Design of an Adaptive Predictive Controller for a Continuous Stirred Tank Reactor

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Abstract: -An adaptive predictive controller has been designed in this paper. The model predictive controller design is based on the linear model and by employing adaptation mechanism; it can be applied to the nonlinear systems. Identification of the linear model parameters in each sample time from a recursive least square method is the suggested technique for adaptation. This method is applied to a *CSTR*[1] as a nonlinear *MIMO* system with considering measurable disturbances. Simulations are performed for normal operating condition and a case in which system is caused with disturbance.

Key-words:-Model Predictive Control, Adaptive, CSTR

1 Introduction

Process Industries for instance chemical and petrochemical plants have special specifications that application of simple control algorithms couldn't cover the total aspects of the control and economic demands. Some of these specifications are nonlinearity, interaction, numerous variables and presence of disturbances. Whereas application of Advance Process Control in an automated hierarchical structure for these systems besides of obtaining the control and product requirements, has involved greatly economic benefits.

Backbone of any *APC* (Advance Process Control) method is Model Predictive Control [1]. *MPC* (Model Predictive Control) refers to a class of algorithms that compute a sequence of manipulated variable adjustments in order to optimize the future behavior of a plant. Originally developed to meet the specialized control needs of power plants and petroleum refineries, *MPC* technology can now be found in a wide variety of application areas including chemicals & petrochemical, food processing, automotive, aerospace, metallurgy, and pulp and paper [2,3,4]. Several authors have published excellent reviews about *MPC* theoretical issues [5, 6]. [7] Provides a vendor's perspective on industrial *MPC* technology and summarizes likely future developments.

Weakness of linear models in considering the nonlinearity of systems caused studying the application of nonlinear models in *MPC*' considered to be started from 1990's. Until now many implementations of *MPC* & *NMPC* (Nonlinear MPC) methods have been reported. [2,4] present an overview of commercially available *MPC & NMPC* technologies.

Adaptation of linear models with occurrence of new conditions according to variety of operating points in nonlinear systems is a solution for extending linear methods in design of controllers for nonlinear systems.

In this paper we concentrate on a *CSTR* as a highly nonlinear system. Our model predictive controller is based on an *ARMAX* (Auto Regressive Moving Average with external Input) model when its parameters are changed properly according to the new operating conditions. Measurable disturbances are considered in design of the controller. The adaptation mechanism is based on identification of the parameters of the linear model according to the inputs, outputs and measurable disturbances values with recursive least squares *(RLS)*.

In section 2 the mentioned adaptive model predictive controller is proposed. A review of model predictive control with considering measurable disturbances is presented in section 2.1. Adaptation mechanism is explained in 2.2. In section 3 model of the selected Continuous Stirred Tank Reactor is introduced. Simulation results are presented in section 4 which shows the validity of the designed algorithm and discussions on the application of the mentioned scheme on *CSTR*. Finally Section 5 draws some concluding remarks.

2 Adaptive Model Predictive Control

2.1 Model Predictive Control

The concept of model predictive control involves the repeated optimization of a performance objective such as (1) over a finite horizon extending from a future time N_1 up to a prediction horizon N_2. Given set points r(k+j), a reference ω(k+j) which is produced by pre-filtering and is used within the optimization of the *MPC* cost function. The control variable u(k+j), over the control horizon N_u , is obtained from solving the cost function.

$$J = \sum_{i=N_1}^{N_2} \left\| \hat{\mathbf{y}}(t+i) - \boldsymbol{\omega}(t+i) \right\|_{\mathbf{Q}}^2 + \sum_{j=1}^{N_u} \left\| \Delta\mathbf{u}(t+j-1) \right\|_{\mathbf{R}}^2 \quad (1)$$

$\hat{\mathbf{y}}(t+i)$ is the predicted outputs vector and $\Delta\mathbf{u}$ is the increment of input vectors. ***R and Q*** are the weighting matrix and N_1, N_2 *and* N_u must be tuned as controller parameters.

