The Optimum Kinematic Design of a Spatial Nine-Degree-of-Freedom Parallel Manipulator

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Abstract: - This paper considers the kinematic optimization of a parallel manipulator actuated in redundant form, using a numeric conditioning of the Jacobian matrix. The kinematic model proposed by Zanganeh and Angeles (1994) consists of nine prismatic in-parallel actuators, i.e., three actuators, called external legs, connect the moving platform directly to the base platform by spherical joints, and the six actuators, termed the upper internal and the lower internal legs, are coupled in pairs by three concentric spherical joints at the internal point. First, the end effector velocity problem is solved, and then the identification of a neutral configuration of manipulator is developed. Finally, the platform geometry parameters of the manipulator are formulated, and solved as an optimization problem.

Key-Words: - Parallel redundant manipulator, kinematic indices, optimitation design problem.

1 Introduction

The optimal design for parallel structures is an open and difficult question. As Merlet noted in [1], it is very important to design parallel structures in accordance with the task to which they are assigned, in particular for mechanisms where the performance is strictly dependent on the mechanical architecture and its dimensions. In the solution of this problem, the first difficulty is that most performance criteria considered for practical applications are *extremum* to functions that are configuration-dependent (stiffness, force in the actuator, dexterity, etc.). The second difficulty is that of using an appropriate design methodology to obtain an optimum architecture of the mechanism for the assigned task. Numerous studies have investigated this problem, and one alternative to the classic approach is that presented by Merlet; the so-called *parameters space* approach [1].

In this paper, the optimum design problem is considered for a new redundant parallel manipulator with nine degrees of freedom proposed by Zanganeh and Angeles in [2, 3] and shown in Fig. 1.

The model consists of nine prismatic in-parallel actuators l_i, r_i and q_i for $i = 1,2,3$, and $\mathbf{e}_i$, $\widetilde{\mathbf{e}}_i$ and $\hat{\mathbf{e}}_i$ for $i = 1, 2, 3$ are the unit vectors along $\overrightarrow{A_iB_i}$, $\overrightarrow{B_iO}$ and $\overrightarrow{A_iO}$, respectively . Moreover, the three actuators $\{l_i\}_1^3$, called *external legs*, connect the moving platform (denoted as MP) directly to the base platform (denoted as BP) by spherical joints at $\{A_i\}_1^3$ and $\{B_i\}_1^3$. The six actuators $\{q_i\}_1^3$ and $\{r_i\}_1^3$, termed the *upper internal* and the *lower internal* legs respectively, are coupled in pairs by three concentric spherical joints at point O.

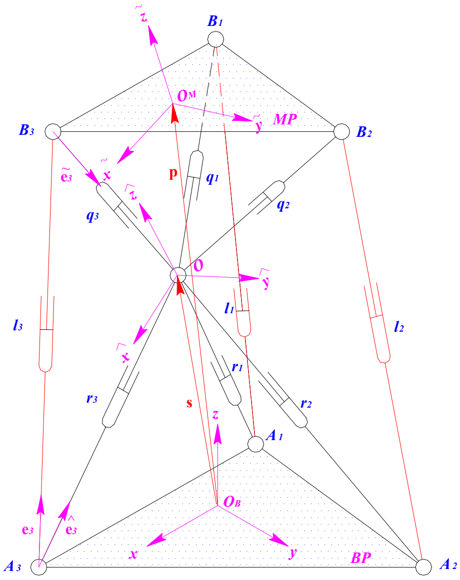

Fig. 1: The model of a redundant parallel manipulator

The full details of the kinematics model, including the direct and inverse position analyses, were derived in [2], the velocity and acceleration relations were studied in [3]. On the basis of the kinematic chain, it is possible to construct an entire class of prototypes which differ in the geometry of the platform and in the position of the joints but which fundamentally refer to the same structure. For example, the two prototypes shown in Fig. 2 and Fig. 3 differ in the position of the external and internal legs, which can be coupled or non-coupled, resulting in either a triangular or a hexagonal platform geometry.

Fig. 2: First prototype of the manipulator

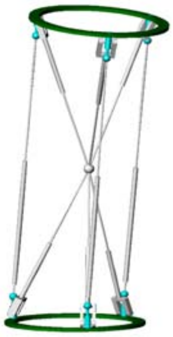

Fig. 3: Second prototype of the manipulator

This paper reports a study of the kinematic performance of the redundant parallel manipulator, considering the numeric *conditioning* of the Jacobian matrix. An appropriate design methodology is used to determine an optimum architecture in terms of kinematic performance for the assigned task.

