Measurement and Simulation of Pipeline Media Leakage

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Abstract: - The paper deals with analysis, measurement and simulation of the leakage from a plastic pipeline that transports the compressible fluid. The theoretical part of this paper deals with the solution of both algebraic and a partial non-linear differential equations of the first order. The equations describe flow of compressible fluids in plastic pipelines. The system of equations was solved using the numerical methods by both Euler and Runge-Kutta of the 4^{th} order. Mathematical model was implemented in Matlab environment. The comprehensive laboratory model of the system for verification of the corresponding mathematical model was developed. The goal was to simulate perforation of the pipeline resulting in media loss. The novel methods of media leakage detection focused on plastic pipeline and compressible media problem are presented. The first one is based on analysis of transient response that arises at the time of pipeline damage. The second one is based on adaptive identification of the system.

Key-Words: - measurement, simulation, leakage, partial differential equations, plastic pipeline, Matlab environment, transient response

1 Introduction

There can come to the damage of pipeline during its application. The leakage represents the economic loss and it can simultaneously come to contamination of the environment. Thus the improvement of the method for detection and localization of the leakage spot is the necessary part of the whole pipeline system organization. Modeling and simulation of the dynamical processes plays a very important role in engineering solution. Using mathematical model for modeling the pipeline damage and media leakage has been shown in this paper. The goal is to present the novel methods of media leakage detection.

2 Mathematical Description of the Problem

The mathematical model describes flowing of the compressible fluid in the segment of plastic pipeline. The fundamental model is described by three partial nonlinear first order differential equations.

1. Newton equation

$$\frac{\partial p}{\partial x}+\rho\, g \sin\Theta+\rho\frac{\partial v}{\partial t}+\rho\, v\frac{\partial v}{\partial x}+\frac{\rho\,\lambda\, v|v|}{2d_n}=0 \qquad (1)$$

2. Bernoulli equation

$$\frac{\partial}{\partial t}\left\{\rho\,(c\,T+\frac{v^2}{2})\right\}+\frac{\partial}{\partial x}\left\{\rho\, v(c\,T+\frac{v^2}{2})\right\}+ \frac{\partial}{\partial x}(p\,v)+\frac{\partial}{\partial x}(\rho\, v\, g\, z)-\frac{\gamma(T_{ok}+T)}{S}=0 \qquad (2)$$

3. Equation of continuity

$$\rho\, S\frac{\partial v}{\partial x}+\rho\, v\left(\frac{\partial S}{\partial p}\frac{\partial p}{\partial x}+\frac{\partial S}{\partial T}\frac{\partial T}{\partial x}+\frac{\partial S}{\partial x}\right)+ S\, v\left(\frac{\partial \rho}{\partial p}\frac{\partial p}{\partial x}+\frac{\partial \rho}{\partial T}\frac{\partial T}{\partial x}\right)+\rho\left(\frac{\partial S}{\partial p}\frac{\partial p}{\partial t}+\frac{\partial S}{\partial T}\frac{\partial T}{\partial t}+\frac{\partial S}{\partial x}\right)+ +S\left(\frac{\partial \rho}{\partial p}\frac{\partial p}{\partial t}+\frac{\partial \rho}{\partial T}\frac{\partial T}{\partial t}\right)=0 \qquad (3)$$

The model includes equations that respect the elasticity of pipeline (4) and elasticity of transferred fluid (5).

$$S=S_0\exp\left(\frac{d_n}{E\,d}(p-p_0)\right)\exp\left(2\,\alpha\,(T-T_0)\right) \qquad (4)$$

$$\rho=\frac{\rho_0}{\exp\left(\frac{1}{K}(p_0-p)\right)\exp\left(\beta\,(T-T_0)\right)} \qquad (5)$$

The model was extended by equations describing the media leakage (6), (7), (8).