Suppose an *ARMAX* model in the form of (2). Which is given in this equation y,u,w,x are the output vector, input vector, measurable disturbance and noise vectors respectively. ***A,B,D,C*** are the corresponding matrix polynomials. *I(t)* is an assumed constant disturbance and ***Dd*** is its matrix polynomial. *I(t)* and ***Dd*** are used for modeling the constant terms that are generated by linearization of the nonlinear system.

$$\mathbf{A}(z^{-1})y(t) = \mathbf{B}(z^{-1})u(t-1) + \mathbf{D}(z^{-1})w(t) + \mathbf{Dd}(z^{-1})I(t) + \mathbf{C}(z^{-1})x(t) \quad (2)$$

Where

$$\mathbf{A}(z^{-1}) = I_{l\times l} + A_1 z^{-1} + \ldots + A_{n_a} z^{-n_a}$$

$$\mathbf{B}(z^{-1}) = B_0 + B_1 z^{-1} + \ldots + B_{n_b} z^{-n_b}$$

$$\mathbf{D}(z^{-1}) = D_0 + D_1 z^{-1} + \ldots + D_{n_d} z^{-n_d}$$

$$\mathbf{Dd}(z^{-1}) = Dd_0 + Dd_1 z^{-1} + \ldots + Dd_{n_{Dd}} z^{-n_{Dd}}$$

$$\mathbf{C}(z^{-1}) = I_{l\times l} + C_1 z^{-1} + \ldots + C_{n_c} z^{-n_c}$$

(***C*** is assumed identity matrix i.e. white noise).

In the following analysis we assume that prediction equations derived to the free and forced response after solving a Diophantine equation. The following method is very similar to the Generalized Predictive Control (*GPC*). These equations are obtained after some extensions of the equations from [3].

$$\begin{aligned} \hat{y}(t+1) &= \mathbf{G}_j(z^{-1})u(t) + \mathbf{H}_j(z^{-1})w(t+1) + \mathbf{f}_1 \\ \hat{y}(t+2) &= \mathbf{G}_j(z^{-1})u(t+1) + \mathbf{H}_j(z^{-1})w(t+2) + \mathbf{f}_2 \\ &\vdots \\ \hat{y}(t+N_2) &= \mathbf{G}_j(z^{-1})u(t+N_2-1) + \mathbf{H}_j(z^{-1})w(t+N_2) + \mathbf{f}_{N_2} \end{aligned} \quad (3)$$

Equation (3) can be represented in a compressed form as (4):

$$\hat{\mathbf{Y}} = \mathbf{GU} + \mathbf{HW} + \mathbf{f} \quad (4)$$

All terms of the (4) aren't used in (1) but a portion of them, based on prediction and control horizons as (5) can be used.

$$\hat{\mathbf{Y}}_{N_{12}} = \mathbf{G}_{N_{12u}} \mathbf{U}_{N_u} + \mathbf{H}_{N_{12}} \mathbf{W}_{N_{12}} + \mathbf{f}_{N_{12}} \quad (5)$$

$$\hat{\mathbf{Y}}_{N_{12}} = \left[\hat{y}(t+N_1)^T \quad \hat{y}(t+N_1+1)^T \quad \cdots \quad \hat{y}(t+N_2)^T \right]^T$$

$$\mathbf{U}_{N_u} = \left[u(t)^T \quad u(t+1)^T \quad \cdots \quad u(t+N_3-1)^T \right]^T$$

$$\mathbf{W}_{N_{12}} = \left[w(t+N_1)^T \quad w(t+N_1+1)^T \quad \cdots \quad w(t+N_2)^T \right]^T$$

$$\mathbf{f}_{N_{12}} = \left[\mathbf{f}_{N_1}^T \quad \mathbf{f}_{N_1+1}^T \quad \cdots \quad \mathbf{f}_{N_2}^T \right]^T$$

With considering (5) in the index of performance as in (1) and solving it, increment of the control input is achieved as (6):

$$D\boldsymbol{U}_{N_u} = \left(\boldsymbol{G}_{N_{12u}}^T \boldsymbol{Q} \boldsymbol{G}_{N_{12u}} + \boldsymbol{R} \right)^{-1} \boldsymbol{G}_{N_{12u}}^T \boldsymbol{Q} \left(\boldsymbol{r} - \boldsymbol{H}_{N_{12}} \boldsymbol{W}_{N_{12}} - \boldsymbol{f}_{N_{12}} \right) \quad (6)$$

At the same time (6) can be solved by Quadratic Programming while constraints on system variables exist.

