A NUMERICAL MODELING OF MAGNETIC FIELD PERTURBATED BY THE PRESENCE OF SCHIP'S HULL

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Abstract ---In this paper we present a method for behavior computation of the magneto static field through the permeable thin layers. For the determination of induction and magnetic field in any point in the space 3D, a formulation in magnetic scalar potential is adopted, by applying the finite element method on the structure with thin layer and the boundary integral method to the surrounding medium. We propose two numerical approaches of the fictitious magnetic charges introduced on the surface of the thin layers.

Keywords : Magneto static field; Thin Layers; Finite elements; Boundary integral

Introduction :

The modeling of the structures with thin layers in order to know the behavior of the magnetic field trough their borders, is the several industrial application object (systems of telecommunications, medical imagery, identification and signatures,...). These applications differ in the mode of the magnetic field source, the nature of material composing the thin layers and its structure. We propose an electromagnetic system composed of a structure with thin layers and its environment. This system presents an open problem that extends to infinity. We consider a structure with thin layer of permeability μ, embedded into a magnetic source field H^s. Its thickness (e_p) is very small than the other dimensions in space R^3. A cut of the system is represented by figure 1. Some work on this same model has summers made with other numerical approaches [1] [2] [3].

Formulation :

The equations that govern it interpret the magneto-static without current. In any point of the system we have $rot\, h = 0$. We supposed that the material does not comprise any initial magnetization; one deduces from it that induction and the magnetic field are related to the magnetic polarization induced by the relation: $b = \mu_0 (h + M)$ and we have : $div \mu (h_r + h_s) = 0$; $rot\, h_s = 0$

The decomposition of the total magnetic field in $h_r + h_s$ is theoretical. The determination of the magnetic field of reaction h^r makes it possible to better highlight the results which will be presented.

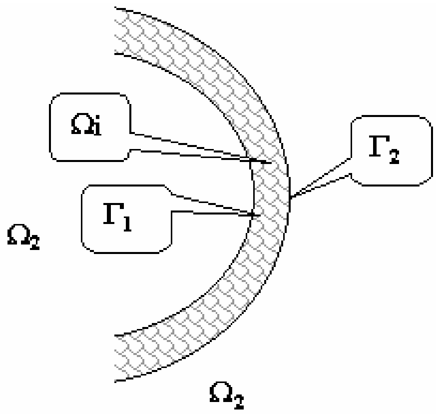


Fig. 1 scheme of a cut of the structure with thin Layers

Subsequently we indicate the field of reaction hr by a simple letter h. Then, the magnetic field derives from a scalar potential φ : $h = grad\, \varphi$. The unknown quantity is the reduced potential magnetic scalar φ.
To establish a variational formulation of the problem we use the second property which verifier owes magnetic induction :

$$div\, \mu (h^s - grad\ \phi) = 0$$

$$\int_{\Omega_i} grad\phi . grad\psi dv \quad - \quad \int_{\Omega_i} h^s . grad\psi dv$$

$$- \int_{\Gamma_1} \frac{\partial \phi}{\partial n_1} \psi d\gamma \quad + \quad \int_{\Gamma_2} \frac{\partial \phi}{\partial n_2} \psi d\gamma \quad = \quad 0$$

where Ψ is an unspecified function test defines in same space as that of state function φ.

A volume Ω with thin layer is characterized by a very small thickness ep compared to its other dimensions geometrical. Thus, one can confuse the two borders Γ_1 and Γ_2, and ep becomes a parameter. The two borders

Γ_1 and Γ_2 will be confused at a median border Γ. Ω becomes surface Γ field occupied by the structure in R^3 of permeability magnetic $\mu = \mu_0\ \mu_r$. Ωe indicates the medium external of magnetic permeability μ_0. By holding account that n is the outgoing normal with Γ the two integrals relating to the two borders become:

$$\int_{\Gamma 1}\mu\left(\frac{\partial\phi}{\partial n_1}+h^s.n_1\right)\Psi \ +\int_{\Gamma 2}\mu\left(\frac{\partial\phi}{\partial n_2}+h^s.n_2\right)\Psi$$

$$=\mu\ \int_{\Gamma}\left(\left[\frac{\partial\phi}{\partial n}\right]+\left[h^s.n\right]\right)\Psi$$

