

# ROBUST CONGESTION CONTROL OF TCP/IP FLOWS

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Abstract: The growth in the TCP/IP traffic and the importance of the industrial applications using Internet (Flexible Control and Instrumentation, Telecontrol, Telemedecine,...) as communication medium have made necessary to know better the dynamic behavior of this dynamic system (TCP/IP flow). The objective is to reduce the congestion and thus render better service to the user with constant investments.

Different models and different techniques of congestion control are proposed. These techniques have in common the necessity to estimate the congestion level of a link and to return this information to the transmission sources. The results obtained has proved that to reduce the congestion it is necessary to anticipate it by slowing down the sources before it occurs.

In our paper we present a robust congestion control algorithm based on Smith Predictor (**SP**). The advantage of using SP is its capacity to predict the state of a dynamic system (congestion in our case) and thus reducing the effect of input delay inherent of TCP/IP flows.

Keywords: TCP/IP Model, Robust Control, Remote Control and Instrumentation

## 1. INTRODUCTION

During the last ten years the Internet Application (Remote Flexible Control and Instrumentation, Telecontrol, Telemedecine,...) has retained the attention of Control researchers for modeling and congestion control of TCP/IP networks.

The congestion in TCP happens when the demand in resource allocation is greater than the network capacity or similiary when the packet flow in a link is greater than the link capacity. In this case packet drop or retransmission deteriorates the quality of service offered by the network.

The congestion-control mechanism, necessary in an under-charged network, becomes indispensable in an over-charged network. Without the congestion-control mechanism, we may not only miss the quality of service but also fail in performing the service which means that the network may enter a blocking state. The industry applications need a better control of the pass band by robust congestion control of TCP/IP flows.

Different congestion-control mechanisms exist.

They differ by their level action ( in the network layer) and their algorithms. They may be classified into two main families:

- End to end congestion-control
- Closed loop congestion-control

The first class uses only the information sent by the end extremity to update the packet flow rate. The second congestion control algorithm update the source flow rate to maintain a predefined constant level of gateway buffer and so offering a better quality of service to the users.

The flow model [5] used in our paper is linearised around a nominal state which is given by:

- nominal flow of the source
- nominal level of buffer

The controllers proposed in the literature such as RED (Random Early Detection) or PI give good performance under certain conditions. They lose their efficiency in the case of variable delays (Round Trip Time (RTT) variation) or number of sessions. This is due to their characteristics which are the impossibility to attenuate the delay effects on the system. So the system becomes unstable if the input delay or/and the parameters of the networks change beyond some limits (number of session, ..).

In this paper, we propose an approach which is able to attenuate the influence of variable delay increasing the robustness of the system. We use a simplified dynamic model represented by  $n$  identical sources and a single link combined with the dynamic model of the buffer [9] [12]. The linearised model is decomposed into a delayed nominal part and the high frequency uncertainties [6].

The paper is organised as follows. In the following section we give the problem formulation. In section three we give the synthesis of the robust controller based on Smith Predictor algorithm. In the fourth section we give the simulation results, conclusion and perspectives.

## 2. PROBLEM FORMULATION

The dynamic model of TCP/IP flows developed by [5] using a fluid flow model is described by the following non-linear differential equations (1):

$$\begin{aligned} \dot{W}(t) &= \frac{1}{R(t)} - \frac{W(t)}{2} \frac{W(t-R(t))}{R(t-R(t))} p(t-R(t)) \\ \dot{q} &= \begin{cases} -C + \frac{N(t)}{R(t)} W(t), & q > 0, \\ \max\{0, -C + \frac{N(t)}{R(t)} W(t)\}, & q = 0 \end{cases} \end{aligned} \quad (1)$$

where:

- $W$  : is the average TCP window size (in packets)
- $q$  : is the average queue length (in packets)
- $R(t)$  : is the round-trip time:  $R(t) = \frac{q(t)}{C} + T_p$
- $C$  : is the link capacity
- $N$  : is the load factor
- $p$  : is the probability of packets mark
- $T_p$  is the propagation delay

To obtain an **LTI** dynamic model we do following hypothesis:

- the linear model is obtained by linearising (1) around the operating point  $(W_0, q_0, p_0)$ .