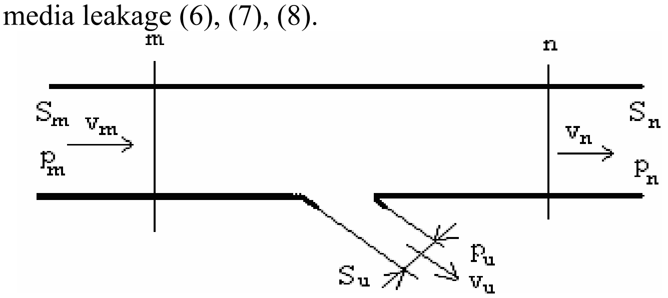

Fig. 1: Media leakage.

$$v_u = \frac{S_m \rho_m v_m - S_n \rho_n v_n}{S_u \rho_u} \tag{6}$$

$$p_n = \rho_n \left(\frac{p_m}{\rho_m} - v_n^2 + v_m v_n \right) \tag{7}$$

$$\begin{aligned} & v_m^2 \left(\left(S_m \rho_m \right) \left(S_m \rho_m - S_n \rho_n \right) \right) + \\ & + v_m \left(S_n \rho_n v_n \left(S_u \rho_u - 2 S_m \rho_m \right) \right) + \\ & + \left(S_m \rho_m v_m \right)^2 - \left(S_u \rho_u \right)^2 \left(\frac{p_m}{\rho_m} - \frac{p_u}{\rho_u} \right) = 0 \end{aligned} \tag{8}$$

where:

c = internal energy of liquid
d = wall thickness of the pipeline
dn = internal diameter of the pipeline
E = modulus of elasticity
g = acceleration of gravity
K = liquid elasticity bulk modulus
l = length of pipeline
p = pressure of liquid
p_0 = relative pressure
S = cross sectional area of the pipeline
S_0 = relative cross sectional area of the pipeline
t = time
T = temperature of liquid
T_{ok} = ambient temperature
T_0 = relative temperature
v = flow velocity of liquid
x = coordinate along pipeline axis
z = elevation of the pipeline
α = bulk expansivity of pipeline
β = bulk expansivity of liquid
γ = heat transfer coefficient
λ = friction factor of pipeline
ρ = density of liquid
ρ_0 = relative density of liquid
Θ = pipeline gradient

The system of equations was solved by using the numerical methods in Matlab environment. The temperature processes were omitted, they would be significant mainly in pipeline with a gas or with a steam. The terms $\delta v/\delta t$ and $\delta p/\delta t$ were expressed and integrated by both Euler and Runge-Kutta numerical methods.

$$\frac{\partial v}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} - g \sin\Theta - v\frac{\partial v}{\partial x} - \frac{\lambda\, v\left|v\right|}{2\, d_n} \tag{9}$$

$$\frac{\partial p}{\partial t} = -\frac{\rho S \frac{\partial v}{\partial x} + \rho v \left(\frac{\partial S}{\partial p}\frac{\partial p}{\partial x} + \frac{\partial S}{\partial T}\frac{\partial T}{\partial x} + \frac{\partial S}{\partial x} \right)}{\rho \frac{\partial S}{\partial p} + S \frac{\partial \rho}{\partial p}} + \frac{S v \left(\frac{\partial \rho}{\partial p}\frac{\partial p}{\partial x} + \frac{\partial \rho}{\partial T}\frac{\partial T}{\partial x} \right) + \rho \left(\frac{\partial S}{\partial T}\frac{\partial T}{\partial t} + \frac{\partial S}{\partial x} \right) + S \frac{\partial \rho}{\partial T}\frac{\partial T}{\partial t}}{\rho \frac{\partial S}{\partial p} + S \frac{\partial \rho}{\partial p}} \tag{10}$$

The initial and margin condition have to be completed to the equations. Initial conditions in this case are mathematical form of:

- a flow rate distribution along the length of pipeline
- a medium pressure distribution along the length of pipeline
- a elevation distribution along the length of pipeline
- a pipeline cross section distribution along the length of pipeline
- a medium density distribution along the length of pipeline

As a fixed margin condition were chosen: pressure at the beginning of pipeline and flow velocity in the end of pipeline. These values correspond with real values that were measured on physical model.

For verification of the corresponding mathematical model was developed the comprehensive laboratory model of the system.

3 Physical Model of the System

Presented problems were measured on experimental apparaturs which scheme is shown in Fig. 2. It composes of a pump, a reservoir, a flexible pipeline of the length of about 80 meters, a valve, 2 pressure sensors and flowmeters placed at the beginning and in the end of pipeline. In the middle of pipeline was simulated the leakage.