2.2 Adaptation Mechanism for Nonlinear Systems

For using the equations in section 2.1 for nonlinear systems we must linearize the nonlinear describing equations of the system in each step time. This scheme is very efficient. In order to increase the speed of the controller, linearization could be performed in greater time scales but in this case accuracy of the linear model may be reduced according to the selected time scales. After linearization and discretization of the linear state space model, it is converted to the transfer function representation as an *ARMAX* model form. In this case we have a model as form as (2) with measurable disturbances and a constant term as transfer function of the assumed constant disturbance that be generated from linearization. Now equations (1-6) can be used for calculation of the control input and this procedure is repeated in each time step [8]. This approach is time consuming and if an accurate model of system is not available or system parameters are time varying, this approach can not work properly. Furthermore equations (1-6) can be used in an adaptive model predictive controller.

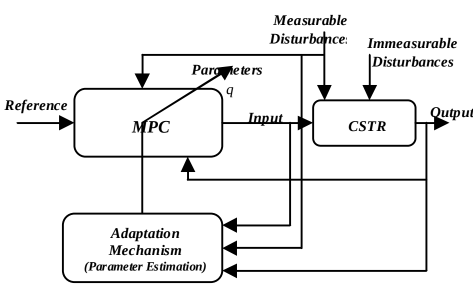

Fig. (1); Block diagram of the controller and *CSTR*

Therefore, identification of the system transfer function parameters could be done by an adaptation mechanism. In this case parameters of the system are identified with well known recursive least squares method [9].

$$y(t) = q^T j + \mathbf{e}(t)$$
(7)

Where ***e(t)*** is a white noise vector.

$$j(t) = [-y(t-1)^T, -y(t-2)^T, \cdots, -y(t-n_a)^T, u(t-1)^T, u(t-2)^T, \cdots, u(t-n_b-1)^T, w(t)^T, w(t-1)^T, \cdots, w(t-n_d)^T, \underbrace{1,1,\cdots,1}_{n_{Dd}}]^T \quad (8)$$

$$q = \left[A_1 A_2 \cdots A_{n_a} B_0 B_1 \cdots B_{n_b} D_0 D_1 \cdots D_{n_d} Dd_0 Dd_1 \cdots Dd_{n_{Dd}}\right]^T \quad (9)$$

In each step time new parameters of the linear transfer function is calculated by *RLS* with considering the previous inputs, outputs and measurable disturbances. (10), (11) represent the *RLS* equations for estimating the parameters. In these equations, $\hat{q}$ is the estimated parameters matrix, p is the error covariance matrix and λ is forgetting factor.

$$\hat{q}(t) = \hat{q}(t-1) + (l + j(t-1)^T p(t-2) j(t-1))^{-1} p(t-2) j(t-1)[y(t) - \hat{q}(t-1)^T j(t-1)]^T \quad (10)$$

$$p(t-1) = \frac{1}{l} p(t-2) - \frac{1}{l}(l + j(t-1)^T p(t-2) j(t-1))^{-1} p(t-2) j(t-1) j(t-1)^T p(t-2) \quad (11)$$

These parameters are used in the *MPC* block. Linearization in first steps is performed to obtain the appropriate initial values in identification algorithm. If number of model parameters is many, parameter identification causes bad modeling and therefore this approach can not work properly.

3 A Continuous Stirred Tank Reactor

The *CSTR* system is a highly nonlinear process and has several interesting features for process control engineers. In the jacketed chemical reactor (*CSTR*) shown in Fig. (2), a second-order exothermic reaction (2A$\rightarrow$ B) takes place, in which 2 components A react irreversibly and at specific reaction rate k to form a product B [10]. The reaction rate constant k follows the Arrhenius (12). According to this equation, the effect of temperature, $T_r(t)$, on the specific reaction rate k is usually exponential. This exponential temperature dependence represents one of the most severe nonlinearities in chemical engineering systems.

$$k = k_0 e^{\frac{-a}{(T_r + 460)}} \quad (12)$$

The mathematical model for this *CSTR* involves a *mass balance on* A, in which the flow of moles of component A into the system, minus the flow of moles of A out of the system, plus the rate of formation of moles of A component form chemical reactions is equal to time rate of change of moles of A component inside system. This concept is expressed by (13). The first law of thermodynamics puts forward the principle of conservation of energy. The mathematical model must include an *enthalpy balance on reacting mass*, and an *enthalpy balance on jacket* (water is flowing through the jacket). In this case, the flow of internal energy into the system, minus the flow of internal energy out of the system, plus the heat added to the system by reaction is equal to the rate of change of internal energy inside the system. The balance on reacting mass is given by (14) and the balance on the jacket by (15). For more details about the model and its parameters you can refer to [10].