The jump of the normal component of the field source being null, the formulation of the interior problem is written then :

$$e_p\int_{\Gamma} grad\ \ \phi.grad\ \ \psi\ d\gamma+\ \mu\ \int_{\Gamma}\left[\frac{\partial\phi}{\partial n}\right]\Psi$$

$$=\ -\ e_p\int_{\Gamma_i} h^s.grad\ \psi\ d\gamma \qquad (1)$$

The second term of the integral equation is called term of edge. This stage, it should be noted on the one hand that we cannot solve the problem without identifying the jump of the normal component of the magnetic field on Γ. On the other hand the solution of the interior problem could not be the solution of the problem posed in open space R^3. Only the term of edge which enables us to take account of the behavior of the field in the external medium. To treat this term of edge we first of all will formulate the external problem and will express its solution according to the traces of the field on the border Γ. The system of equations which govern the behavior of the magnetic field in the medium external of the structure :

$$divh\ =\ 0\quad\Rightarrow\quad\Delta\varphi\ =\ 0$$

We indicate by ψ the surface potential on Γ associated with the magnetic field of reaction. To know the function φ only on the border Γ would be enough with the determination to φ in Ωe, and thus to the magnetic field of reaction in the external medium. The problem consists in finding a potential φ such as:

$$\Delta\varphi\ =\ 0\qquad sur\,\Omega e$$

$$\varphi\ =\ \psi\qquad sur\,\Gamma$$

With through Γ, the jump of the normal component of magnetic induction and the jump of the tangential component of the magnetic field are null. The equivalent conditions, imposed on the scalar potential φ, are thus:

$$[\frac{\partial\varphi}{\partial n}]\ =\ q\ \ ;\ \ [\frac{\partial\varphi}{\partial t}]\ =\ 0$$

With : $q\ =\ [n.M]$. The magnetic potential can be to calculate starting from the surface density of magnetic load q. Indeed, by using the technique of the simple potential layer [4], shows that the solution of exterior problem can be written in the following integral form:

$$\varphi(x)=\frac{1}{4\pi}\int_{\Gamma}\frac{q(y)}{|x-y|}dy \qquad (2)$$

The magnetic charge q, introduced on Γ, are fictitious. It is also an intermediate unknown used to couple the exterior and the interior problem. To keep the potential φ like only unknown factor on Γ, we will have to express q according to φ :

$$[\partial\varphi/\partial n]=R\varphi\,.$$

Our goal now is to calculate R.

Numerical Implementation :

One proceeds then in the way indicated below. One adopts a triangular grid on Γ and one associates a value φi of φ for each node i of the grid. On a triangle, one writes: $\varphi=\sum_{i=1}^{3}\varphi_i\lambda_i$.

In this work we propose two forms of approximations of the magnetic charges on Γ. In a first time, we take the charges like constant functions by triangle. We write then : $q=\sum_{ti=1}q_i\eta_i$. With q_i is the value of q on triangle t_i and $\eta_i=\begin{cases}1\,on\ ti\\0\ \text{elsewhere}\end{cases}$

A variational formulation of (2) makes it possible to carry out the condition of coupling; i.e. to calculate the operator *R* :

$$\int_{\Gamma}\varphi\,\eta_k=\int_{tk}\sum_{i=1}^{3}\varphi_i\lambda_i=\sum_{i=1}^{3}\varphi_i\frac{S_{tk}}{3}=\sum_{I}B_{kI}\varphi_I$$

$$\int_{\Gamma}q'dx\int_{\Gamma}\frac{q\,dy}{|x-y|}=\int_{tk}dx_k\sum_{j}q_j\int_{tj}\frac{dy}{|x_k-y_j|}=\sum_{J}\sum_{K}D_{kJ}q_J$$

where q' is defined like test function :

$$q'=\sum_{tj=1}(q_j=1)\eta_j\ .$$

In the matrix form we thus obtain : $q = (D^{-1})B\ \varphi$.