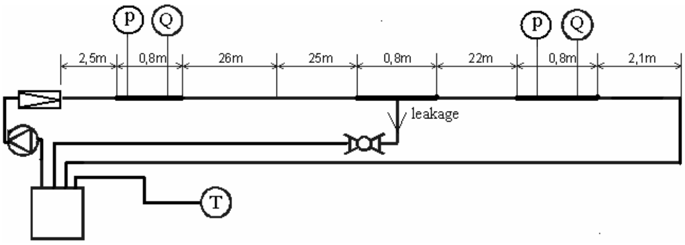

Fig. 2: Scheme of the physical model

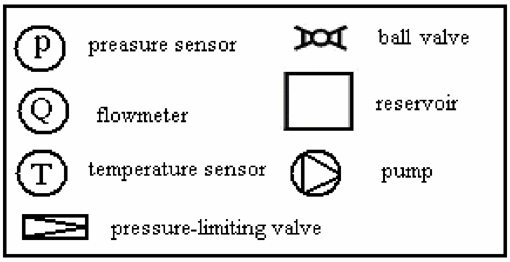

Fig. 3: List of components

There was an accurate mathematical model of the system set up. Because some of the parameters were not given by the producer of the pipeline, they had to be measured experimentally. There were mainly two parameters: The friction factor of the pipeline λ and the modulus of elasticity E.

3.1 Measurement of Dependence between Extensibility of pipeline and the Pressure.

There are two courses of the dependence between extensibility of pipeline and the pressure. Theoretical (expected) course – this is given by equation (4) of the model. Real course – this reflects measured values-Fig.4.

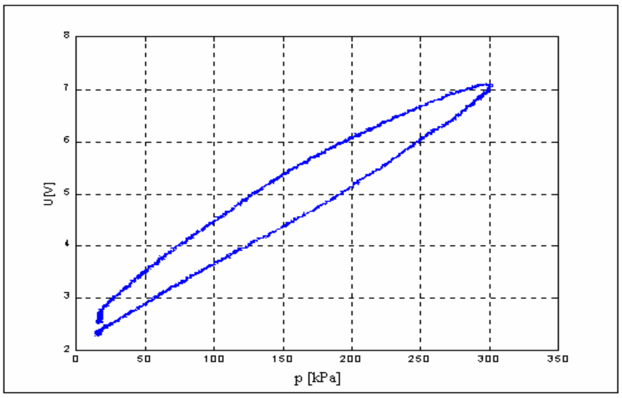

Fig. 4: Dependence between extensibility of pipeline and the pressure.

Measurement of dependence between extensibility of pipeline and the pressure led to verdict that equation (4) is convenient only for really elastic materials with inconsiderable non-linearities, for example for steel. Pipeline used in this experiment consists of plastic material; there is a net of fixed threads within the wall whose effect will not show but at higher pressure or pressure peaks. This is reflected by real measured course in Fig. 4. The material has also significant deformation losses.

3.2 Measurement of the friction factor of pipeline

Using the pressure-limiting valve a different conditions were set. The pressure and flow rate at the beginning and in the end were measured. Friction factor is done by the term:

$$\lambda = \frac{2d_n \Delta p}{\rho l v} \tag{11}$$

The result is shown in the table below.

P1 [kPa]	v1 [m/s]	P2 [kPa]	v2 [m/s]	λ [-]
201,943	1,650	119,952	1,703	0,0250
201,479	1,653	119,257	1,708	0,0250
202,446	1,646	120,960	1,700	0,0250
206,145	1,618	126,630	1,675	0,0252
223,406	1,498	154,542	1,575	0,0250
309,239	0,934	281,635	1,013	0,0250

Table 1: Measurement of the friction factor

4 Measurement of Media Leakage

The simulation of pipeline perforation was done. The leakage was simulated 53 meters from the beginning of pipeline The measurement of the pressure in the end and the flow velocity at the beginning, as in previous case, was done. There was constant pressure held at the beginning of pipeline.

	P1 [kPa]	P2 [kPa]	v1 [m/s]	v2 [m/s]
Leakage 0%	130,597	106,070	1,200	1,207
Leakage 12%	130,072	105,386	1,260	1,098
Leakage 15%	131,041	105,640	1,300	1,103
Leakage 24%	130,042	105,164	1,323	1,002
Leakage 27%	130,505	105,094	1,349	0,987

Table 2: Measurement of media leakage

5. Modeling of Media Leakage.

Numerical model solves the equation and there are its results: the pressure, the velocity and the internal diameter along the pipeline and along the time. These courses are shown in the figures below.