- we consider that  $N(t) \equiv N$  and  $C(t) \equiv C$
- If  $R(t)$  appears as an argument of a function we consider it constant and equal to  $R_0$  ( $R(t) = R_0$ )

From the equation (1) and the precedents hypothesis we obtain:

- **The nonlinear time invariant model:**

$$\begin{aligned} \dot{W}(t) &= \frac{1}{\frac{q(t)}{C} + T_p} - \frac{W(t)}{2} \frac{W(t-R_0)}{\frac{q(t-R_0)}{C} + T_p} p(t-R_0) \\ \dot{q} &= \begin{cases} -C + \frac{N}{R_0} W(t), & q > 0, \\ \max\{0, -C + \frac{N}{R_0} W(t)\}, & q = 0 \end{cases} \end{aligned} \quad (2)$$

- **The LTI model:**

$$\begin{aligned} \Delta \dot{W}(t) &= -\frac{N}{R_0^2 C} (\Delta W(t) + \Delta W(t-R_0)) - \\ & - \frac{1}{R_0^2 C} (\Delta q(t) b + \Delta q(t-R_0)) - \frac{R_0 C^2}{2N^2} \Delta p(t-R_0) \\ \Delta \dot{q}(t) &= \frac{N}{R_0} \Delta W(t) - \frac{1}{R_0} \Delta q(t) \end{aligned} \quad (3)$$

where:

- $q_0$  : is equal to the desired length of the queue
- $p_0$  : is the desired marking probability
- $W_0 = \frac{R_0 C}{N}$  and  $R_0 = \frac{q_0}{C} + T_p$
- $\Delta W = W - W_0$ ,  $\Delta q = q - q_0$  are  $\Delta p = p - p_0$  are the perturbed states ( $(\Delta W, \Delta q)^T$ ) and control variable ( $\Delta p$ ) about the operating point.

For practical purposes we decompose the dynamic model (3) in a nominal part and a high frequency part [6]. The nominal part will be used as a synthesis model and the high frequency part will be treated as model error. After some block-diagram manipulations we obtain the block-diagram representation given by figure (1).

We may remark that our LTI dynamic system is an input delayed system with model uncertainties under multiplicative form. The mathematical expression of model uncertainties is given by:

$$\Delta(s) = \frac{2N^2 s}{R_0^2 C^3} (1 - e^{-sR_0}) \quad (4)$$

From the LTI open loop model we may formulate the **AQM Control Problem** as *Find the AQM Robust Control capable to stabilise the queue length at a predefined value with efficient queue length and robustness with respect to variation of the propagation delay* [6].

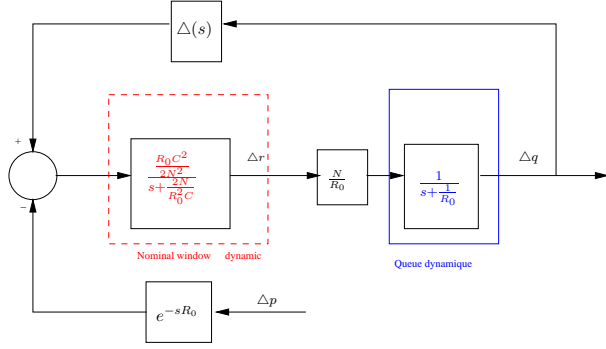


Fig. 1. Block diagram representation of model uncertainties of TCP/IP flow under multiplicative form

### 3. ROBUST AQM CONTROLLER

As stated in the previous section our problem is to synthesize a robust controller capable:

- to regulate the queue length at a predefined value
- to reduce the effect of the input delay
- to be robust to propagation delay