In the second time, we take the charges like linear functions on each element of the grid. We write on each triangle: $q=\sum_{i=1}^{3}q_i\lambda_i$. In this case the operator *R* takes another form. in fact, the variational formulation of (2) is written:

$$\int_{\Gamma}\varphi\,q'=\sum_{tk}\int_{tk}\sum_{i=1}^{3}\varphi_i\lambda_i\sum_{j=1}^{3}q'_j\lambda_j=\sum_{I}\sum_{J}S_{IJ}\varphi_J$$

$$\int_{\Gamma} q' dx \int_{\Gamma} \frac{q\,dy}{|x-y|} = \sum_{tk} \int_{tk} \sum_{j=1}^{3} (q_j = 1) \lambda_j \sum_{tl} \int_{tl} \sum_{i=1}^{3} q_i \lambda_i \Big/ |x-y|$$

$$\sum_{I} \sum_{J} Q_{IJ}\, q_J$$

where q' is defined like test function :

$$q' = \sum_{j=1}^{3} (q'_i = 1) \lambda_j .$$

In the matrix form we thus obtain :

$$q = (Q^{-1}) S\, \varphi,$$

where I and J indicate the total numbers of the nodes of the grid. These calculations enables us to write the variational formulation of the problem with only unknown φ. The variational formulation becomes:

$$e_p \int_{\Gamma} grad\ \phi . grad\ \psi\, d\gamma + \mu \int_{\Gamma} (Q^{-1}) S_{\Psi}$$

$$= - e_p \int_{\Gamma_i} h^s . grad\ \psi\, d\gamma$$

From this formulation, we calculate φ and in particular his trace on Γ, and consequently the density of charge q. Once to calculate q, we calculate φ outside the structure with thin layers, the magnetic field in any point of the system and the induced magnetization M of the structure with thin layers.

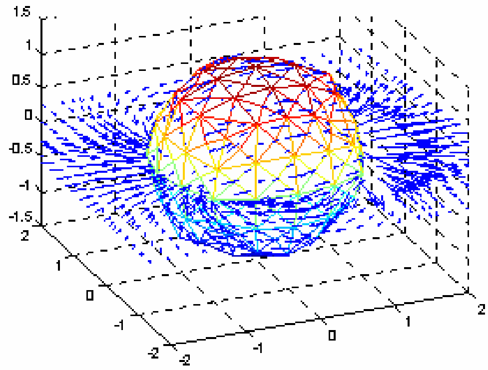
Computation Results :

µr	X	Y	Z	CHXT	HAX
1000	0.0	0.0	4.0	40.10	41.08
	2.0	0.0	3.4	40.02	40.67
	0.0	2.0	3.4	40.10	40.67
	3.4	0.0	2.0	39.86	39.86
10000	0.0	0.0	4.0	40.11	41.23
	2.0	0.0	3.4	40.02	40.77
	0.0	2.0	3.4	40.11	40.77
	3.4	0.0	2.0	39.85	39.84

Table I: Comparative table between the analytical and numerical values (With the approximation constant charges). The model is a hollow sphere of ray unit and Ep thickness. The source field is following X direction. the values HAY and HAZ are null. We did not present the values HYT and HZT in order to simplify the table. These values are almost null.

µr	X	Y	Z	CHXT	HAX
1000	0.0	0.0	4.0	37.82	41.08
	2.0	0.0	3.4	37.24	40.67
	0.0	2.0	3.4	37.87	40.67
	3.4	0.0	2.0	38.53	39.86
10000	0.0	0.0	4.0	38.30	41.23
	2.0	0.0	3.4	38.16	40.77
	0.0	2.0	3.4	39.08	40.77
	3.4	0.0	2.0	39.00	39.84

Table II: Comparative table between the analytical and numerical values (With the approximation linear charges).



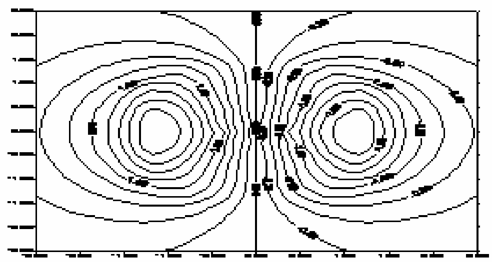


Fig.2-1 lines of magnetic field and equipotential magnetic on the sphere and outside the sphere

