Generalized model of mode competition under external optical feedback in semiconductor lasers

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Abstract: - We present an innovative model of mode competition in semiconductor lasers subjected to an arbitrary amount of external optical feedback. The model introduces the optical feedback as time-delay of mode intensities in the external cavity and formulates time-delay multimode rate equations. The mode competition is described by taking into account both the symmetric and asymmetric cross-suppressions of modal gain. The model is applied to depict in details influence of optical feedback on the mode competition-induced effects, such as mode hopping, chaos and pulsation, in mode dynamics and laser operation.

Key-Words: - Chaos, mode competition, multimode, nonlinear gain, optical feedback, pulsation, semiconductor laser, single mode.

1 Introduction

Phenomena of competition among longitudinal modes in semiconductor lasers are origins of many nonlinear effects, such as mode hopping and mode jittering [1]-[3]. Under optimum design of laser structure these phenomena can also results in stable single or multimode oscillations [4]. The competition phenomena come from coupling among modes due to cross-suppression of the gain of each mode by the other modes. The cross-suppression effects of gain are either symmetric or asymmetric with respect to the central mode of the gain spectrum. The latter type depends on the mode wavelength spacing, cavity length and linewidth enhancement factor [5], and is pronounced in long-wavelength InGaAsP lasers [1]. On the other hand, in most applications of semiconductor lasers, such as fiber-optic communication systems, the device is subjected to an amount of external optical feedback (OFB) which may stabilize or degrade device performance depending on the OFB parameters [6]-[9]. Therefore OFB is one of the most studied problems in laser dynamics, but yet it has not been fully understood. Furthermore, inadequate attention has been paid to trace the influence of OFB on the mode competition phenomena and the associated nonlinear effects.

In this paper, we present an innovative model to the mode competition under an arbitrary amount of OFB. The model is generalization of the single-mode model of laser operation under OFB recently proposed by the authors in [8] to the multimode case. The model counts OFB as time delay of the cavity-mode intensities due to round trips in the external cavity formed between the laser front facet and an external reflector. The model then modifies the laser rate equations of cavity modes into time delay ones. The mode competition is described basing on a solid theoretical model that takes into account both symmetric and asymmetric cross-suppressions of modal gain [3],[5]. We depict in details influence of OFB on the mode competition-induced effects in mode dynamics and laser operation. The results indicate that even when the solitary laser oscillates dominantly in a single mode, the operation changes to unstable multimode when OFB induces chaotic mode dynamics.

In the next section, we introduce the theoretical multimode model of laser operation under arbitrary amount of OFB. In Section 3, results of numerical simulation of laser dynamics and OFB are presented. The conclusions appear in Section 4.

2 Theoretical Model

The present model of semiconductor lasers under OFB is schematically shown in Fig. 1. Dynamics of the mode competition under OFB are described by the time-delay rate equations of the photon number $S_p(t)$ and optical phase $\theta_p(t)$ of longitudinal modes and the rate equation of the injected electron number $N(t)$:

$$\frac{dS_p}{dt} = \left\{ G_p - G_{th0} + \frac{c}{n_D L_D} \ln\left|U_p\right| \right\} S_p + \frac{a\xi N/V}{\left[2\left(\lambda_p - \lambda_0\right)/\delta\lambda\right]^2 + 1} \quad , p = 0, \pm1, \pm2, ... \tag{1}$$

$$\frac{d\theta_p}{dt} = \frac{\alpha a \xi}{2V}\left[N - N_g\right] - \frac{c}{n_D L_D}\varphi_p \tag{2}$$

$$\frac{dN}{dt} = -\sum_p A_p S_p - \frac{N}{\tau_s} + \frac{I}{e} \tag{3}$$

where G_p is the optical gain of mode p which is defined to take into account the self-suppression induced by the same mode and both the symmetric and asymmetric gain suppressions induced by the other modes $q \neq p$,

$$G_p = A_p - BS_p - \sum_p \left[D_{p(q)} - H_{p(q)}\right] S_q$$

A_p is the linear gain, B is the coefficient of self-suppression, and $D_{p(q)}$ and $H_{p(q)}$ are coefficients of symmetric and asymmetric gain suppressions due to modes $q \neq p$, respectively. These coefficients are given by [3],[5]:

$$A_p = \frac{a\xi}{V}\left[N - N_g - bV\left(\lambda_p - \lambda_0\right)^2\right] \tag{4}$$

$$B = B_c\left(N - N_s\right) \tag{5}$$

$$D_{p(q)} = \frac{4}{3} \frac{\mathrm{B}}{\left(2\pi c \tau_{in}/\lambda_p^2\right)^2 \left(\lambda_p - \lambda_0\right)^2 + 1} \tag{6}$$

$$H_{p(q)} = \frac{3}{8\pi}\left(\frac{a\xi}{V}\right)^2 \left(N - N_g\right) \frac{\alpha \lambda_p^2}{\lambda_q - \lambda_p} \tag{7}$$