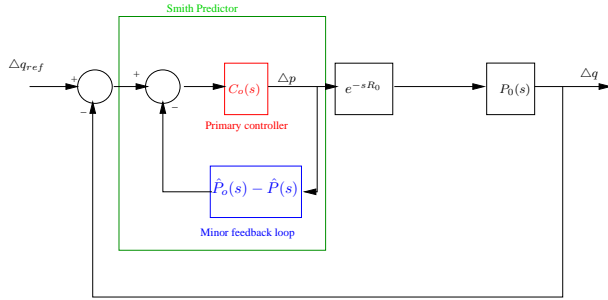


Fig. 2. Block diagram representation of Smith Predictor associated with a system with model uncertainties under multiplicative form

The schema block of closed loop system via the **Smith Predictor** is given in the figure (2) where:

- $P(s)$  is a linear transfer function of the form  $P(s) = P_0(s)e^{-\theta s}$
- $P_0(s)$  is a stable rational transfer function
- $\hat{P}(s)$ ,  $\hat{P}_0(s)$  are respectively the nominal values of  $P(s)$  and  $P_0(s)$

We remark that (**SP**) controller  $C(s)$  is a combination of a primary controller  $C_0(s)$  ( usually a **PI** or a **PID**) and an **error prediction** component. Whereas the error prediction component has a predefined structure and parameters, the parameters and structure of the primary controller has to be defined. We will treat the case where the primary controller is a **PID** controller whose parameters have to be defined.

From the figure (2) we may see also, that the signal output of minor feedback loop ( see figure (2)) contains the output signal prediction which

has as advantages to eliminate the gain reduction (overcorrection) due to the input delay.

In the rest of this paper, we will consider only the propagation delay variations  $\Delta\theta$  or  $\Delta R_0$  and suppose that the other parameters are correctly estimated. For clearness of exposition we give some definitions and theorems necessary for synthesis of AQM robust controller based on **SP**.

**Definition** A system that is asymptotically stable in the ideal case (in our case  $P(s) = \hat{P}(s)$ ) but became unstable for infinitesimal modeling mismatches is called a **practically unstable system**.

Suppose that  $C_0(s)$  is a primary controller which stabilize the undelayed system  $P_0(s)$ . In this case the closed-loop transfer function is given by:

$$Q(s) = \frac{C_0 \hat{P}_0(s)}{1 + C_0 \hat{P}_0(s)} \quad (5)$$

**Theorem 1 [10]** For the system with an **SP** to be closed-loop practically stable, it is necessary that:

$$\lim_{\omega \rightarrow \infty} |Q(j\omega)| < \frac{1}{2} \quad (6)$$

**Remark 1** In our case ( only mismatches in the propagation delay) the condition (6) is also sufficient.

**Theorem 2 [10]**

(a) the closed-loop system is asymptotically stable for any  $\Delta\theta$  if

$$|Q(j\omega)| < \frac{1}{2}, \forall \omega \geq 0 \quad (7)$$

(b) If

$$|Q(j\omega)| \leq 1, \forall \omega \geq 0 \wedge \lim_{\omega \rightarrow \infty} |Q(j\omega)| < \frac{1}{2} \quad (8)$$

then there exists a finite positive  $(\Delta\theta)_m$  such that the closed-loop system is asymptotically stable for all  $|\Delta\theta| < (\Delta\theta)_m$ .

**Remark 2**  $\Delta\theta$  gives the mismatch between the estimated and real input delay.

From the **theorem 1,2** and for a particular structure chosen of primary controller  $C_0(s)$  we may give the following lemma.

**Lemma** The primary controller given by  $C_0(s) = \frac{K}{s} P_0(s)^{-1}$  render the closed loop system:

- (1) practically stable
- (2)  $\exists (\Delta\theta)_m > 0$  such that the closed-loop system is asymptotically stable for all  $|\Delta\theta| < (\Delta\theta)_m$ .

