Topological Analysis of Nonreciprocal Electrical Network with Help of Singular Elements

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Abstract: - At present time the windows environment and graphical output devices facilitate the widespread use of non-numerical, graphical and topological methods. In this paper a possibilities for implementation of the topological network analysis method with help of singular elements (nullators and norators) are described and the project of the graphical design environment based on generation of set of network multi-skeletons is presented. This method is well suited for the calculation of active analog systems transfer characteristics and for the symbolic analysis as well.

Key-Words: - topological analysis, electrical network, nullator, norator, singular element

1 Introduction

Evaluation of network functions with the help of topological formulas has been considered to be an attractive approach in modeling of electrical systems. The method is based on application of Kirchhoff theorem [2][3]. This theorem has been proofed for reciprocal networks, i.e. networks consisting of bilateral (regular) one-ports as resistors, capacitors or inductors. For the case of nonreciprocal active systems the two graph method was proposed [1]. An alternative solution has been shown [5], that two singular elements - nullator and norator - have to be added and concept of regular skeleton have to be formulated in the case of nonreciprocal active systems. The method with singular elements is advantageous namely in the cases when there is fewer nonreciprocal elements (as transistors or operation amplifiers) then reciprocal elements (as resistors, capacitors or inductors) in the system.

Network functions as immitances or transfer functions are equal to ratio of two network determinants. The generalized formula for evaluation of network determinants (values of network graphs) requires explicit listing of the all regular skeletons of multi-graph. There has been considerable interest in the design of efficient algorithms for skeletons (named also spanning tree) enumeration and a number of algorithms with varying efficiencies have been reported in the literature. In this paper we will use method of successive transformations of graphs with singular elements (nullators and norators).

2 Problem Formulation

Generally an electronic system is a complex network containing different components. At the beginning of the system modeling it is necessary to determine the state variables of the two kinds of equations:

1. Signal continuity and equilibrium equations in the ports of a system. These equations are not dependent on technology. In electronic systems theory the variables are port voltages and port currents, and the equations of continuity and equilibrium are known as Kirchhoff laws.
2. Individual models of components. This models depends on the applied technology.

Basic problem of an analog system modeling is computation of the signal at the output port in response to the excitation at the input port, i.e. computation of transfer function:

$$T = \left.\frac{\text{output signal}}{\text{input signal}}\right|_{\text{boundary condition}} \tag{1}$$

In the simple case of a lumped RLC network the set of components is a set of resistors, inductors and capacitors or generally a set of reciprocal one-ports with one parameter - resistivity or conductivity or inductivity or capacity. In such case a graph concept is useful aid for computation of the transfer function (1). In the case of the system with active and nonreciprocal components e.g. with operational amplifiers (OA), it is necessary to generalize the graph method. One useful procedure of generalization is stated in [5] and briefly described in [6]. In this method for modeling of nonreciprocal properties of active components the singular elements (norator and nullator) are exploited and generalized topological formulas will be stated.

2 Generalized Topological Formula

Graph

$$\Gamma(V_\Gamma, E_\Gamma, N_\Gamma)$$

where

V_Γ is set of n nodes, (2)

E_Γ is set of m edges,

N_Γ is set of m_0 nullator- norator pairs.

is a structure which consists of three sets $V_\Gamma, E_\Gamma, N_\Gamma$ and two relations R_E and R_N. Set of edges E_Γ is an association of regular edges E_r (impedances Y or admitances Z), nullator edges E_0 and norator edges E_∞ i.e.:

$$E_\Gamma = E_r \cup E_0 \cup E_\infty$$

where

E_r is set of regular edges,

E_0 is set of nullator edges,

E_∞ is set of norator edges.

Relation R_E: (edge-nodes relation)

$$\mathrm{R}_E = \{(\mathrm{v}^+, \mathrm{v}^-, e);\ \mathrm{v}^+, \mathrm{v}^- \in V \text{ for all } e \in E\}\ .$$

Relation R_N: (nullator-norator edge relation)

$$\mathrm{R}_N = ([E_0]_\Gamma : [E_\infty]_\Gamma)$$

where

$[E_0]_\Gamma$ is vektor of nullators of graph Γ,

$[E_\infty]_\Gamma$ is vektor of norators of graph Γ.

Generation of multi-skeletons is based on process of the successive transformations. Let two particular sub-graphs of graph Γ exists

1. $\Gamma_0(V_\Gamma, \{E_s \cup E_0\}, N_T)$,
2. $\Gamma_\infty(V_\Gamma, \{E_s \cup E_\infty\}, N_T)$. (3)

Where

$E_s \in E_r$,

$\mathrm{N}_T = ([E_0]_T : [E_\infty]_T)$

is relation norator-nulator edge after the transformations.

Graphs (3) have the following properties.

1. Graphs

$$\Gamma_0(V, \{E_s \cup E_0\}, N_T)$$
$$\Gamma_\infty(V, \{E_s \cup E_\infty\}, N_T)$$

are connected in all connected parts of the graph $\Gamma(V, E, N)$.

