The Fastest Determination of Precise Magnitude and Phase of Harmonic Signal in Case of 50 Hz Power Line Disturbance

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Abstract: - The fast and precise vector determination is necessary for various applications, especially for an impedance measurement. This paper shows the way for the fastest determination of magnitude and phase of harmonic signal in one halt-period in the case of synchronous sampling. The second part of paper shows a procedure of the fastest rejection of the 50 Hz power line disturbance by the way of separation of a disturbance harmonic signal from the measured signal. It is shown, that this procedure requires only three halt-periods of measured signal. The presented method is substantially faster than any classical way of analog or digital signal filtering.

Key-Words: - Fastest measurement, precise vector measure, synchronous sampling, disturbance rejection

1 Introduction

We need to measure the precise vector value (magnitude and phase) in some tasks, especially for the fast and precise impedance measurement. Usually a way to precise measurement of vector value consists in various signal processing techniques as averaging, filtration etc. However these signal procedures require long time of evaluation and therefore they cannot be used if very fast measurements are needed.

The main problem consists in question of the minimum time in a case of precise measurement of magnitude and phase shift of harmonic signal. The new special method has to be developed in a case of the very high speed measurements of 120 Hz harmonic wave signal with 50 Hz disturbing signal, because any filtration of disturbing signal brings considerable extension of measuring time.

2 The fasted determination of magnitude and phase shift of harmonic signal

There are two possibilities in the process of determination of the magnitude and phase values, the synchronous and asynchronous sampling. The synchronous sampling is preferable because the asynchronous sampling method requires more measuring time in principle. Therefore the synchronous process was used for problem solution .

As the first step must be solved the question, what is the minimum time for precise measurement of magnitude and phase shift of a harmonic wave signal. The minimum sampling frequency is given by sampling theorem

$$f_s < 2f_m \ . \qquad (1)$$

If we consider the condition of periodicity of the sampled signal for synchronism and for exactness of Discreet Fourier Transformation (DFT), the sampling frequency has to be

$$f_s = nf_m \ , \qquad (2)$$

where the n is integer number 4 or higher. We will consider the minimum value n=4.

In this case the complex value of the measured vector $\hat{V}$ can be expressed by DFT

$$\hat{V}_1 = \frac{1}{2}\sum_{k=0}^{3} v(k)e^{jk} \ , \qquad (3)$$

where

$$V_1 = \sqrt{\left(\mathrm{Re}(\hat{V})\right)^2 + \left(\mathrm{Im}(\hat{V})\right)^2} \ , \qquad (4)$$

$$\varphi_1 = arctg\left(\mathrm{Re}(\hat{V}) / \mathrm{Im}(\hat{V})\right). \qquad (5)$$

The above mentioned considerations determine the minimum measuring time: one period T and four samples. Nevertheless we can use the symmetry of the harmonic function

$$\cos(\omega t + \varphi) = -\cos(\omega t + \varphi + \pi) \ . \qquad (6)$$

Therefore the sampled values are

$$s(k) = -s(k+2) \quad (7)$$

and the expression (2) of the measured vector $\hat{V}$ can be calculated from two samples

$$\hat{V}_1 = \sum_{k=0}^{1} v(k)e^{jk} = v(0) + jv(1) . \quad (8)$$

In this case the minimum measuring time takes half of period (T/2) with two samples.

3 Fast rejection of the harmonic disturbing signal

In any cases the harmonic disturbing signal is dominant. We consider the practical case when the impedance measurement signal with frequency 120 Hz is disturbed by 50 Hz power line signal. In this case the rejecting of disturbing signal by classical filters brings long time setting of the response. Therefore the numerical methods of 50 Hz signal rejecting have to be used.

The main idea of derived processing method is based on above mentioned symmetry of the measured 120 Hz signal. It can be describes by following equations. The measured signal can be expressed as

$$v_{120}(t) = V_{120}\cos(240\pi t + \varphi_{120}) \quad (9)$$

and the disturbing 50Hz signal as

$$v_{50}(t) = V_{50}\cos(100\pi t + \varphi_{50}) . \quad (10)$$

We can express the disturbed 120 Hz signal as sum of both signals

$$v_{\Sigma 1}(t) = v_{120}(t) + v_{50}(t) . \quad (11)$$

Is it not possible to reject simple and directly the disturbing 50 Hz signal, but we can separate this signal from $v_{\Sigma 1}(t)$ by rejecting of ideal measured 120 Hz signal $v_{\Sigma}(t)$. It is possible if we consider the symmetry of this signal (6). Therefore the resulting signal $v_{\Sigma 1}(t)$ has to be summed with the same signal with phase shift π. In this case we obtain new signal

$$\begin{aligned} v_{\Sigma+\pi}(t) &= v_{120}(t) + v_{50}(t) + v_{120}(t+\pi) + v_{50}(t+\varphi_\pi) = \\ &= v_{50}(t) + v_{50}(t+\varphi_\pi) . \end{aligned} \quad (12)$$

