also verified by experiment (for more information, see references [17] - [19]).

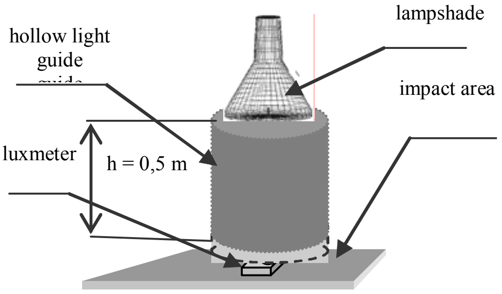

Fig. 11. Geometrical illustration of a hollow light guide

The geometrical situations that were modelled and verified are shown in the Figs 10 and 11.

3.3.2 Building the numerical model

The formulation of the basic thermal model is based on the first law of thermodynamics

$$q + \rho \ \ c \ \ \boldsymbol{v} \cdot divT - div\left(k \ \ gradT\right) = \rho c\left(\frac{\partial T}{\partial t}\right) \qquad (6)$$

where q is the specific heat, ρ is the specific weight, c is the specific solidification heat, T is the temperature, t is the time, k is the coefficient of calorific conduction, v is the the velocity of flow. This model can, with respect to the application of Snell's principles, be simplified into the form

$$q - div\left(k \ \ gradT\right) = \rho c\left(\frac{\partial T}{\partial t}\right) \qquad (7)$$

According to the Stefan-Boltzmann principles, heat transfer by way of radiation between surfaces with relative indexes i, j is formulated as

$$q_{ri} = \sigma \ \ \varepsilon_i \ \ A_{i,j} \ \ S_i\left(T_i^4 - T_j^4\right) \qquad (8)$$

where q_{ri} is the specific heat transferring from surface with index i, σ is the Stefan-Boltzmann constant, ε_i is the emissivity of surface, $A_{i,j}$ is the projection factor of surface with index i to surface with index j, S_i is the area of surface with index i, T_i, T_j are the temperature of surfaces i, j. Projection factor $A_{i,j}$ is determined as

$$A_{i,j} = \frac{1}{S_i}\int_{S_i}\int_{S_j}\frac{\cos\vartheta_i \cos\vartheta_j}{\pi r^2}dS_j dS_i \qquad (9)$$

When the projection factor is determined, it is possible to use the Gallerkin principles for converting this problem into model (7). Marginal and initial conditions must be respected.

$$\left[K\right]\left\{T\right\} = \left\{Q\right\} \qquad (10)$$

where K is the coefficients matrix, T is the columnar matrix of sought temperatures, Q is the columnar matrix of heat sources. Thermal flow T_f is determined from temperature T as

$$T_f = -\left(k \, gradT\right) \qquad (11)$$

By the radiation principle, the elements of column matrix of heat sources Q can be determined as

$$Q_{i,j} = S_i A_{i,j} \varepsilon_i \, \sigma\left(T_i^4 - T_j^4\right) \qquad (12)$$

and adjusting for mathematical model (7) yields

$$Q_{i,j} = \overbrace{S_i A_{i,j} \varepsilon_i \, \sigma\left(T_i^2 - T_j^2\right)\left(T_i + T_j\right)}^{K}\left(T_i - T_j\right) \qquad (13)$$

The heating model will be used for the modelling of light problem using the Snell principles in optics. Light source with lighting intensity *E (lx)* corresponds to equivalent heat quantity density of heat flow q'', light flow *Φ(lm)* corresponds to equivalent quantity of heat flow q'. The resulting light flow is defined by equation (14):

$$\Phi_e = \frac{T_{f,e}}{S_{n,e}} \qquad (14)$$

where Φ_e is the flow of light on the element, T_f,e is the equivalent of the thermal flow through the element, $S_{n,e}$ is the normal surface to the element (more can be found in [19]).

3.3.3 Results of the R-FEM method

The result of modelling by the R-FEM method is shown in the Fig. 12.

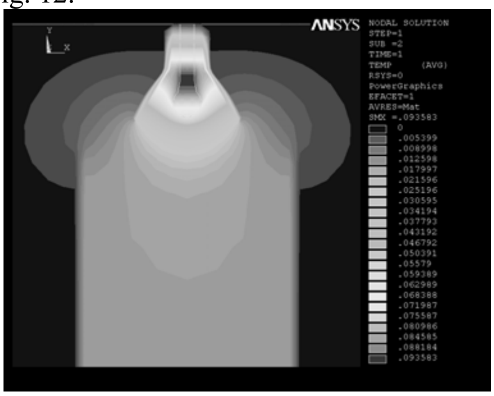

Fig. 12. The result of the R-FEM method for Fig. 11

3.3.4 Verification of the R-FEM

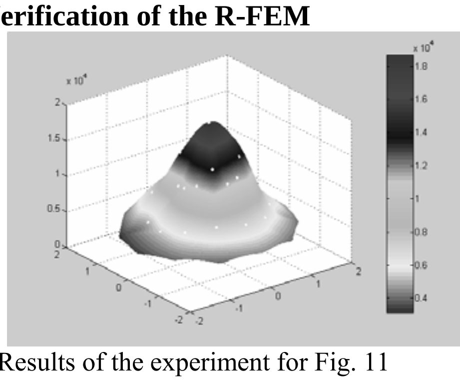

Fig. 14. Results of the experiment for Fig. 11

The results of the verification of the R-FEM via experiments are given in Fig. 11. There are differences between the values obtained by modelling and experimental measurement, ranging from 5 – 15 %, depending on the distribution of the net of elements. When the elements of the net are of a lower density, the differences are also lower. This problem requires the net of elements to be optimized.

3.3.5 Advantages of the R-FEM

One of the biggest advantages of this method is the wide spectrum of its usage. We can design the interior and also exterior scenes with its specifications in the materials quality, climatic dissimilarities and geometrical dimension varieties. We can use all types of sources of light with their diversity of the colour distribution in the light spectrum. The designers are not limited by the geometrical dimension varieties, colour distribution in the light spectrum, material qualities or climatic dissimilarities.

The other advantage is that the method is very accurate. The degree of accuracy can be chosen by choosing the method of generating nets of elements and the solution algorithm because all this is provided by the ANSYS standard program tools.

3.4 The Full FEM wave solution

FEM is the short form for the Finite Elements Method modelling. The light problems that fulfill the condition $\lambda_S << \max(D) \wedge \lambda_S < 10 \,.\, \max(D)$, where λ_S is the source of light wavelength and D is one of the geometrical dimensions of the modelling assignment, will be solved by the FEM using the full wave equation, which was used to define light emissions. This method of the solution yields highly accurate results, but is demanding as regards geometrical declaration, and time-consuming (for example, for incoherent sources of light the calculation is too long). The Full FEM wave solution is suitable for a specific purpose.

4 Conclusion

This article describes numerical methods of modelling lighting problems, which are used in Computer Graphics and in Lighting Engineering. It also describes the R-FEM method, which has been verified and found to be a great asset for the modern trends in modelling lighting problems. It can solve specific light problems that up to now have only been solved by using many simplifications. The research described in the paper was financially supported by FRVŠ grant No IS 432169/2003 and by MSM research plan No 00216305XX.

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