Mass Balance on A:

$$\frac{\partial C_A}{\partial t} = \frac{W}{Vr}(C_{Ai} - C_A) - kC_A^2 \quad (13)$$

Enthalpy Balance on reacting mass:

$$\frac{\partial T_r}{\partial t} = \frac{W}{Vr}(T_{ri} - T_r) - \frac{UA}{VrC_p}(T_r - T_{jo}) + \frac{(-DH)}{rC_p}kC_A^2 \quad (14)$$

Enthalpy Balance on Jacket:

$$\frac{\partial T_{jo}}{\partial t} = \frac{UA}{M_j C_{pj}}(T_r - T_{jo}) - \frac{W_j}{M_j}(T_{jo} - T_{ji}) \quad (15)$$

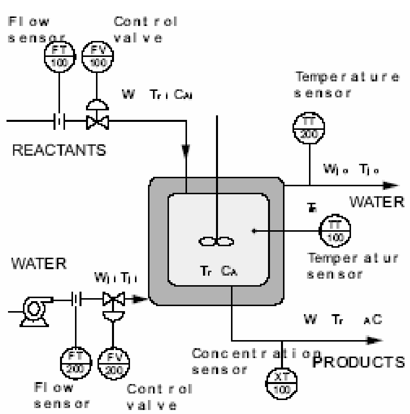

Fig (2); Continuous Stirred Tank Reactor

In this system, we consider the following variables will change over time:

Control Input variables:

- W (t): Feed mass flow rate (lb/min)
- W_j(t): Water cooling rate at jacket (lb/min)

Outputs:

- C_A(t): Concentration of reactant A in reactor and exit stream (lb/ft^3)
- T_r(t) : Reactor temperature (^oF)
- T_{jo}(t): Outlet jacket temperature (^oF)

Measurable disturbances:

- C_{Ai}(t): Concentration of reactant A in feed (lb/ft^3)
- T_{ji}(t) : Inlet jacket temperature (^oF)
- T_{ri}(t) : Input reactants temperature (^oF)

Tracking the desired set points of concentration of reactant A and outlet jacket reactor temperature is the goal of design of predictive controllers. Since reactor temperature must be in the safe region, it is considered in design of the controllers.

The model given by (12-15) can be expressed as (16).

$$\frac{\partial x(t)}{\partial t} = f(\boldsymbol{x}(t), \boldsymbol{u}(t), \boldsymbol{w}(t)) \quad (16)$$

Where:

$$\boldsymbol{x}(t) = \begin{bmatrix} C_A(t) \\ T_r(t) \\ T_{jo}(t) \end{bmatrix}, \boldsymbol{w}(t) = \begin{bmatrix} C_{Ai} \\ T_{ri} \\ T_{ji} \end{bmatrix}$$

$$\boldsymbol{y}(t) = \begin{bmatrix} C_A \\ T_{jo} \end{bmatrix}, \boldsymbol{u}(t) = \begin{bmatrix} W(t) \\ W_j(t) \end{bmatrix}$$

After linearizing equation (16) by Taylor series expansion around the operating point (17) is obtained. In this situation we have a linear state space model:

$$x(t+1) = A\mathbf{x}(t) + B\mathbf{u}(t) + N\mathbf{w}(t) + N_0 \quad (17)$$

A,B,N are the system matrix, control input matrix and measurable disturbance input matrix respectively. N_0 is a constant $n \times 1$ matrix (n is number of state variable) which will be generated from linearization around the operating point because of nonzero initial values.

4 Simulation Results

In this section, the designed algorithm is applied to the *CSTR*. The values of *CSTR* parameters are from [10].

The *CSTR* system has 2 outputs, 3 measurable disturbances and 2 inputs. For utilizing the identification method in adaptation mechanism, we want to use (7) in the *RLS* algorithm and vectors in φ must have same dimension. Here we add a zero input as third input and consider T_r as third output.