2 Architecture design

The kinematic performance of a manipulator is closely related to the numerical stability of the Jacobian matrix and thus to its numerical condition. For this purpose, several indicators of the kinematic performance of manipulators (kinematic indices) in terms of the numerical condition of Jacobian matrices, have been introduced:

- Yoshikawa, 1985 introduced *manipulability*, based on the determinant of the product of the manipulator Jacobian by its transpose [4].
- Paul and Stevenson (1983) assessed the kinematic performance of spherical wrists (the absolute value of the Jacobian determinant) [5].

These performance indices, are configuration-dependent, i.e. depending on joint variables. But, it is well known that motion performance also depends on the manipulator architecture, and this dependence is notably greater in parallel than in serial manipulators because there are much larger differences in the architecture of the former than in that of the latter. Thus the choice of architecture is essential for good kinematic performance.

The kinostatic performance index used in this paper is the *condition number* of the Jacobian matrix, first applied to robotics by Salisbury e Craig [6]. The *condition number* calculated in our optimum design problem for the redundant parallel manipulator, is less than 3.9110, a value which allows the architecture to be considered close to the isotropy architecture condition.

3 The Conditioning Index

Singularities or an ill-conditioned Jacobian matrix **J** have to be avoided in solving the *twist* vector **t** from the relation,

$$\mathbf{J}\dot{\boldsymbol{\theta}} = \mathbf{t} \qquad (1)$$

where $\dot{\boldsymbol{\theta}}$ is the vector velocity of the actuated joints and $\mathbf{t} = [\boldsymbol{\omega}^T, \ \dot{\mathbf{p}}^T]^T$ with $\boldsymbol{\omega}$ and $\dot{\mathbf{p}}$ are the angular velocity and the vector velocity of the end-effector respectively. In fact, if **J** is singular or ill-conditioned, it is, respectively, impossible to determine **t** or to determine **t** accurately. The singularity or the ill-conditioning of the Jacobian matrix is a result of both the configuration and the architecture of the manipulator. Moreover, the architecture singularities cannot be avoided by trajectory planning or control, but can only be eliminated by design. Similarly, ill-conditioning resulting from the architecture also has to be eliminated by design. The degree of the influence of an architecture on the numerical conditioning of **J** is termed *architecture conditioning* and the literature contains several indicators to measure architecture conditioning. One of the classic indicators is a *condition number* of the *Jacobian* matrix.

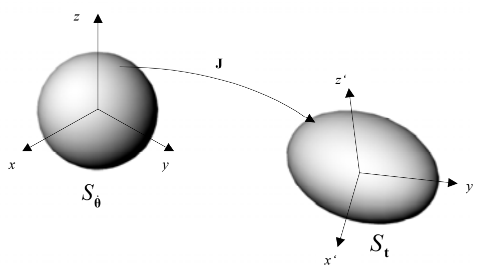

Fig. 4: Liner transformation of the unit sphere into an ellipsoid associated to **J**

The Jacobian matrix **J** transforms the unitary sphere defined in joint velocity space $S_{\dot{\boldsymbol{\theta}}}$ into an ellipsoid, rotated or mirrored, in the Cartesian velocity space $S_{\mathbf{t}}$ as indicated in Fig. 4. If the Jacobian matrix **J** has all homogeneous elements, the *condition number* *k*(**J**) can be calculated as the ratio between the largest and the smallest of the single values,

$$k(\mathbf{J}) = \frac{\sigma_M}{\sigma_m} \qquad (2)$$

This scaling can assume a value between one and infinity; the unitary minimum occurs when the singular values are all the same, in this case the transformation is from one sphere to another of different radius and the matrix is said to be isotropic so that the manipulator is defined isotropic. Conversely, in singular matrices the smallest singular value equals zero and thus the *condition number* becomes infinite. This index of kinostatic performance can be seen as a parameter indicating the distortion of the sphere in joint velocity space; the greater this distortion, i.e. the greater the *condition number*, the worse the *conditioning* of the Jacobian matrix. It should be noted that the *condition number* is an indicator of the amplification of the computational errors in the solution of the linear equation systems associated with the matrix itself, Forsythe and Moler [7], Dahlquist and Björck [8]. The *condition number* of the Jacobian matrix, or rather its reciprocal, between zero and 1, is therefore an index of kinematic performance for manipulators which allows a better computer calculation.

