Computation complexity of time-frequency representation of signals

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Abstract: - Multidimensional time-frequency representations of signals are images of changes in time of signal spectrum and thus play an important role when they are analysed and processed. There is a number of possible ways of time-frequency representation of signals. Their properties depend on the transformation algorithm used, and they differ not only in the resolution of changes in the time and the frequency domain but also in the computation complexity during the realization. Today, rapid processing of great amounts of data in a very short period of time is often required. It is, above all, in systems processing signals in real time that minimum delay in processing the signal is of much concern and therefore it is important to tackle the problem of the computation complexity of the algorithms used.

Key-Words: - Digital signal processors, wavelet transform, Fourier transform, FFT, DWT, digital signal processing, spectrogram, scalegram.

1 Introduction

In the spectral analysis of signals it is often of advantage to know the changes in signal spectrum vs. time. Signal frequency changes vs. time give multidimensional time-frequency representations of signals such as spectrogram and scalegram.

2 Realization of spectrogram

Spectrogram is a time sequence of short-time signal spectra, which is realized as a sequence of short-time Fourier transforms (STFT) of time-limited signal frames. The signal is divided into a sequence of frames that overlap by a conveniently chosen overlap, as shown in Fig.1.

All such signal frames are weighted by a window function for the removal of fringe distortions caused by the finite length of frame, and then processed by the fast Fourier transform (FFT) algorithm to give the spectral representation. It is obvious from Fig.1 that the number of frames, M, into which an input signal of length d is decomposed, depends on the length of individual frames, N, and on their overlap p according to relation (1).

$$M = (d - N) / p \, . \qquad (1)$$

A fast decomposition algorithm of type DIT is used when implementing the FFT in a digital signal processor [1]. Essentially, this is a progressive decomposition of the signal into even and odd components. If the length of the frame being processed is $N = 2^n$ samples, then n-times repeated decomposition can be used to decompose the frame into sequences of only 2 samples, whose DFT can be expressed by relation (2) from [1].

$$\begin{aligned} S(1) &= s(1) + s(2) \\ S(1) &= s(2) + W \cdot s(1)\,, \\ W &= -e^{-j\pi n} \end{aligned} \qquad (2)$$

where $S(k)$ is the k-th spectral component,
$s(n)$ is the n-th signal sample,
W is the transformation operator.

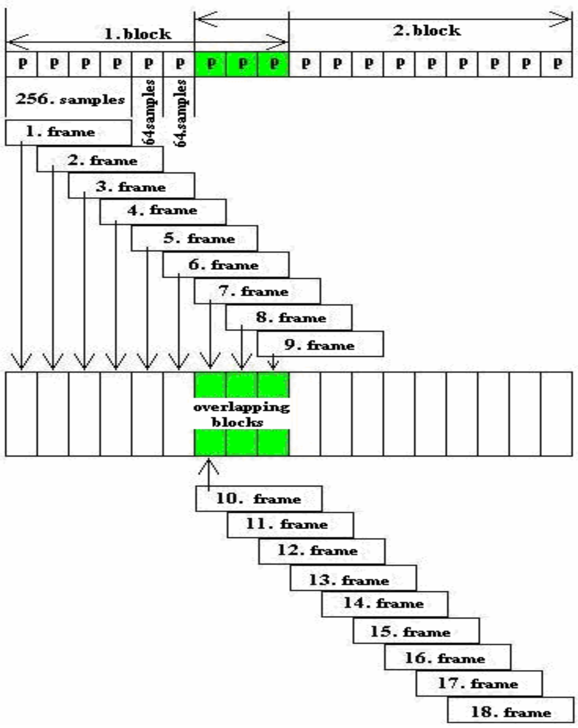

Fig.1 Signal decomposition into frames

In the literature, these two simple mathematical operations are often referred to as **"butterfly".** The signal-flow graph of one such butterfly is shown in Fig.2.

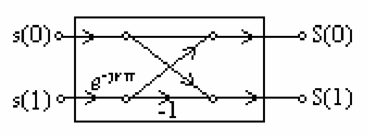

Fig.2 Signal-flow graph of a butterfly

In digital signal processors, butterfly operations are realized using two simple instructions for multiplying two operands with accumulation, which are often referred to as MAC instructions (*Multiply AcCumulate*). The whole DIT algorithm operates section-wise at several decomposition levels, with each section being formed by parallel computation of $N/2$ butterflies, as illustrated in Fig.3.