The assumptions for simulations are:

- The process noise is random and applied to the input of system with variance 1, zero expectation value and amplitude of 10% of the input control signal. (Signal/Noise ratio is constant and equal to 90%).
- The measurable disturbances have the nominal expectation values and a random noise with variance 1, zero expectation value and amplitude of 10% of the nominal value are added to them. . (Disturbance per disturbance noise is also constant and equal to 90%).
- Reference trajectories are exponential functions that follow the set points smoothly as (18):

$$\mathbf{w}(t) = \mathbf{SP} + (\mathbf{y_0} - \mathbf{SP})e^{-at}\ , a = 0.005(1/\text{sec}) \quad (18)$$

SP is the set points vector and $\mathbf{y_0}$ is the output initial value vector.

- Set points of this *CSTR* are:

$C_A = 3.5955\ lb/ft^3$

$T_{jo} = 120.0222\ °F$

- The Inputs are limited to the [0-1400] (lb/min).

With respect to the system dynamic and with a trial and error operation, the parameters of the controller are achieved as follows:

$$N_1 = N_u = 1, \quad N_2 = 3, \quad T_s = 1 \text{ sec}$$

$$R = \begin{bmatrix} 5 & 0 \\ 0 & 2 \end{bmatrix}, \quad Q = \begin{bmatrix} 4000000 & 0 \\ 0 & 4000 \end{bmatrix}$$

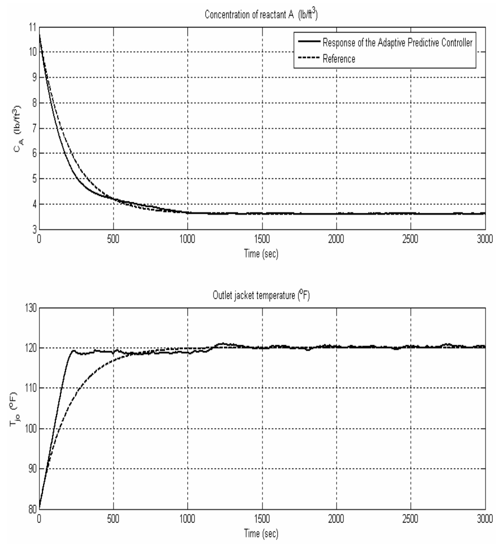

Fig. (3) Responses of the *CSTR* after application of the adaptive predictive controller. Dashed line is for adaptive predictive controller of the nonlinear system and dotted line for reference

Fig. (3) illustrates responses of the controller that be applied to the *CSTR.*

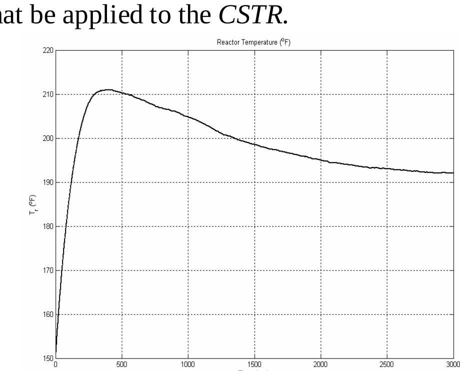

Fig. (4); third state (T_r)

Fig.(4) shows T_r as the third state of the system. Fig.(5) represents control input signals.

In this case, the controller works properly. Outputs are tracking the set points with less than 1% steady state error and T_r also is good.

If the model of *CSTR* is affected by disturbance or parameters of the system are time varying, identification of the model parameter is a good solution in consideration with linearization but choosing initial values for identification of the model parameters are important. At hence we obtained initial values from linearization for first steps in identification.

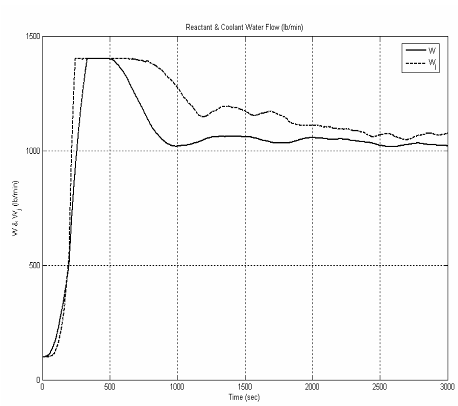

Fig. (5); Control Input signals

Now we consider the system that its measurable disturbances are changed. In this case we suppose concentration of reactant A in feed is increased 50% (i.e. C_{Ai} =16.2 *lb/ft*^{3}) at time t=1500 sec.

Two cases are discussed:

Case 1: the value of this measurable disturbance is changed but the measured value of C_{Ai} that be send to the controller is as same as before.