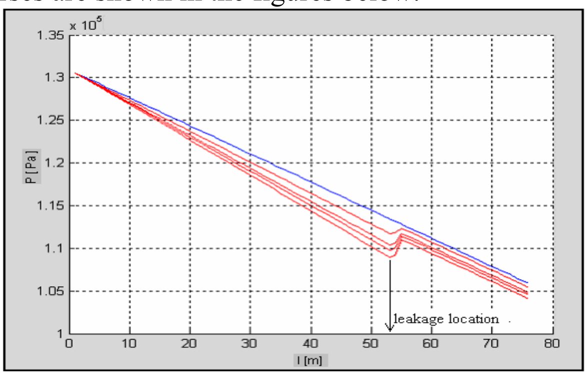

Fig. 5: Pressure along the length of pipeline for 4 different leakages.

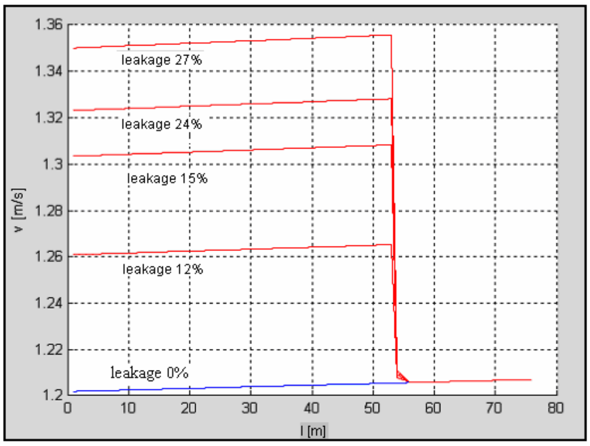

Fig. 6: Velocity along the length of pipeline

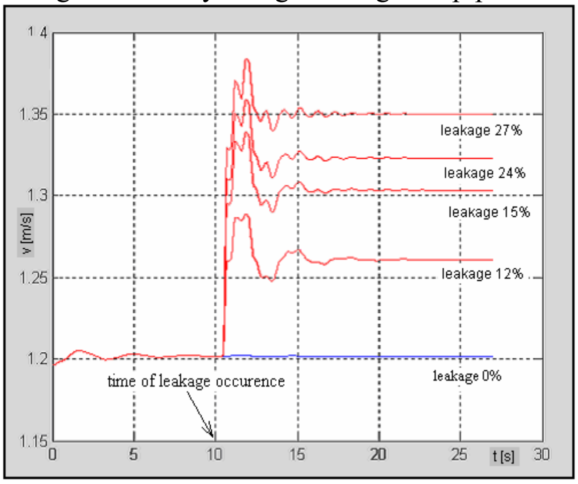

Fig. 7: Velocity along the time

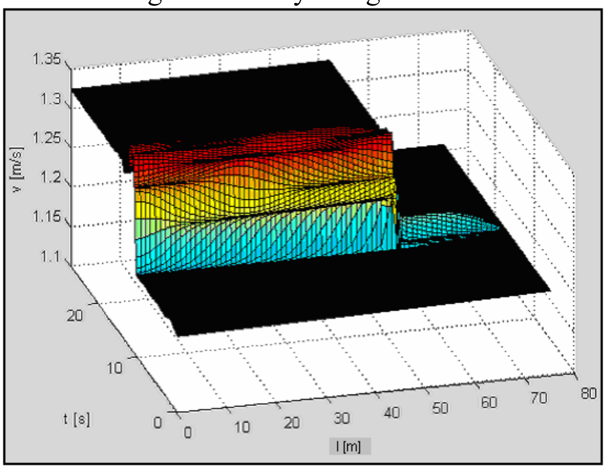

Fig. 8: Velocity – 3D.

6. Comparison

Despite of the accurate description of plastic material property is missing, the model provides sufficient result compared to measured data. The deformation losses arise particularly in high pressure. We compare amount of media leakage (computed and measured) and flow velocity (computed and measured) at the beginning of pipeline.