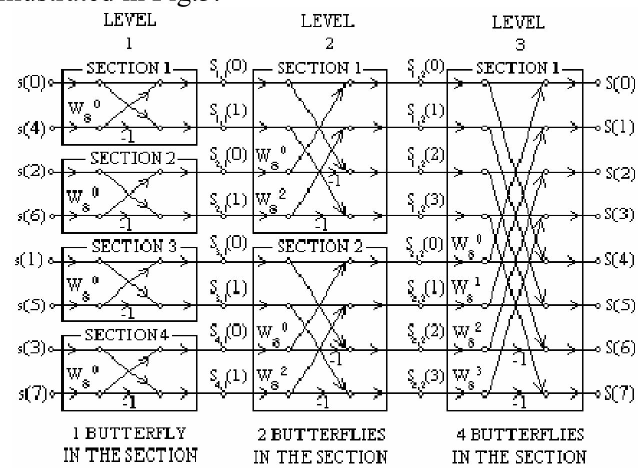

Fig.3 Signal-flow graph of the FFT algorithm

It follows from Fig.3 that processing a frame of length N proceeds by $\log_2 N$ decomposition levels, with just $N/2$ butterflies being realized at each level. Each level is formed by just two MAC instructions and thus the number of instructions necessary to realize the whole FFT algorithm for one signal frame of length N is given by relation (3).

$$I_r = N \cdot \log_2 N. \tag{3}$$

Multiplying relation (1) by relation (3) gives a relation for the total number of MAC instructions necessary to realize the spectrogram matrix

$$I = \frac{N(d-N)}{p} \cdot \log_2 N. \tag{4}$$

When processing the signal into the spectrogram it is convenient to choose the length of the frame being processed to be ca. 30 ms since within this interval of time the speech signal can be considered stationary. If at sampling frequency $f_s = 8000$ Hz we are seeking a frame whose length is $N = 2^n$ samples and which should take ca. 30 ms, we must choose the frame length $N = 256$ samples. For the spectrogram resolution in the time domain to be sufficiently high we choose the frame overlap to be $p = 64$ samples. To make the calculation easier, it is convenient to choose the input signal length $d = 6656$ samples. According to relation (1) we then divide the signal into $M = (6656 - 256)/64 = 100$ frames, and the total number of MAC instructions necessary to process the whole spectrogram according to relation (4) will be

$$I = \frac{256(6656-256)}{64} \cdot \log_2 256 = 204800.$$

3 Realization of scalegram

Scalegram is a graphical representation of detailed coefficients of the discrete wavelet transform (DWT). Detailed coefficients S_{DWT} represent the similarity between the signal and the applied wavelet of length a shifted by time τ. Their computation is given by relation (5) from [2].

$$S_{DWT}(j,k) = \sum_{n=0}^{N-1} s_n \cdot_j w_{k-n}, \quad a = 2^j, \quad \tau = k \cdot 2^j, \tag{5}$$

where s_n is the n-th signal sample,
$S_{\mathrm{DWT}}(j,k)$ is the k-th coefficient of j-th decomposition level,
w_{k-n} is the k-th sample of the applied wavelet of length a, shifted by τ.

The most frequent realization method suitable for implementation in a digital signal processor is the fast pyramidal algorithm, with which the signal decomposition proceeds by individual levels through a system with several sampling frequencies (*Multirate system*). The core of the algorithm is formed by a bank of two orthonormal FIR filters (filters of the QMF type), which at each decomposition level decomposes the signal into the upper and the lower half of frequency band. The output signals of the filter bank are doubly decimated. The signal obtained from the lowpass filter forms a vector of approximation coefficients, which enter the next decomposition level while the signal on the output of highpass filter forms a vector of detailed coefficients, which form the algorithm output. The algorithm vs. time is illustrated schematically in Fig.4.
The QMF filters employed are digital filters of the type of FIR, whose frequency responses are mutually mirrored about $f_s/4$ and form the orthonormal base of the whole frequency spectrum ($0 \div f_s/2$). Thus we have here one filter of the type of highpass filter and one of the type of lowpass filter, whose cut-off frequencies are $f_m = f_s/4$, which split the signal spectrum into the upper and the lower part. The response of FIR filter to the

signal corresponds to the convolution of the signal and the impulse response of the filter.