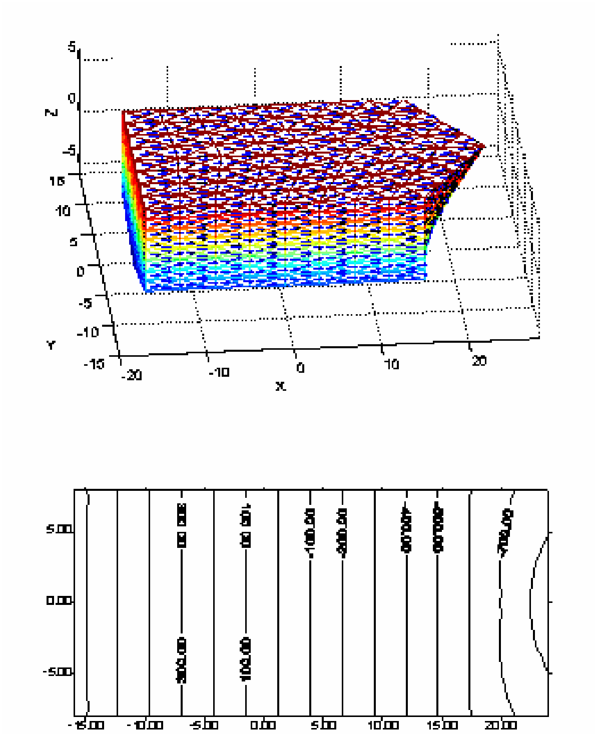


Fig. 2-3 lines of magnetic field and magnetic equipotential in the model I Ship type with linear charges

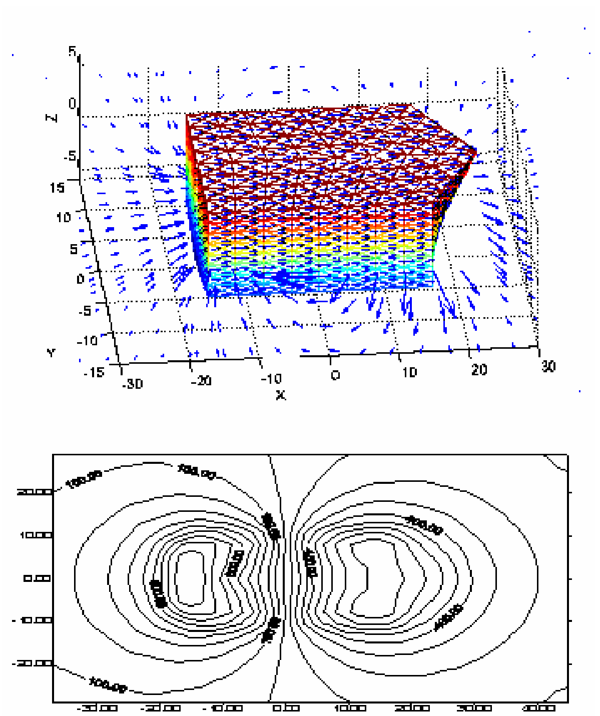


Fig. 2-4 lines of magnetic field and magnetic equipotential in the model I Ship type and outside the model.

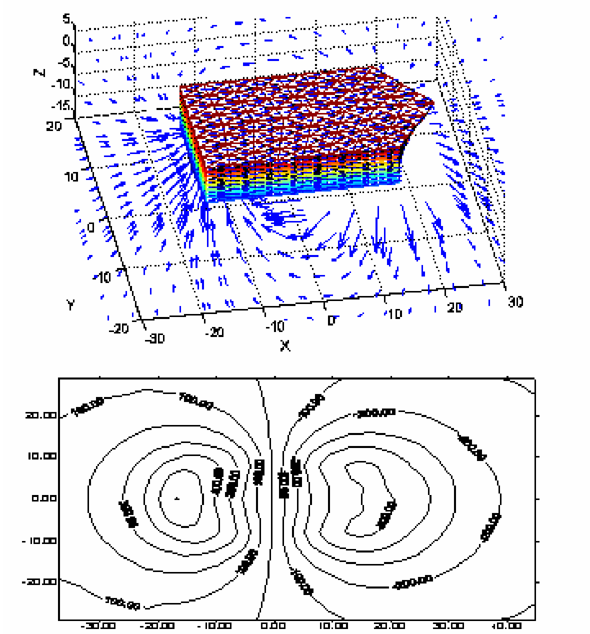


Fig. 2-5 lines of magnetic field and magnetic equipotential in the model I Ship type and outside the model.