The last term in Eq. (4) represents inclusion of spontaneous emission into mode p. OFB is described as time delay of the cavity-mode intensities due to round trips in the external cavity; it is represented by the functions Up,

$$U_p = 1 - \sum_{m=1} \left\{ \left(K_{ex}\right)^{\mathrm{m}} \left[\left(1 - R_f\right)/R_f\right]^{m\text{-}1} \times \exp\left\{-jm\psi_p\right\} \sqrt{S_p(t-\tau)/S_p(t)} \times \exp\left\{ j\left[\theta_p(t-\tau) - \theta_p(t)\right]\right\}\right\} = \left|U_p\right| e^{-j\varphi_p} \tag{8}$$

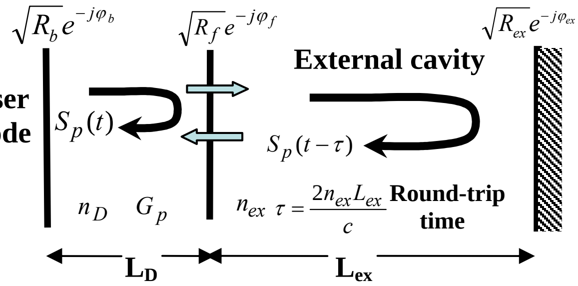

Fig. 1. Model of laser diode under OFB.

where ψ_p is a phase term determining the phase difference between the optical feedback and the field of mode p when reflected back into the laser cavity at the front facet. m is an index of round trips. K_{ex} determines the amount of OFB and is given by the ratio of the external reflector reflectivity R_{ex} to that of the front facet R_f and the coupling ratio η of OFB to the laser cavity:

$$K_{ex} = \left(1 - R_f\right)\sqrt{\frac{\eta R_g}{R_f}} \tag{9}$$

3 Simulation Results and Discussion

The rate equations (1)-(3) are solved numerically by the fourth-order Runge-Kutta method using a time step as small as 5 ps. Eleven longitudinal modes are considered. Mode p=0 is assumed to center the dispersion curve of gain. Modes with positive index, p>0, are assumed to lie on the long-wavelength side, while those on the shorter side are characterized with negative indices, p<0. The laser output spectra are calculated by averaging the modal photon numbers $S_p(t)$ over a time period of 2-4 μs during which the laser dynamics are stabilized. Definition and typical numerical values of the parameters in the above equations for Fabry-Perot AlGaAs lasers are as follows. a=2.75x10^{-12}s^{-1} is a tangential coefficient of gain. ξ=0.2 is field confinement factor. V=150μm^{-3}, L_D=300μm and n_D=3.59 are volume, length and refractive index of the laser cavity. L_{ex}=5cm and n_{ex}=1 are length and refractive index of the external cavity, respectively. τ_s=2.79ns and τ_{in}=0.1ps are interband and intraband relaxation times. b=2.83x10^{19}m^{-3}Å^{-2} is width of the linear gain dispersion curve. α=2 is linewidth enhancement factor. B_c=6.2x10^{-5}s^{-1} is coefficient of gain suppression. G_{th0}=2.82x10^{11}s^{-1} is threshold gain in the solitary laser. $\delta\lambda$=23nm is the half width of the spontaneous emission. The present calculations are made for a single round trip, and the phase term ψ_p is set zero. These parameters result in continuous