	Leakage		ERROR	Velocity of media flow v_1	ERROR
	[l/s]	[%]	[%]	[m/s]	[%]
Meas.	0,130	12,86		1,260	
Model	0,131	13,25	-0,39	1,261	-0,08
Meas.	0,159	15,15		1,300	
Model	0,157	15,03	0,12	1,298	0,15
Meas.	0,258	24,26		1,323	
Model	0,257	24,18	0,08	1,322	0,08
Meas.	0,291	26,83		1,349	
Model	0,293	27,00	-0,17	1,349	-0,01

Table 3: Measured data and computed data comparison

7. Methods of Leakage Localization

7.1 Adaptive identification of the system

Media leakage localization is based on measurement of status values (pressure and flow velocity) at the beginning and in the end of pipeline. The task is solved using adaptation of mathematical model, where the demanded parameter is the place of leakage. The difference between measurement data and computed data is an objective function. The purpose of the adaptation is minimizing an objective function by changing the parameter demanded. This parameter of the model minimizing an objective function is designated as a feasible place of media leakage.

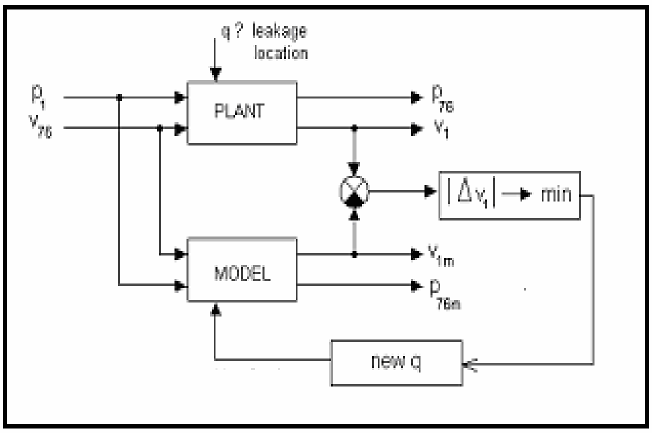

Fig. 9: Adaptive identification

Designed optimizing algorithm demands maximum four changing of parameter for finding the problematic place. In the Fig. 10 is shown the result of optimizing algorithm. The x-coordinate of the point of intersection of the blue line and x-axis determines the localization of media leakage.

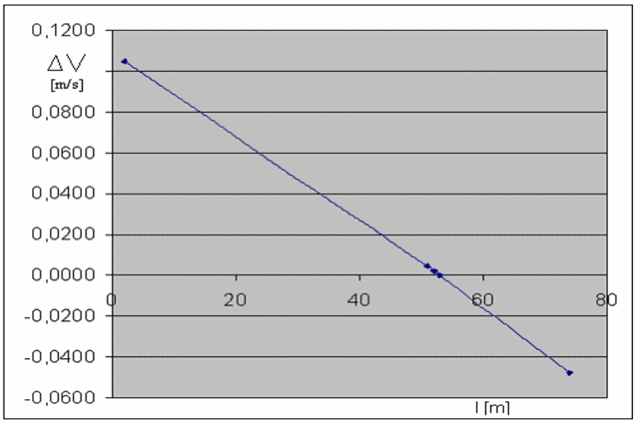

Fig. 10: Number of iterations

Demanded place was designated 53 meters from the beginning of pipeline in three cases and one case was designated 52 meters from the beginning of pipeline. This corresponds to a real media leakage location constructed on physical model according the Fig. 2.

Leakage	Leakage localization – distance from the beginning of pipeline [m]
12%	53
15%	52
24%	53
27%	53

7.2 Transient response analysis

The second method of media leakage localization that is described below is based on analysis of transient response. In the moment the leakage arises the transient response grows up. The transient response has a characteristic pressure and streaming wave. The wave expands from the place of leakage both ways and can be recorded by measurement devices at the beginning and in the end of pipeline.