As noted by Lipkin and Duffy [9], the *condition number* of **J** becomes deprived of physical significance when the Jacobian matrix is dimensionally heterogeneous: ordering the single values of different dimensions from the largest to the smallest would result in the *condition number* varying with the unit of measurement of the dimensions of the manipulator. This incongruity can be resolved by adopting a different kinematic model proposed by Gosselin and Angeles [10]. Here the problem is overcome by defining a *characteristic link length* l_c which divides the elements of the Jacobian matrix by length sizes. Different characteristic lengths are proposed in the literature: unitary length, mean length, natural length, each appropriate for a particular individual objective.

Using a characteristic link length l, the original Jacobian matrix is homogenized as,

$$\mathbf{J}_h = \mathbf{J}diag(\frac{1}{l},\frac{1}{l},\frac{1}{l},1,1,1) \tag{3}$$

Now the indicator of numerical conditioning is defined as,

$$k(\mathbf{J}) = \left\|\mathbf{J}_h\right\|\left\|\mathbf{J}_h^{-1}\right\| \tag{4}$$

In [11], Ma gave many examples of architectures in which there are singularities and ill-conditioning which can then cause problems in manipulator performance.

The architecture design with the smaller condition number of **J** has the better kinematic performance. The architecture has the best performance when the condition number of its Jacobian matrix is equal to 1 and such architecture is called *isotropic architecture*, i.e.,

$$\mathbf{J}_h^T\mathbf{J}_h = \sigma\mathbf{1} \tag{5}$$

where σ is a positive scalar and **1** is the 6×6 identity matrix. Clearly, it is impossible to achieve an isotropic architecture at every configuration because the Jacobian matrix is also configuration dependent. Hence an architecture is considered isotropic as long as the corresponding Jacobian matrix can become isotropic in one configuration.

4 The Optimum Kinematic design

Architecture conditioning is measured by a kinostatic performance index, i.e. the *condition number* of the **J** matrix, introduced above. In the problem considered here, the index $k(\mathbf{J})$ depending on the geometry parameters of the two platforms must be calculated and then optimised by means of design variables.

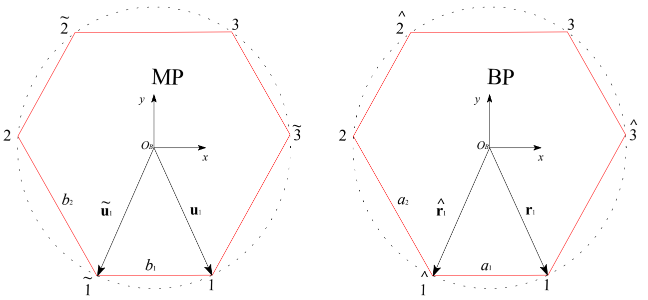

Fig. 5: Geometry and parameters of manipulator platforms

The geometry of the base and moving platforms, shown in Fig. 5, are, for simplicity, hexagons with three equal long sides alternating with three equal short sides. The architecture of the platform manipulator is therefore fully defined by the parameters a_1, a_2, b_1, b_2.

The problem of optimum design is considered for a particular platform configuration, termed the *neutral configuration*, i.e. the two mass centers of the two platform and the spherical joints at point O, are aligned along the z axis. The configuration of the moving platform is defined by a rotation matrix **Q** of a frame $O_M\_\tilde{x}\,\tilde{y}\,\tilde{z}$,fixed into MP, respect to inerzial frame $O_B\_x\ y\ z$, and a position vector **p** of the center mass of the MP. The internal point O is individuated by **s** vector as showed in Fig. 1. The *neutral configuration* is defined as,

$$\mathbf{p} = \begin{bmatrix} 0 \\ 0 \\ z_p \end{bmatrix}; \qquad \mathbf{s} = \begin{bmatrix} 0 \\ 0 \\ z_s \end{bmatrix}; \qquad \mathbf{Q} = \mathbf{1} \tag{6}$$

where **1** is a 3×3 identity matrix and $z_p > z_s$. In this particular configuration, the moving platform is not oriented with respect to the base. Displacements are along the z axis and since z_p varies, the configuration for a given manipulator is not unique. The parameters z_s and z_p are also design variables in the optimization problem.