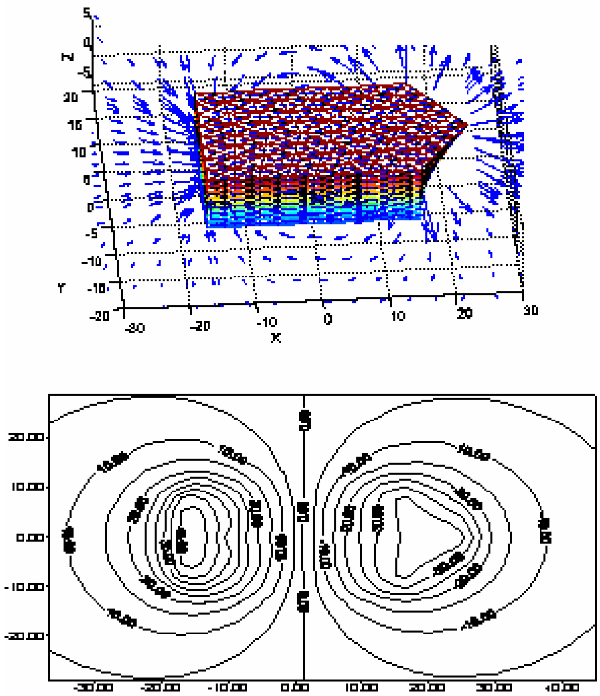


Fig. 2-6 lines of magnetic field and magnetic equipotential in the model I Ship type and outside the model.

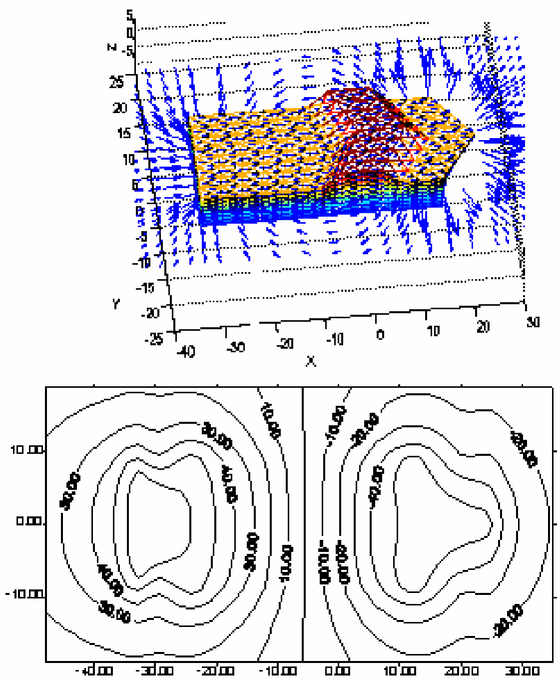


Fig. 2-7 lines of magnetic field and magnetic equipotential in the model II Ship type and outside the model.

The results presented here are at different altitude Z and the magnetic source field is directed along the ship.

Conclusion :

Each one of these two approximations corresponds to a computer code. A first numerical implementation is carried out with charges taken constant by triangle on the border of the structure with thin layers. A second numerical implementation is carried out with charges taken like linear functions on each triangle of the border. To test the good functioning of these computer codes, we considered a spherical model in order to make a comparative study between the numerical results and analytical calculations (see table I and II). The numerical results relating to different models are represented by
the equipotential sketched lines and the lines of the magnetic field (see figures). We notice that the flux of the magnetic field is well channeled by the thin layers, the lines of magnetic field are closed again in the area outside. In addition, the figures show that the lines equipotential take forms in conformity with the physical form of the structure of the thin layers.

The principal difficulty of our problem resides especially on the calculation of the term of edge. The approach that we proposed proves more flexible and less expensive from point of view capacity of the memory and time CPU. The calculation of the term of edge, of the model with constant charges, utilizes less large matrices compared to those of the model with linear charges. However, the model, with linear charges, gives numerical results in conformity and a faster convergence.

We note that the approach with linear charges on Γ gives a good representation of the magnetic potential and in consequence of the magnetic field on our physical system. This shows a certain regularity of the field, inside and outside the structure with thin layers, and checks the theory concerning the conformity of the magnetic field in the space of the acceptable fields.

This code can be with the profit of several applications we can project calculations on other fields such as the magnetic shielding and magnetic compatibility.

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