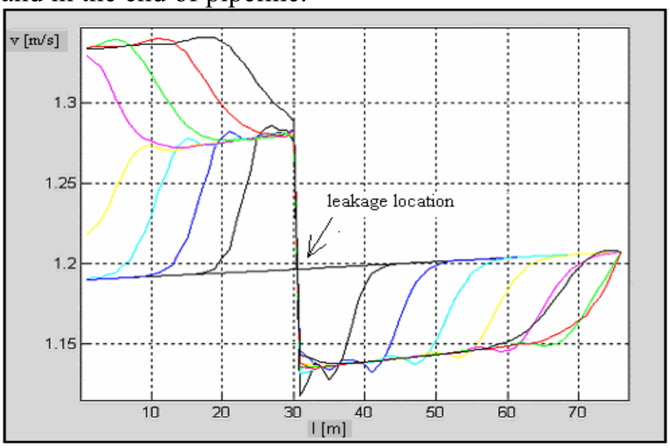

Fig. 11: Streaming wave - along the length of pipeline

In the Fig. 12 and Fig. 13 there is shown time course of streaming and pressure wave at the beginning and in the end of pipeline. At the each wave there is possible to identify the points of local extremes. The time, when the extremes arise, depends on the spacing the spot of leakage from the measurement points. The main aim of this task is to realize what the velocity of wave propagation is. If the velocity is known, then is no problem to calculate a distance which the wave had to cover.

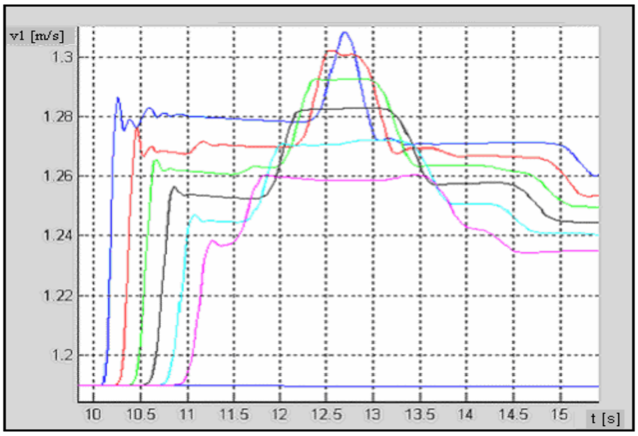

Fig. 12: Streaming wave at the beginning of pipeline for different location of media leakage

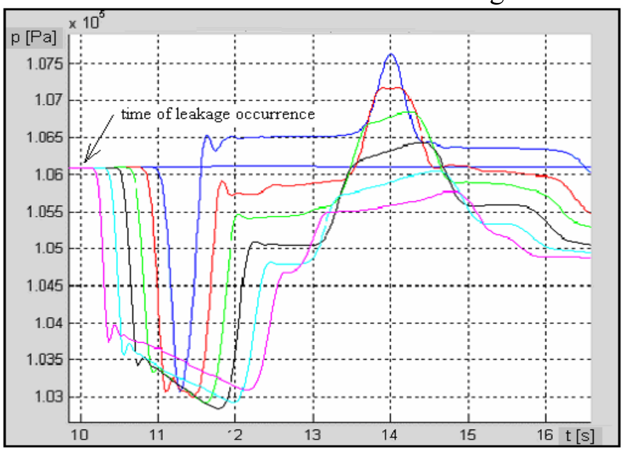

Fig. 13: Pressure wave in the end of pipeline for different location of media leakage.

There was calculated velocity of pressure and streaming wave propagation as 53 m/s from analysis of wave reflection. This value agrees with the velocity of the spread of hydraulic head calculated in [2].

4 Conclusion

The work deals with leakage media localization and identification. It is focused on compressible fluid and plastic pipeline. During problem solution there were done:

- The mathematical model of plastic pipeline including media leakage was designed and verified.
- The physical model of the plant was made out. The purpose was to simulate a media leakage and measure the state variables.
- Two methods for media leakage localization were

designed

The methods described in this paper are designed for detection from the small up to medium leakages. It means the leakages, where the pumps are able to refill the media loss and when the working of the system is not endangered immediately. The described method use common telemetric values measured in real time – pressure, velocity flow and temperature in the initial and final point of pipeline. We can presume good results of these methods only in case of high accuracy of sensors. Bad accuracy of measuring puts the level down concerning the focusing for media leakage spot. The work shows possibilities of measurement engineering and mathematical modeling for media leakage determination and constitutes a contribution for an economical service and environmental protection.

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