The optimization design problem consists of finding a configuration ($\mathbf{Q}$, $\mathbf{p}$, $\mathbf{s}$) and an architecture of the manipulator such that the index $k(\mathbf{J})$ is minimum. In mathematical formulation means,

$$\frac{\partial k}{\partial x_p} = \frac{\partial k}{\partial y_p} = \frac{\partial k}{\partial x_s} = \frac{\partial k}{\partial y_s} = \frac{\partial k}{\partial \theta_x} = \frac{\partial k}{\partial \theta_y} = \frac{\partial k}{\partial \theta_z} = 0 \quad (7)$$

The neutral configuration is the most stable local configuration for the condition number. The configuration of the manipulator can be expressed in terms of nine variables, i.e. the three rotation angles θ_x, θ_y, θ_z, about the x, y, and z axes, respectively, of the inertial frame of the BP, and the Cartesian coordinates of the points $\mathbf{p}$ and $\mathbf{s}$. Therefore, indicating the geometric parameters of BP and MP by a_1, a_2, b_1, b_2, respectively, it is possible to verify that the condition number k is dependent on nine variables that achieve the neutral configuration of the manipulator. Using eq.(2), the Fig.6 shows the variation of $k(\mathbf{J})$ from a neutral configuration with respect to four, i.e., of the nine variables reported above.

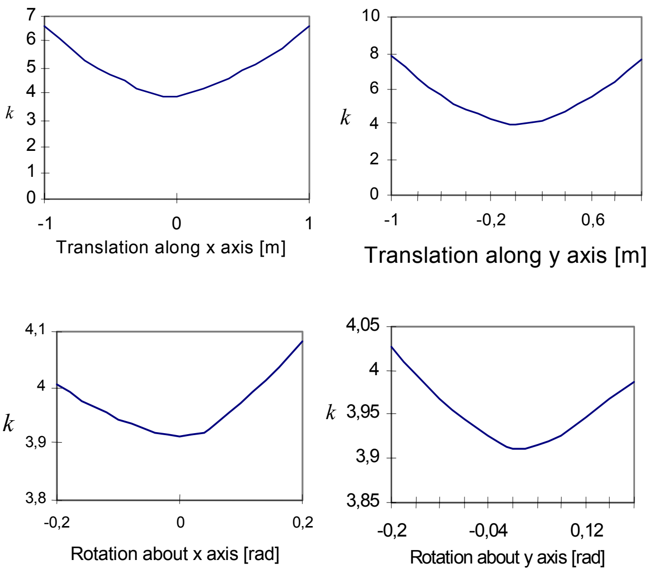

Fig. 6: Condition number k vs. displacement

The optimum design problem, with design a_1, a_2, b_1, b_2, z_p, z_s, can now be mathematically formulated as: determine the minimum of the objective function $k(\mathbf{J})$,

$$\min_{(a_1, a_2, b_1, b_2, z_p, z_s)} k(\mathbf{J}) \quad (8)$$

subject to the following constraints,

$$a_1 \geq 0 \quad a_2 \geq 0 \quad b_1 \geq 0; \quad b_2 \geq 0 \quad (9a)$$

$$z_p > 0 \quad z_s > 0 \quad z_p > z_s \quad (9b)$$

The four constraints, eq.(9a), ensure a non-negative geometrical dimension for the platform; the first two constraints of eq.(9b) relating to the points $\mathbf{p}$ and $\mathbf{s}$, are to avoid the singular configuration where both plates are coincident. The third condition of eq.(9b) avoids the point of confluence of the internal articulations O lying on one of the bases.

To derive the objective function, the *Jacobian* matrix of the manipulator must be derived in terms of the design variable. The geometry of platforms is shown in Fig. 5.