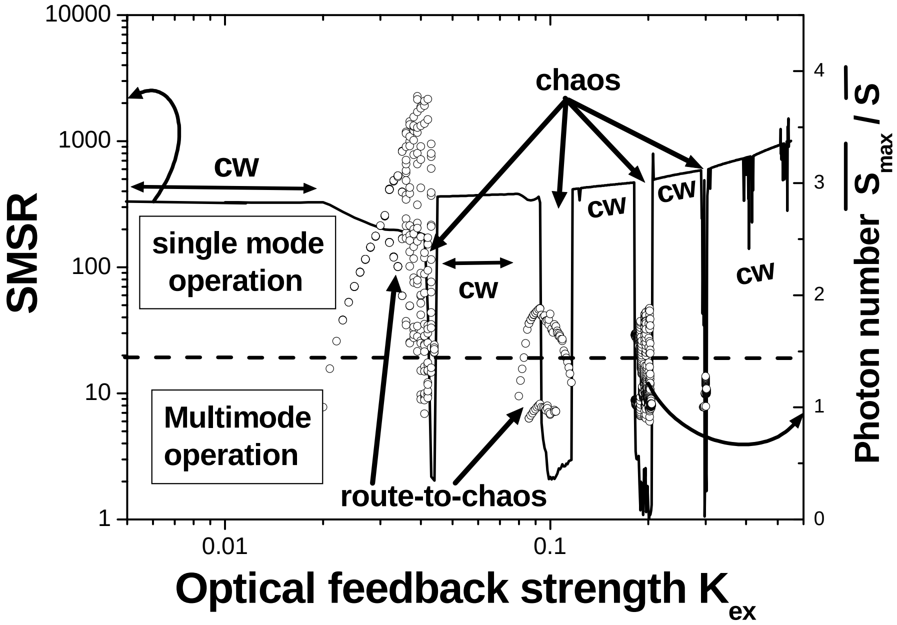

Fig. 2. Variation of SMSR with OFB strength K_{ex} and the bifurcation diagram (right-hand axis).

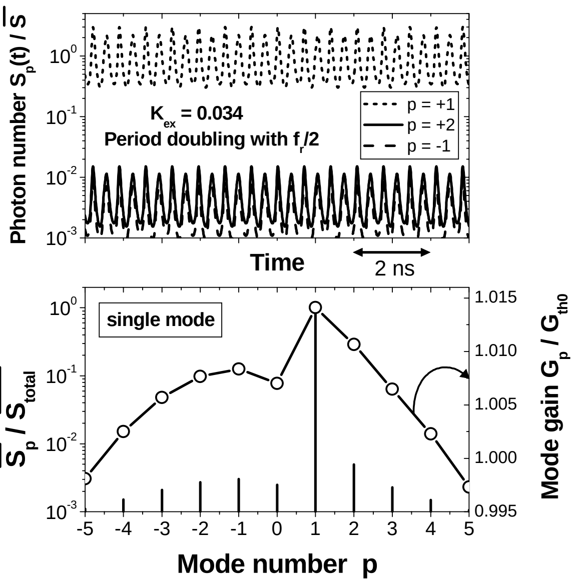

Fig. 3. (a) Time trajectory of $S_p(t)$, and (b) the corresponding averaged gain and output spectrum when K_{ex}=0.034. The time trajectory corresponds to period-doubling route-to chaos (with the relaxation frequency), while the output spectrum indicates single-mode (p=+1) operation.

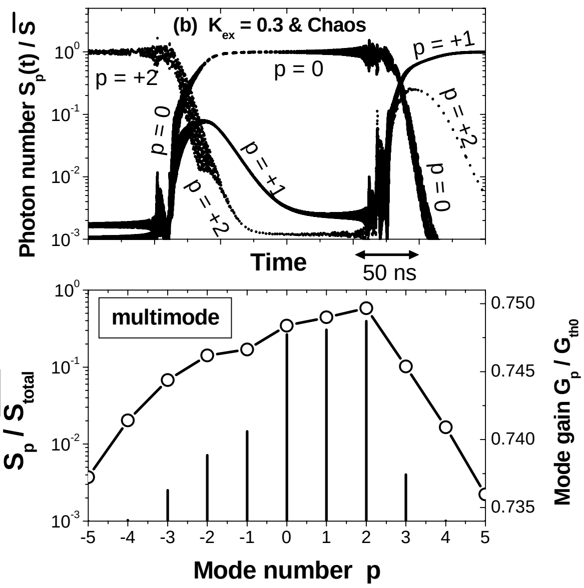

Fig. 4. (a) Time trajectory of $S_p(t)$, and (b) the corresponding averaged gain and output spectrum when K_{ex}=0.3. The time trajectory shows chaotic dynamics, while the output spectrum indicates multimode operation.

wave (cw) nearly single-mode oscillation (p=+1) of the solitary laser when the current is $I=2I_{th0}$.