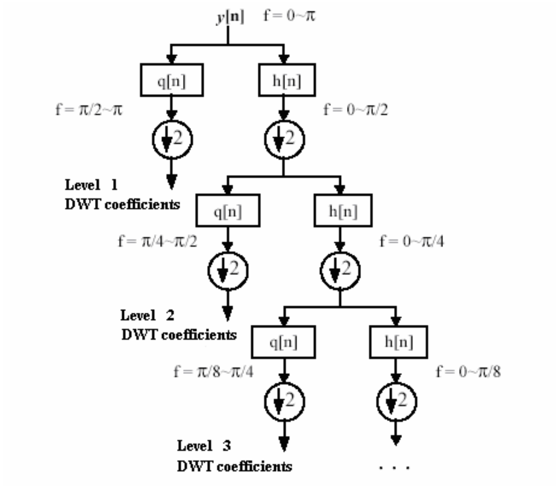

Fig.4 Schematic diagram of pyramidal algorithm

The convolution of the signal and the impulse response of an FIR filter of R-th order is described by relation (6) from [2].

$$y(n) = \sum_{k=0}^{R} x(k)h(n-k)\,. \tag{6}$$

It is evident from relation (6) that the sum of the products of signal samples and (R+1) values of the impulse response of filter in digital signal processors is realized by (R+1) MAC instructions. The number of MAC instructions necessary to perform the whole convolution of a signal of length d and the impulse response of filter is given by relation (7).

$$I_k = d \cdot (R+1)\,. \tag{7}$$

In the implementation in digital signal processors, decimation is only realized by changing the method of memory addressing. It is sufficient to change the value N of the addressing unit register, which in the modulo addressing mode realizes the shifting of address pointer by N positions. The whole decimation is thus realized by a single move (MOV) instruction, which in parallel data processing does not take any time.

Each decomposition level is always formed only by the convolution of a signal of length d and the impulse response of the highpass and the lowpass FIR filter of R-th order, with the signal length being halved on each subsequent level. The number of MAC instructions necessary to realize one decomposition level is given by relation (8).

$$I_S = 2d_n \cdot (R+1)\,. \tag{8}$$

The number of MAC instructions required to realize the whole pyramidal algorithm that decomposes the signal into K decomposition levels is given by relation (9).

$$I = 2d \cdot (R+1) \cdot \sum_{i=0}^{K-1} 2^{-i}\,. \tag{9}$$

When processing a signal section of length d = 6656 samples by the fast pyramidal algorithm, using FIR filters of order R =31, into N = 8 decomposition levels, the processor will by relation (9) perform a total of

$$I = 2 \cdot 6656 \cdot (32) \cdot \sum_{i=0}^{8-1} 2^{-i} = 422656 \text{ MAC}$$

instructions, which is approximately twice as much as in the case of spectrogram processing.

However, the algorithm can also work with FIR filters of order R = 1. In that case to process a signal the whole algorithm needs only

$$I = 2 \cdot 6656 \cdot (2) \cdot \sum_{i=0}^{8-1} 2^{-i} = 26416 \text{ MAC instructions,}$$

which is almost eight times less than in the case of spectrogram processing.

4 Conclusion

For low-order QMF filters the computation complexity of scalegram is several times smaller than that of spectrogram. It is therefore obvious that in applications that do not require a high resolution of multidimensional representation of signals it is, in view of the considerable reduction of time and computer demands, of advantage to use for the time-frequency representation of signals the detailed DWT coefficients determined by the fast pyramidal algorithm. This concerns particularly signal compression applications, which do not require a very high resolution of details in signal representation. Using the scalegram in signal representation is not suitable when separating signal from noise, for example by the method of thresholding multidimensional representations, where much depends on the resolution of details of multidimensional representation if the speech activity area of the signal is to be correctly distinguished from noise.

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