The kinematic equations of the parallel manipulator are derived in [3] and reported below,

$$\mathbf{p} = \mathbf{s} + \mathbf{v} \quad (10)$$

$$\mathbf{p} + \mathbf{u}_i - l_i \mathbf{e}_i - \mathbf{r}_i = \mathbf{0} \quad \text{for } i=1, 2, 3 \quad (11)$$

$$\mathbf{s} - r_i \hat{\mathbf{e}}_i - \hat{\mathbf{r}}_i = \mathbf{0} \quad \text{for } i=1, 2, 3 \quad (12)$$

$$\mathbf{v} + \tilde{\mathbf{u}}_i + q_i \tilde{\mathbf{e}}_i = \mathbf{0} \quad \text{for } i=1, 2, 3 \quad (13)$$

These vectors are expressed referring to the inertial frame associated with the base platform. Further, differentiating eqs.(10 & 13) with respect to time leads to the relations,

$$\dot{\mathbf{p}} = \dot{\mathbf{s}} + \dot{\mathbf{v}} \quad (14)$$

$$\dot{\mathbf{p}} + \boldsymbol{\omega} \times \mathbf{u}_i - \dot{l}_i \mathbf{e}_i - l_i \dot{\mathbf{e}}_i = \mathbf{0} \quad \text{for } i=1, 2, 3 \quad (15)$$

$$\dot{\mathbf{s}} - \dot{r}_i \hat{\mathbf{e}}_i - r_i \dot{\hat{\mathbf{e}}}_i = \mathbf{0} \quad \text{for } i=1, 2, 3 \quad (16)$$

$$\dot{\mathbf{v}} + \boldsymbol{\omega} \times \tilde{\mathbf{u}}_i + \dot{q}_i \tilde{\mathbf{e}}_i + q_i \dot{\tilde{\mathbf{e}}}_i = \mathbf{0} \quad \text{for } i=1, 2, 3 \quad (17)$$

and projecting along the unit vectors associated with actuated legs, the following scalar equations are obtained,

$$\mathbf{e}_i^T \dot{\mathbf{p}} + \mathbf{e}_i^T \boldsymbol{\omega} \times \mathbf{u}_i - \dot{l}_i \mathbf{e}_i^T \mathbf{e}_i - l_i \mathbf{e}_i^T \dot{\mathbf{e}}_i = 0 \quad (18)$$

$$\hat{\mathbf{e}}_i^T \dot{\mathbf{s}} - \dot{r}_i \hat{\mathbf{e}}_i^T \hat{\mathbf{e}}_i - r_i \hat{\mathbf{e}}_i^T \dot{\hat{\mathbf{e}}}_i = 0 \quad (19)$$

$$\tilde{\mathbf{e}}_i^T \dot{\mathbf{v}} + \tilde{\mathbf{e}}_i^T \boldsymbol{\omega} \times \tilde{\mathbf{u}}_i + \dot{q}_i \tilde{\mathbf{e}}_i^T \tilde{\mathbf{e}}_i + q_i \tilde{\mathbf{e}}_i^T \dot{\tilde{\mathbf{e}}}_i = 0 \quad (20)$$

Simplifying eqs.(18-20) gives,

$$\mathbf{e}_i^T \dot{\mathbf{p}} + \mathbf{e}_i^T \boldsymbol{\omega} \times \mathbf{u}_i - \dot{l}_i \mathbf{e}_i^T \mathbf{e}_i = 0 \quad (21)$$

$$\hat{\mathbf{e}}_i^T \dot{\mathbf{s}} - \dot{r}_i \hat{\mathbf{e}}_i^T \hat{\mathbf{e}}_i = 0 \quad (22)$$

$$\tilde{\mathbf{e}}_i^T \dot{\mathbf{v}} + \tilde{\mathbf{e}}_i^T \boldsymbol{\omega} \times \tilde{\mathbf{u}}_i + \dot{q}_i \tilde{\mathbf{e}}_i^T \tilde{\mathbf{e}}_i = 0 \quad (23)$$

and also

$$\dot{l}_i = (\mathbf{u}_i \times \mathbf{e}_i)^T \boldsymbol{\omega} + \mathbf{e}_i^T \dot{\mathbf{p}} \quad (24)$$

$$\dot{r}_i = \hat{\mathbf{e}}_i^T \dot{\mathbf{s}} \quad (25)$$

$$\dot{q}_i = -\tilde{\mathbf{e}}_i^T \dot{\mathbf{v}} - (\tilde{\mathbf{u}}_i \times \tilde{\mathbf{e}}_i)^T \boldsymbol{\omega} \quad (26)$$