2. Graphs

$$\Gamma_0(V, \{E_s \cup E_0\}, N_T)$$
$$\Gamma_\infty(V, \{E_s \cup E_\infty\}, N_T)$$

have no loops.

The permutations of singular elements after transformations are defined:

$\pi_0 = \begin{pmatrix} [E_0]_\Gamma \\ [E_0]_T \end{pmatrix}$ is a permutation of nullators;

$\pi_\infty = \begin{pmatrix} [E_\infty]_\Gamma \\ [E_\infty]_T \end{pmatrix}$ is a permutation of norators.

We say that graph $\Gamma_s(V_\Gamma, \{E_s \cup E_0 \cup E_\infty\}, N_T)$ is *the regular skeleton* of the graph $\Gamma(V_\Gamma, E_\Gamma, N_\Gamma)$. Let $S(\Gamma)$ is the set of all regular skeletons.

Let all regular edges $e \in E_r$ have values

$$\mathrm{val}(e \in E_r) = \{y_e, z_e\}$$

where in general case y_e and $\mathrm{z_e}$ are complex operators. In the simple case of RCL network:

$$\mathrm{val}(e \in E_r) = \begin{cases} \{y_e = \mathrm{G} + \mathrm{pC}, \quad z_e = 1\} \\ \qquad \text{for admitances,} \\ \{\mathrm{z}_e = \mathrm{R} + \mathrm{pL}, \quad y_e = 1\} \\ \qquad \text{for impedances.} \end{cases}$$

Value of the graph $\Gamma(V_\Gamma, E_\Gamma, N_\Gamma)$ is defined as

$$\Delta = \sum_{\forall s \in S(\Gamma)} \mathrm{zn}(\pi_0) \cdot \mathrm{zn}(\pi_\infty)\ (\prod_{\forall e \in E_s} y_e \prod_{\forall e \in E_\Gamma - E_s} z_e) \qquad (4)$$

where function $\mathrm{zn}(\pi) = +1 \text{ or } -1$ is the sign of permutation π. Sign of permutation π is plus $(+1)$ if number of inversions of permutation is even and minus (-1) if number inversions of permutation is odd [4]. In applied process of transformations a sign is generated automatically as number of successive inversions.

As it is shown in [5] the formula for computation of the transfer function (1) in case of short-circuit (SC) or open circuit (OC) boundary conditions can be expressed in the form

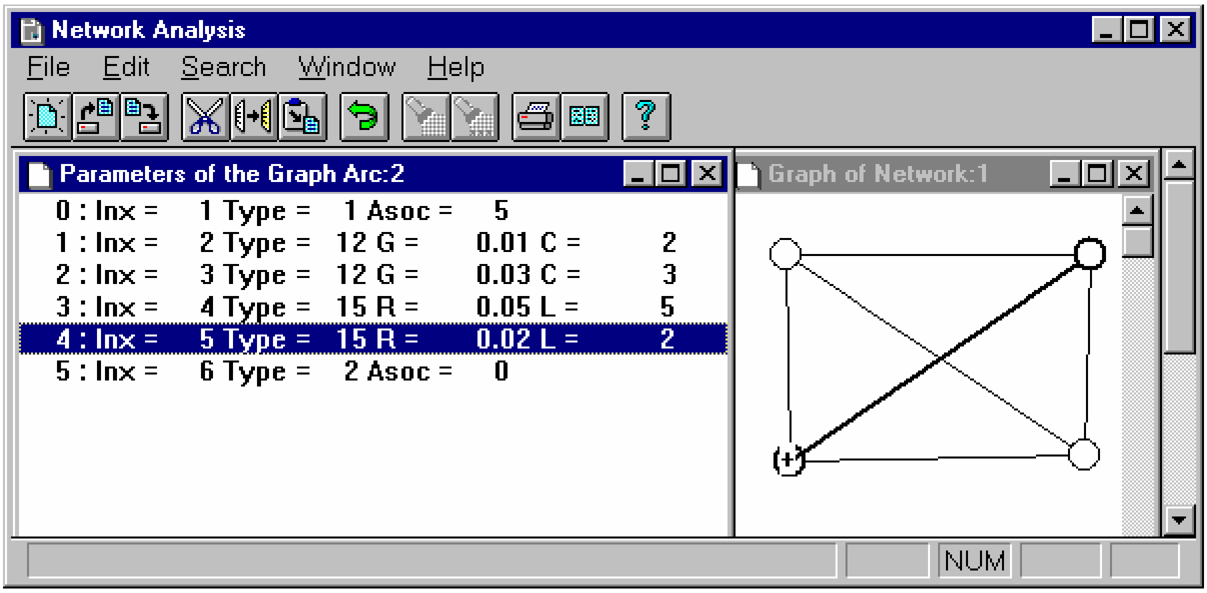
$$T = \frac{\Delta_{(\text{output : input})}}{\Delta_{\text{input, output}}} \qquad (5)$$

where
"input" = "boundary condition at the input port"
and
"output" = "boundary condition at the output port"
both are equal SC or OC.

"(output : input)" express that boundary condition at the output port is replaced by the nullator and boundary condition at the input is replaced by the norator.