**Proof:**

If we replace in (5) the expression of primary controller  $C_0(s) = \frac{K}{s}P_0(s)^{-1}$  we obtain:

$$Q(s) = \frac{P_0(s)C_0(s)}{1 + P_0(s)C_0(s)} = \frac{K}{K + s} \quad (9)$$

From (9) we may write:

$$\lim_{\omega \rightarrow \infty} |Q(j\omega)| = 0 (< \frac{1}{2}) \quad (10)$$

and the system is practically stable.

From (9) we have also:

$$|Q(j\omega)| < 1, \forall \omega \geq 0 \wedge \lim_{\omega \rightarrow \infty} |Q(j\omega)| < \frac{1}{2} \quad (11)$$

so we prove the second part of lemma .

**Remark 3** In [10] it is shown that a conservative estimation of  $\Delta\theta$  may be given by:

$$(\Delta\theta)_m = \frac{\pi}{3\omega_0} \text{ where } \omega_0 \geq K\sqrt{3}.$$

where  $\omega_0$  is the frequency above which  $|Q(j\omega)| < \frac{1}{2}$

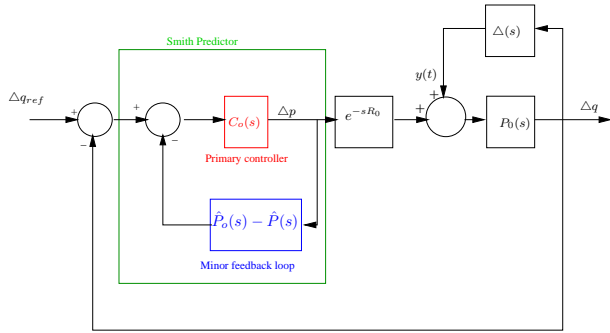


Fig. 3. Association of model uncertainties under multiplicative form and the closed loop of TCP/IP flow nominal model

From figure (2) and figure (1) we may derive the block diagram representation of our robust closed loop system given in figure (3). The objectifs of **SP** robust controller is to stabilise the closed loop system under input delay variation. The following theorem gives the sufficient condition over the primary controller  $C_0(s)$  parameter **K** which assure the robust stability of close loop system (TCP/IP flow).

**Theorem 3:** The close loop system is robust stable if we choose:

- (1)  $C_0(s) = \frac{K}{s}P_0(s)^{-1}$
- (2)  $K = \min(\frac{\sqrt{2}}{2R_0}, \min(\frac{1}{R_0}, \frac{2N}{R_0C}))$

**Proof:**

The input/output closed-loop transfer function of system without model uncertainties is given by:

$$F_{cl}(s) = \frac{P_0(s)C_0(s)e^{-sR_0}}{1 + P_0(s)C_0(s)} \quad (12)$$

If we replace the expression of primary controller  $C_0(s) = \frac{K}{s}P_0(s)^{-1}$  in (12) we obtain:

$$F_{cl}(s) = \frac{Ke^{-sR_0}}{K + s} \quad (13)$$

We have also:

$$Q(s) = \frac{P_0(s)C_0(s)}{1 + P_0(s)C_0(s)} = \frac{K}{K + s} \quad (14)$$

From the figure (2) the transfer function between the input perturbation signal  $y(t)$  and the output signal  $\Delta q(t)$  is:

$$F(s) = \frac{P_0(s)}{1 + P_0(s)C(s)e^{-R_0s}} \quad (15)$$

From the expression of  $C_0(s)$ ,  $P(s)$ ,  $P_0(s)$  and (15) we obtain:

$$F(s) = \frac{P_0(s)(s + K(1 - e^{-R_0s}))}{K + s} \quad (16)$$

From the small gain theorem we have the closed-loop stability under multiplicative form of model uncertainties (which is our case) if:

$$|\Delta(s)F(s)| < 1; \forall \omega > 0 \quad (17)$$

Combining (4), (15) and (17) we may write:

$$\left| \frac{\frac{2N^2j\omega C^2}{2N}}{(j\omega + \frac{2N}{R_0^2C})(j\omega + \frac{1}{R_0})} \frac{(s + K(1 - e^{-R_0j\omega}))}{R_0^2C^3(K + j\omega)} \right| < 1 \quad (18)$$