This is the pure disturbing 50 Hz signal but with different magnitude and phase. A simple elimination of the phase shift φ_π is possible if we use the same sum but with the opposite phase shift

$$\begin{aligned} v_{\Sigma-\pi}(t) &= v_{120}(t) + v_{50}(t) + v_{120}(t-\pi) + v_{50}(t-\varphi_\pi) = \\ &= v_{50}(t) + v_{50}(t-\varphi_\pi) . \end{aligned} \quad (13)$$

By summing of both shifted signal we obtain the pure disturbing 50 Hz signal with different magnitude but with the same phase as the original disturbing signal

$$v_{\Sigma 2}(t) = v_{\Sigma+\pi}(t) + v_{\Sigma-\pi}(t) . \quad (14)$$

The relations of the considered vectors are sketched in the vector diagram in Fig. 1. The phase shift α can be easily expressed for known frequencies. In the case signal frequencies 120 Hz and 50Hz is given as

$$a = \pi\frac{50}{120} = \pi / 2.4 = 75^o \ . \quad (15)$$

If we know the magnitude $V_{\Sigma 2}$ (14), the value of V_{50} can be derived (see Fig.1) from equations

$$V_{50} = \frac{V_{\Sigma+\alpha/2}}{2\cos(\alpha/2)} \ , \quad (16)$$

$$V_{\Sigma+\alpha/2} = \frac{V_{\Sigma 2}}{2\cos(\alpha/2)} \ , \quad (17)$$

$$V_{50} = \frac{V_{\Sigma 2}}{4(\cos(\alpha/2))^2} \ . \quad (18)$$

In our case of frequencies 120 Hz of measured signal and 50Hz of disturbing signal ($\alpha = 75^o$) we obtain correction coefficient k

$$V_{50}/V_{\Sigma 2} = k = 0.3971976 \ . \quad (19)$$

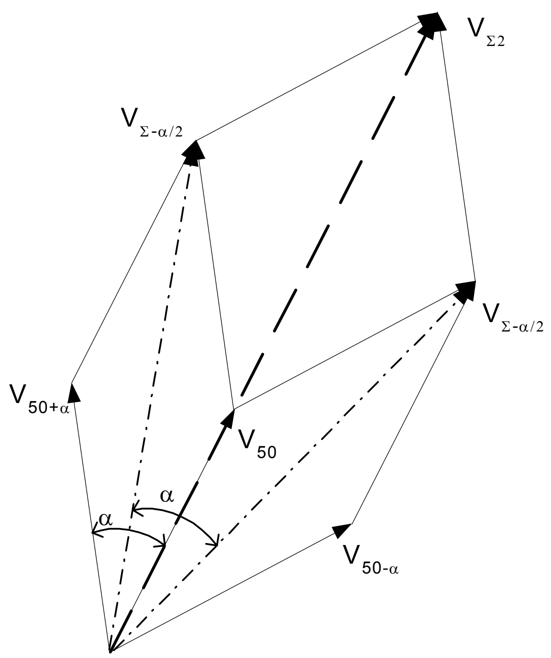

Fig. 1: The vector diagram: Summing of phase shifted disturbing vectors – eq. (10)-(14).

Graphic illustration of the equations (9) – (14) is shown in Fig. 2. The disturbing signal has the same magnitude for visual simplicity.