Case 2: the value of this measurable disturbance is changed but in this case the measured value of C_{Ai} is as same as the actual value which is applied to the *CSTR*.

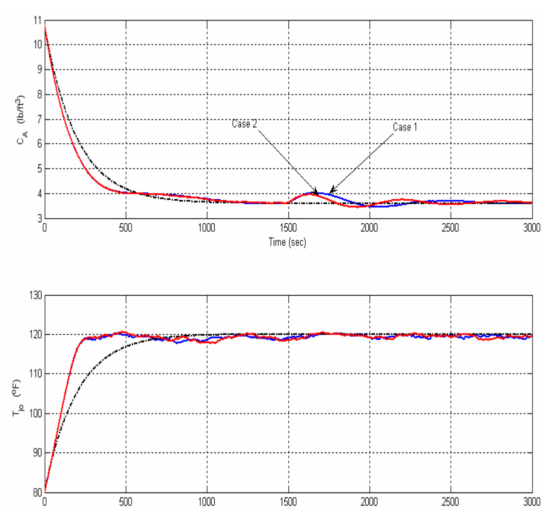

Fig. (6); C_A & T_{jo} after application of the adaptive predictive controller to the *CSTR* which its measurable disturbance is changed (case 1 & 2). Dashed line is for adaptive predictive controller of the nonlinear system and dotted line for reference

Fig.(6) illustrates responses of the *CSTR*, after applying the controller while concentration of reactant A in feed is increased 50% in two cases. The assumptions and controller parameters are as same before. After a transient time and with a little variation, the controller can generate control input signals that cause good tracking of the references. The controller has good responses in two cases.

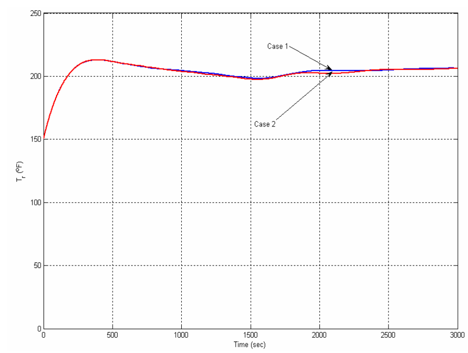

Fig. (7); Third state (T_r) after application of adaptive predictive controller in cases that measurable disturbance is changed.

Fig. (7) represents T_r for two mentioned cases. Fig. (8) also represents control input signals for the controllers in these cases.

As can be seen in Fig. (6), if measurable disturbances are measured well and its violations are reported to the controller, the responses will be better and the reference can be tracked faster. It's because of considering the measurable disturbances in design of the controller.

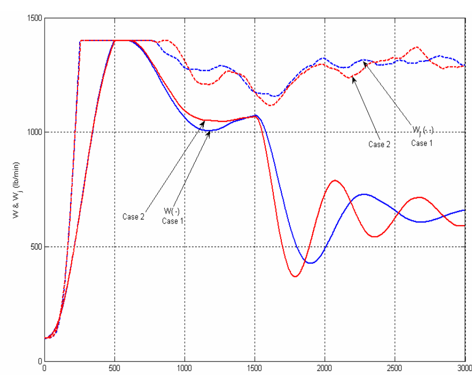

Fig. (8); Control Input signals (measurable disturbance is changed.-cases 1 & 2)

5 Conclusion

In this paper a linear model predictive controller with an adaptation mechanism is extended to deal with nonlinear systems. The adaptation mechanism is based on identification of the model parameters in a recursive manner. In this way, the controller adapts itself with nonlinear model. Furthermore measurable disturbances are considered in derivation of the model predictive controller. If an accurate model of a system is available, linearization is an efficient solution but it is time consuming. When parameters of the system are unknown, then identification of linear system could be extended to nonlinear system. This method is faster but its efficiency is reduced by increasing the number of parameters. Also selection of initial value for identification of model parameters is important. For identification of model parameters we obtained initial values of parameters by linearization the nonlinear model in first steps and then we employ adaptation mechanism.

The adaptive predictive controller is applied to a *CSTR* as a nonlinear *MIMO* system and the results are discussed in normal case and two cases that measurable disturbances are changed. In case 1 this change isn't reported to the controller but in case 2 is reported. In these cases controller is working properly. In case 2, controller works better than case 1.

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