$\forall \omega > 0$

From (18) we may write:

$$\left| 2R_0j\omega \frac{1}{\frac{R_0^2C}{2N}j\omega + 1} \frac{1}{R_0j\omega + 1} \frac{\frac{1}{2K}j\omega + 1}{\frac{1}{K}j\omega + 1} \right| < 1 \quad (19)$$

$\forall \omega > 0$

If we choose as dominant root the  $F(s)$  pole  $-\frac{1}{K}$  then from (19) we may write:

$$\left| 2R_0j\omega \frac{\frac{1}{2K}j\omega + 1}{\frac{1}{K}j\omega + 1} \right| < 1; \forall \omega > 0 \quad (20)$$

Let  $\omega_c = K$  be the unique unity-gain crossover frequency of (20), then we may write:

$$K < \frac{\sqrt{2}}{2R_0} \wedge \min\left(\frac{1}{R_0}, \frac{2N}{R_0C}\right) \quad (21)$$

which prove the theorem.

**Remark 4** The controller proposed  $C(s)$  has a predefined structure based on the TCP flow LTI model and a gain  $K$ . So its tuning is more easy than PI or PID controller.

**Remark 5** Under condition of the unique unit gain crossover frequency the phase margin is at least  $180^\circ$  ( $\phi_m$ ). So the delayed margin may be given by ( $\theta_m = \frac{\phi_m}{\omega_c} \geq \frac{2\pi}{\omega_c}$ ).

#### 4. SIMULATION RESULTS

In this section we present the simulations results obtained by application of the Smith Predictor controller and we compare them with the results obtained by PI controller given in [6].

As simulation parameters we take:

- number of sessions:  $N = 60$
- number of packets/sec:  $C = 3750$
- propagation delay:  $T_p = 0.2$  seconds
- and thus a nominal delay  $R_0 = 0.2467$

From these simulation parameters and applying theorem 3 we obtain:

$$K \in \{2.866, \min(0.52, 4.05)\}$$

thus we may choose  $K = 0.3$ .

The primary controller  $C_0(s)$  is a PID controller whose values are determined by the nominal model and the parameter  $K$ . So from the nominal model and the parameters choosed we obtained the discrete PID coefficients:  $K_p = 1.1725e - 005$ ,  $K_i = 5.4583e - 0.006$ ,  $K_d = 2.5600e - 006$ .

In figure 4 and 5 we give the simulation results obtained by applying Smith Predictor and PI controller [6] on the nominal system. We may see that the queue response time, measured in number of sampling period, under Smith Predictor control is inferior to that needed for the queue controlled by PI controller.

For the second simulation test we take the same parameters and we change the propagation delay at 25-th sampling period from  $R_0$  to  $3R_0$ . The results given in figure 6 and 7 respectively, show

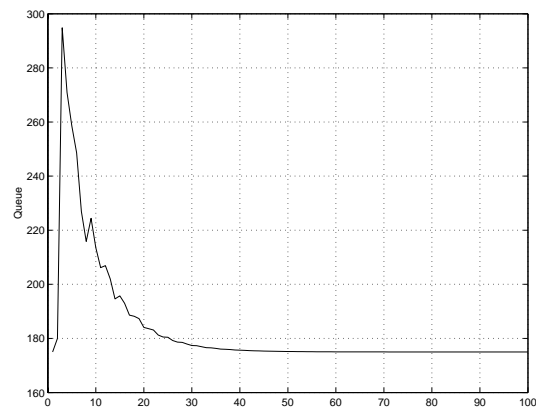


Fig. 4. The queue length dynamic under Smith Predictor in function of number of sampling periods