Tab. 1 shows the numeric results of practical measuring process. The time of sampling can be various and in this case The sample 0 corresponds to phase shift 30^o of measured signal. The extracted signal $kV_{\Sigma 2}$ corresponds precisely to original disturbing signal. Therefore it can be used for rejecting of disturbance from measured signal $V_{\Sigma 1}$, because

$$V_{120} = V_{\Sigma 1} - kV_{\Sigma 2} \quad . \qquad (19)$$

The two resulting calculated samples correspond to n=2 and n=3. By equations (3)-(5) can be calculated resulting magnitude and phase shift:

$$V_{120} = 1,\ \varphi_{120} = 60^o \quad . \qquad (20)$$

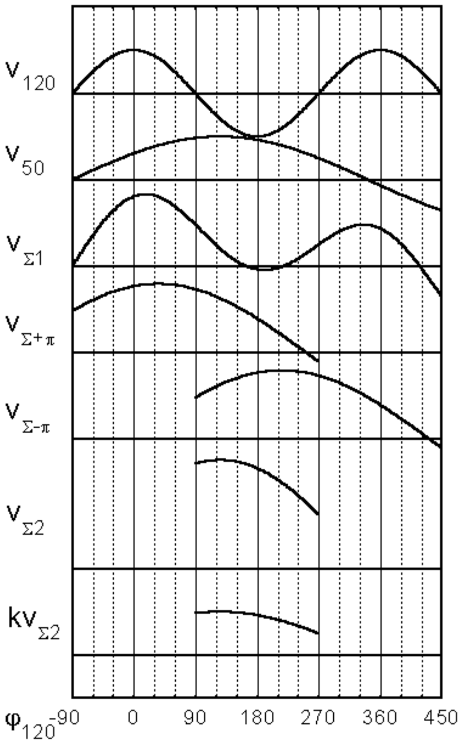

Fig. 2: Time (phase) dependencies of original signal $v_{120}(t)$, disturbing signal $v_{50}(t)$ and process of extracting of disturbing signal – eq. (10)-(16).

It is necessary to consider that calculated phase shift φ_{120} corresponds to zero time at n = 3. Therefore the actual phase of measuring process has to be corrected to further context of signal processing.

It is necessary to determine the minimum measuring time in the case of 50 Hz power line disturbance. As results from equations (12) and (13) show, it is necessary to use further two halt-period parts of signal to extracting of disturbance signal. Therefore the resulting required time of measurement takes three halt-period parts of signal what corresponds to 6 samples, as we can see on Tab. 1.

Tab. 1 Sampled values of signals from Fig. 1

n	0	1	2	3	4	5
φ_{120}	30^o	120^o	210^o	300^o	390^o	480^o
v_{120}	0.5	0.8660	**-0.5**	**-0.8660**	0.5	0.8860
v_{50}	0.2164	0.7660	**0.9990**	**0.8192**	0.3007	-0.3420
$v_{\Sigma 1}$	0.7164	1.6321	0.4990	-0.0469	0.8007	0.5240
$v_{\Sigma+\pi}$	1.2156	1.5852	1.2998	0.4771		
$v_{\Sigma-\pi}$			1.2155	1.5852	1.2998	0.4771
$v_{\Sigma 2}$			2.5152	2.0623		
$kv_{\Sigma 2}$			**0.9990**	**0.8192**		
$v_{\Sigma 1-}\ kv_{\Sigma 2}$			**-0,5**	**-0,8660**		

4 Conclusion

This paper prescribes the new way of fast and precise determination of the magnitude and phase shift of harmonic wave signal. The published signal processing method was successfully applied in area of high speed impedance measurements. By signal process development it was derived that the minimum time consumption of measuring process is one halt-period of signal with two samples in the case of synchronous sampling.

The second part of paper deals with a procedure of the fastest rejection method in the case of the 50 Hz power line signal disturbance by the way of very high speed separation of a disturbance harmonic signal from the measured signal. It is derived, that this procedure requires only three halt-periods of measured signal. The above derived method of signal processing requires substantially smaller time consumption than any classical way of analog or digital signal filtering.

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References:

[1] P. Bonanomi, Phase Angle Measurements with Synchronized Clocks. *IEEE Transactions on Power Apparatus and Systems,* Vol. PAS-100, No. 12, December 1981.

[2] A. G. Phadke et al., .Synchronized Sampling and Phasor Measurements for Relaying and Control,. *IEEE Transactions on Power Delivery,* Vol. 9, No. 1, January 1994.

[3] G. Benmouyal, E. O. Schweitzer, A. Guzmán, Synchronized Phasor Measurement In Protective Relays For Protection, Control, And Analysis Of Electric Power Systems. *29th Annual Western Protective Relay Conference,* Spokane, Washington October 22-24, 2002