Figure 2 plots influence of OFB on the minimum side-mode suppression ratio (SMSR), which is defined as the intensity ratio of the dominant mode to the strongest side mode. In this paper, the single mode oscillation is decided when SMSR exceeds twenty [2]. The figure plots also on the right-hand axis the OFB-bifurcation diagram of the total photon number $S = \sum_p S_p$ (peaks of the time trajectories of S). The figure indicates that the laser maintains cw operation over the low range of OFB, K_{ex}<0.02. Then the laser oscillates periodically with the relaxation oscillation frequency f_r. Then a region of period-doubling route to chaos (K_{ex}=0.02~0.04) is obtained before the laser exhibits chaotic operation (coherence collapse). As the OFB coefficient k_{ex} increases the laser exhibits also cw operation interrupted by chaotic regions. The chaos cycles in the strong OFB region is characterized by quasi-periodic routes-to chaos, e.g., when K_{ex}= 0.08~0.095. The frequency of pulsing regions increases from f_r in the region preceding the first chaos cycle to the resonance frequency of the external cavity in the last chaos cycle. Figure 2 shows that in all regions of cw, periodic pulsation and period doubling, the mode competition keeps single mode oscillation of the laser. The figure indicates also that the SMSR increases with K_{ex} when the laser operates in cw, and it is found that the lasing mode jumps from p=+1 to longer-wavelength modes. This indicates stronger effect of the asymmetric cross-suppression of gain. The SMSR decreases apparently when the laser enters the region of chaotic operation, where the OFB enhances the mode competition so as to bring the modes into a severe mode-hopping region; the laser operates in multi-modes. Similar effect was reported by Ryan et al. [6].

Examples of the time trajectory of the mode photon number and the corresponding time-averaged output spectrum are given in the following figures.

Figures 3(a), 4(a) and 5(a) plot the time trajectories of $S_p(t)$, while Figs. 3(b), 4(b) and 5(b) plot the corresponding time-averaged gain and output spectra in the regions of period-doubling, chaos, and strong OFB-induced pulsation, respectively. In Fig. 3(a), the pulses of $S_p(t)$ posses two peaks with the sub-harmonic 1/2 of the relaxation frequency f_r. The gain and output spectra in Fig. 3(b) indicate a single dominant mode p=+1. Figure 4(a) shows strong hopping between modes p=0, +1 and +2 characterizing the chaotic operation. The corresponding output spectrum indicates multimode oscillation as found in Fig. 4(b). The pulsation characterizing the strong OFB is characterized by very strong dominant mode p=+2 and the external-cavity mode frequency separation, as seen in Figs. 5(a) and 5(b).

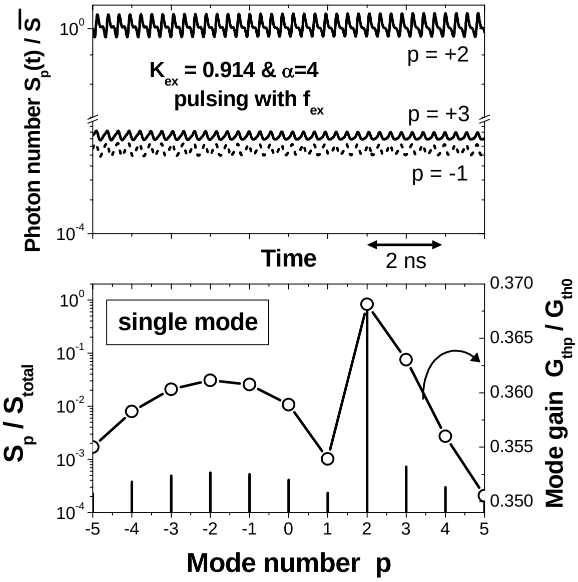

Fig. 5. (a) Time trajectory of $S_p(t)$, and (b) the corresponding averaged gain and output spectrum when K_{ex}=0.914 (strong OFB). The time trajectory shows periodic pulsation, while the output spectrum indicates single mode p=+2 operation.

4 Conclusion

We presented a theoretical model to characterize the mode competition phenomena under an arbitrary amount of external optical feedback in semiconductor lasers. The optical feedback was introduced as time-delay terms in the rate equations mode intensities. Influences of optical feedback on mode oscillation and dynamics were presented in details. The feedback may induce periodic pulsation, period-doubling pulsation, chaos depending on the feedback parameters. Nearly-single mode lasers maintain single-mode oscillation under optical feedback in the operating regions of continuous wave and pulsation. The chaotic dynamics is characterized by mode hopping and unstable multimode oscillation.

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