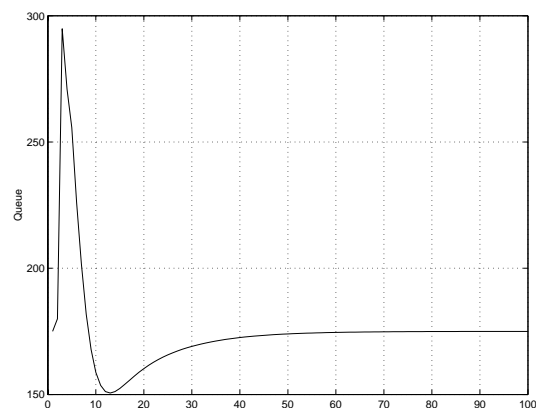


Fig. 5. The queue length dynamic under PI controller in function of number of sampling periods

that the Smith Predictor stabilise the queue dynamics in the case where the PI controller fail.

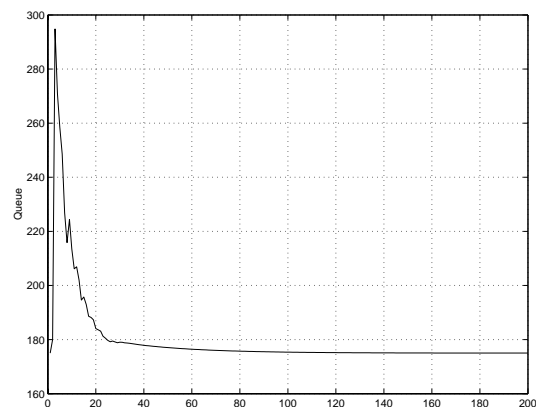


Fig. 6. The queue length dynamic under Smith Predictor in function of number of sampling periods

In the third test we change the system parameters to :  $N_1 = 100, C_1 = 1000, q_{ref} = 175$  and thus we obtain  $R_{01} = 0.3750 < R_0$ . From figure 8 we see that the Smith Predictor stabilise the queue

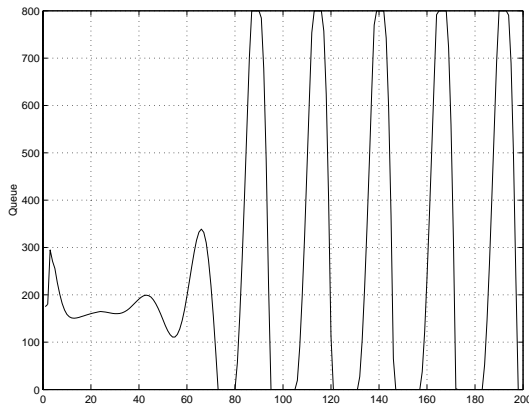


Fig. 7. The queue length dynamic under PI controller in function of number of sampling periods

dynamic but the response time is greater.

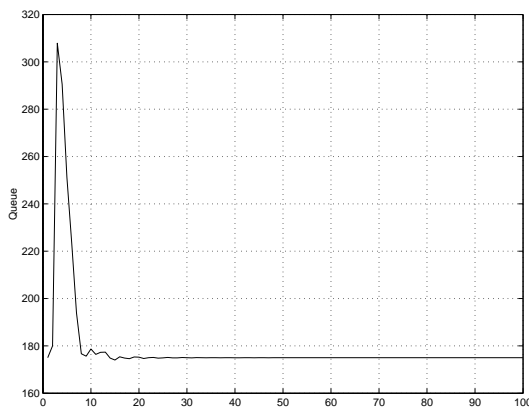


Fig. 8. The queue length dynamic under Smith Predictor in function of number of sampling periods in the case of model parameter change.

From the simulation results we may conclude that the the Smith Predictor performs well in the delay variation situation and has also some robust properties in the case of system parameter variations. This is due to its capacity to predict the system output and so reduce the input delay influence.

As future perspective it is important to work on robust controller capable to consider the input delay variation and system parameters variations at the same time.

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