These relations can be rewritten in the following way,

$$\dot{l} = \mathbf{At} \tag{27}$$

$$\dot{r} = \mathbf{B}\dot{\mathbf{s}} \tag{28}$$

$$\dot{q} = -\mathbf{C}\dot{\mathbf{v}} - \mathbf{D}\boldsymbol{\omega} \tag{29}$$

where

$$\mathbf{A} \equiv \begin{bmatrix} \frac{(\mathbf{u}_1 \times \mathbf{e}_1)}{l_c} & \frac{(\mathbf{u}_2 \times \mathbf{e}_2)}{l_c} & \frac{(\mathbf{u}_3 \times \mathbf{e}_3)}{l_c} \\ \mathbf{e}_1 & \mathbf{e}_2 & \mathbf{e}_3 \end{bmatrix}^T \tag{30}$$

$$\mathbf{B} \equiv [\hat{\mathbf{e}}_1 \quad \hat{\mathbf{e}}_2 \quad \hat{\mathbf{e}}_3]^T \tag{31}$$

$$\mathbf{C} \equiv [\tilde{\mathbf{e}}_1 \quad \tilde{\mathbf{e}}_2 \quad \tilde{\mathbf{e}}_3]^T \tag{32}$$

$$\mathbf{D} \equiv \begin{bmatrix} \frac{(\tilde{\mathbf{u}}_1 \times \tilde{\mathbf{e}}_1)}{l_c} & \frac{(\tilde{\mathbf{u}}_2 \times \tilde{\mathbf{e}}_2)}{l_c} & \frac{(\tilde{\mathbf{u}}_3 \times \tilde{\mathbf{e}}_3)}{l_c} \end{bmatrix}^T \tag{33}$$

Using the characteristic length l_c, the matrices **A** and **D**, indicated in eq.(30) and eq.(33) respectively are written in homogeneous form.

Rewriting eqs.(27-29) in compact form and in homogeneous form gives,

$$\mathbf{Ft} = \mathbf{G}\dot{\mathbf{u}} \tag{34}$$

where

$$\mathbf{F} \equiv \begin{bmatrix} \mathbf{A} \\ \mathbf{E} \end{bmatrix}; \quad \mathbf{G} \equiv \begin{bmatrix} \mathbf{1} & \mathbf{O} & \mathbf{O} \\ \mathbf{O} & -\mathbf{1} & \mathbf{CB}^{-1} \end{bmatrix}; \tag{35a}$$

$$\mathbf{E} = [\mathbf{D} \;\; \mathbf{C}]; \quad \dot{\mathbf{u}} \equiv \begin{bmatrix} \dot{l} \\ \dot{q} \\ \dot{r} \end{bmatrix} \tag{35b}$$

As determined by Zanganeh and Angeles [2], and adopting the notation introduced in that paper, the equation relating the twist of the MP to the actuator velocities, in the form of the forward velocity problem, is,

$$\mathbf{t} = \mathbf{F}^{-1}\mathbf{G}\dot{\mathbf{u}} = \mathbf{J}'\dot{\mathbf{u}} \tag{36}$$

In this form, **J'** is the Jacobian matrix associated with the forward velocity problem, and dimensionally homogenized by a *characteristic link length* l_c.

Consequently the optimization problem can be formulated using the Jacobian matrix obtained by either the inverse or the forward problem. The result is a design problem as close as possible to isotropy architecture. A good Jacobian conditioning will have an effect on the numerical solution of the IKP, rendering the kinematic model of the manipulator stable.

Based on the assumption that the manipulator is in a neutral configuration, the joint position vectors with respect to the base frame shown in Fig. 5, are expressed as,

$$\mathbf{r}_1 = \begin{bmatrix} \frac{1}{2}a_1 \\ -\frac{\sqrt{3}}{6}(a_1 + 2a_2) \\ 0 \end{bmatrix} \quad \mathbf{r}_2 = \begin{bmatrix} -\frac{1}{2}(a_1 + a_2) \\ -\frac{\sqrt{3}}{6}(a_1 - a_2) \\ 0 \end{bmatrix} \quad \mathbf{r}_3 = \begin{bmatrix} \frac{1}{2}a_2 \\ \frac{\sqrt{3}}{6}(2a_1 + a_2) \\ 0 \end{bmatrix} \tag{37a}$$

$$\hat{\mathbf{r}}_1 = \begin{bmatrix} -\frac{1}{2}a_1 \\ -\frac{\sqrt{3}}{6}(a_1 + 2a_2) \\ 0 \end{bmatrix} \quad \hat{\mathbf{r}}_2 = \begin{bmatrix} -\frac{1}{2}a_2 \\ \frac{\sqrt{3}}{6}(2a_1 + a_2) \\ 0 \end{bmatrix} \quad \hat{\mathbf{r}}_3 = \begin{bmatrix} \frac{1}{2}(a_1 + a_2) \\ -\frac{\sqrt{3}}{6}(a_1 - a_2) \\ 0 \end{bmatrix} \tag{37b}$$