3 Implementation of Algorithm for Evaluation of All Regular Skeletons of Graph $\Gamma(V, E, N)$

The program NTW for input of graph topology and graph parameters was realized. Two view windows are in Fig. 1. The output document format of the program NTW is used as input document format of program NTWTOOBV. This program is used for generation of all regular skeletons and evaluation of network determinants as sums of multi-skeletons. In the program is used efficient method of successive de-compositions and transformations of multi-graph. Result of program NTWTOOBV is list of signed multi-skeletons. An example of the program output is in Fig. 2.



(a) (b)

Fig. 1 – Two windows of program NTW (a) Parameter list view window. (b) Graph view window.

4 Conclusion

The theoretical foundation of topological methods is representation of a graph values as explicit function of values of graph edges. There are simple relations between the edges values of a topological model and the components parameters of a real system. The results of these methods are formulae, which express explicitly the dependencies of current and voltages on system parameters values. Such output is in really symbolic or in semi-symbolic form. From the symbolic expressions obtained in such a way, the frequency characteristics may be calculated with help of simple algorithms. By this method it is possible calculate not only short or open circuit parameters but also the scattering parameters [7] or models of devices with distributed parameters [8]. The benefits of suggested methods will be manifested especially in systems composed o subunits, which may be considered as separate subsystems.

There was one disadvantage of topological methods, which hindered the widespread application of those methods in the past; excessively large number of members in symbolic expressions. However, it is possible to overcome this flaw by substituting the values of single skeletons into value of multi-skeletons. These substitutions are based on results which have been developed in study [5] and briefly in the paper [6]. This method is used in the program systems presented in this paper.

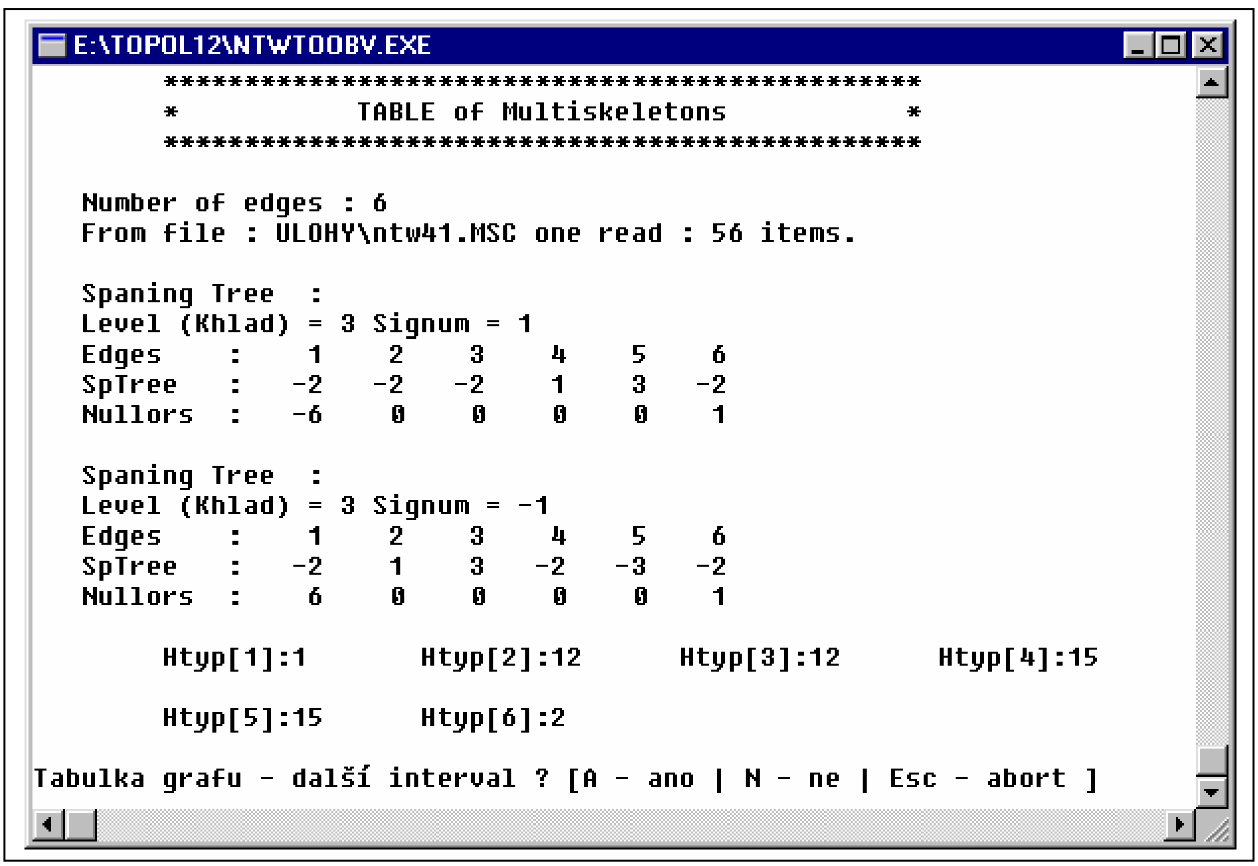


Fig. 2 – The final part of output window of program NTWTOOBV

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