$$\mathbf{u}_1 = \begin{bmatrix} \frac{1}{2}b_1 \\ -\frac{\sqrt{3}}{6}(b_1 + 2b_2) \\ 0 \end{bmatrix} \quad \mathbf{u}_2 = \begin{bmatrix} -\frac{1}{2}(b_1 + b_2) \\ -\frac{\sqrt{3}}{6}(b_1 - b_2) \\ 0 \end{bmatrix} \quad \mathbf{u}_3 = \begin{bmatrix} \frac{1}{2}b_2 \\ \frac{\sqrt{3}}{6}(2b_1 + b_2) \\ 0 \end{bmatrix} \tag{38a}$$

$$\tilde{\mathbf{u}}_1 = \begin{bmatrix} -\frac{1}{2}b_1 \\ -\frac{\sqrt{3}}{6}(b_1 + 2b_2) \\ 0 \end{bmatrix} \quad \tilde{\mathbf{u}}_2 = \begin{bmatrix} -\frac{1}{2}b_2 \\ \frac{\sqrt{3}}{6}(2b_1 + b_2) \\ 0 \end{bmatrix} \quad \tilde{\mathbf{u}}_3 = \begin{bmatrix} \frac{1}{2}(b_1 + b_2) \\ -\frac{\sqrt{3}}{6}(b_1 - b_2) \\ 0 \end{bmatrix} \tag{38b}$$

Examining the condition number curves shown in Fig. 6 for a generic trajectory, it is clear that the minimum condition corresponds to the configuration where both internal point O and the origin of the coordinate frame associated with the moving platform, are both belong to the z axis of the inertial frame associated with the base platform.

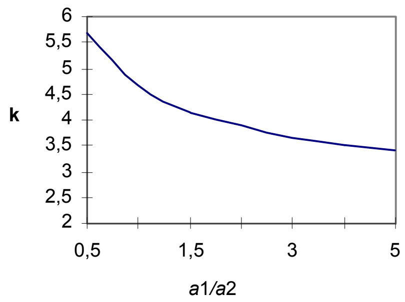

Fig. 7: Variation of k vs. the ratio of a_1/a_2

It has also been established that the objective function decreases on increasing the ratio between the dimensions of the sides of the two platforms (a_1/a_2, and b_1/ b_2) (leaving the other parameters free to vary) as shown in Fig. 7.

5 Numerical optimization

The optimization problem formulated in Section 4 was solved using software based on the technique of *Sequential Quadratic Programming* (SQP). This software is efficient in solving optimum constrain problems.

The six solutions of design variables are:

$$a_1 = 0.0000\ l_c \text{ m} \tag{39a}$$

$$b_1 = 5.8124\ l_c \text{ m} \tag{39b}$$

$$z_p = 3.9828\ l_c \text{ m} \tag{39c}$$

and the corresponding value of the objective function is,

$$k = 3.9110 \tag{40}$$

To obtain a practical dimension for the values found in (15), assuming $l_c = 0.5$, the variables become,

$$a_1 = 0 \text{ m} \quad b_1 = 2.9 \text{ m} \tag{41}$$

The results obtained clearly indicate that the best prototype, in terms of elevated kinematic performance, is that in which the universal joints of the internal legs are united with the joints of the external legs, as shown in Fig. 2. In this way a manipulator with triangular platforms is produced.

It is important to note that parallel platform manipulators with six prismatic joints, widely used for flight simulation cabins, have hexagonal bases (the fixed base twice the size of the mobile base, i.e., $2a_2$=b_1) with one side much shorter than the other. This form is close to the triangular platform architecture which is optimal from the kinematic point of view.

4 Conclusion

This paper presents the kinematic optimization of a redundant parallel manipulator using the *condition number*. The results obtained show that the architecture where the points of attachment of the external legs are rotated with respect to the upper internal legs on the moving platform corresponds to the best kinematic performance. This design solution is very important in preventing kinematic singularities. Moreover, from analyses of condition number curves, there was clear persistence of isotropy configurations in the trajectories examined. Finally, the neutral configuration corresponding to the condition number minimum was determined for given geometric manipulator parameters.

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