Model mismatch and systematic errors in an optical FMCW distance measurement system

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Abstract: - In this paper the problem of optical FMCW (frequency modulated continuous wave) distance estimation is considered. A measurement acquisition system is realized from which the real data are collected. A simple signal model is tested and verified using both simulated and measured data. According to this signal model a distance estimation procedure is introduced. A performance comparison of the distance measurements obtained from real and synthetic data reveals that the achieved measurement accuracy of the real system can be precisely predicted from simulations for the system specifications under consideration and that the distance sensor is therefore accurately modeled. Further we investigate the effects of systematic errors on the distance estimation performance in the acquisition system. It is shown that systematic errors are negligible compared to stochastic errors and the variance of the measurements is close to optimal.

Key-Words: - FMCW, laser radar, distance measurement, optical sensor

1 Introduction

Optical time-of-flight (TOF) distance measurement systems are used in many industrial and military applications. One advantage of optical systems is their ability to obtain measurements without contact. In contrast to microwave radar systems where the carrier frequency is modulated, in optical systems the intensity of the laser beam is modulated. Depending on the modulating signal, pulse- or continuous wave (CW) modulation, various methods exist for estimating the TOF [1]. Most common for optical distance sensors are the pulse method and the CW phase difference method.

In this paper a FMCW-system (frequency modulated continuous wave) is investigated. In contrast to the previous mentioned methods, FMCW systems are not often used for optical distance measurements yet. One advantage of FMCW is its multi target capability, what means two or more axial targets can be detected and their absolute locations or their displacement can be measured, respectively. A verification of the theoretical description for FMCW-systems by the corresponding real measurements has not been previously accomplished. In [2] an FMCW-system is introduced and investigated concerning the stochastic distance estimation errors. While systematic errors are neglected, it is shown that the stochastic error is close to optimal and limited by the Cramer-Rao bound.

In this paper the deterministic errors in the FMCW-system are analyzed, that have their reasons e.g. in frequency depending hardware components, calibration errors or finite sampling effects. In section 2 the principle of distance measurements via FMCW is described. After deriving the signal model, the algorithms for distance estimation are discussed. For simplicity the focus of our investigations lies on the single target scenario. The data acquisition by real measurements with an experimental setup developed at the Ruhr-Universität Bochum as well as by generating synthetic data is presented in section 3. Based on the signal spectrum the model mismatch is investigated. Furthermore, the estimation bias of both experimental and simulated measurements is compared for series of measurements and analyzed concerning systematic errors.

2 FMCW distance measurements

TOF distance measurement systems exploit the well-known relation between distance D and TOF τ

$$D = \frac{1}{2}\frac{c}{n}\tau \tag{1}$$

via the speed of light c. The factor ½ occurs due to light propagation towards the target and back. The refractive index n is assumed to be one in this paper, what is almost correct for measurements in air. With (1) the distance measurement boils down to a time-delay measurement. Depending on the modulating signal, different algorithms exist to obtain the time-delay τ.

2.1 Signal model

In this paper the FMCW method is applied, where a linear chirp is used as modulating signal. Using a homodyne detector reduces the bandwidth of the signal that has to be measured. Furthermore, the chirp linearity is of great importance for the derivation of the signal model. The linear chirp is described by

$$s(t) = \mathrm{rect}\left(\frac{t}{T_c}\right) \cdot \exp\left\{ \mathrm{j}\left(2\pi f_0 t + \pi \frac{B_c}{T_c} t^2\right)\right\} \quad (2)$$

with the chirp duration T_c, the chirp bandwidth B_c and the center frequency f_0. The received signal

$$r(t) = A \cdot s(t - \tau) \quad (3)$$

is time shifted by τ. The amplitude A of the signal depends on the reflection properties of the target and is assumed to be constant over the measurement time. By using a homodyne detector and a low pass filter, and under the assumption $\tau << T_c$ the instantaneous frequency (IF) signal

$$\begin{aligned} g(t) &= s(t) \cdot r(t) \\ &= \sum_{i=1}^{m} A_i \cdot \cos\left\{ 2\pi \frac{B_c}{T_c} \tau_i t + 2\pi f_0 \tau_i - \pi \frac{B_c}{T_c} \tau_i^2 \right\} \cdot \\ &\cdot \mathrm{rect}\left(\frac{t}{T_c}\right) + n(t) \end{aligned} \quad (4)$$

is obtained. Generally, the transmitted signal is reflected at m different axial targets. Hence the IF-signal is the superposition of multiple sinusoidal functions with different time-shift. In (4) $n(t)$ describes the additive measurement noise, which is inevitable in real systems. It is assumed to be Gaussian white noise with variance σ_n^2 and zero mean. By establishing the instantaneous frequency f_{if} and the corresponding phase φ_{if}

$$f_{if} = \frac{B_c}{T_c} \tau, \quad \varphi_{if} = 2\pi f_0 \tau - \pi \frac{B_c}{T_c} \tau^2 \quad (5)$$

the signal model for the IF-signal during the measurement time can be written as

$$g(t) = \sum_{i=1}^{m} A_i \cdot \cos\left(2\pi f_{if_i} t + \varphi_{if_i}\right) + n(t) \,. \quad (6)$$

The distance estimation problem reduces to the classic harmonic retrieval problem or estimating the frequencies and corresponding phases of a multi tone signal [3]. For single tone signals one possibility is the estimation in time-domain by linear regression, as proposed in [4]. This estimation is sensitive to chirp nonlinearities and misses the potential of multi-tone estimation, respectively. In the case of a nonlinear chirp the frequency of the IF-signal is no longer constant over time and the estimation problem gets much more complicated.

Of course the frequency f_{if} as well as the corresponding phase φ_{if} depend on the target distance. Therefore the estimation of frequency and phase of the IF-signal combined with (5) and (1) yields simple distance measurements.

2.2 Algorithm

As the signal model is very simple, the corresponding algorithms should be reduced to a few necessary steps, too. For simplicity the following algorithm is introduced for a single tone estimation. Later it will be easy to extend for multiple tone estimation. The IF-signal for a single target scenario is given by (6) with $m = 1$:

$$g(t) = A \cdot \cos\left(2\pi f_{if} t + \varphi_{if}\right) + n(t) \,. \quad (7)$$

Computing the spectrum of the IF-signal leads to the frequency f_{if} of $g(t)$ at the position of the maximum in the magnitude of the spectrum. The phase φ_{if} is the corresponding phase of the spectrum at f_{if}.

The selected estimation procedure works as follows. To reduce leakage effects, which occur in the FFT because of the fractional relation between the known sampling frequency and the unknown signal frequency, it is essential to use a window function [5]. In this case a hanning-window is applied. Furthermore, the Hilbert-transform is performed to obtain the complex base-band representation. In the ideal case the signal frequency corresponds to the maximum of the spectrum. Thus, an iterative maximum search is applied, based on FFT. Interpolation is used to obtain sub-sample resolution. The position of the maximum gives an estimator for the signal frequency f_{if}. The corresponding phase of the spectrum at f_{if} yields an estimator for the signal phase φ_{if}. According to the described procedure the estimation is processed in the following steps.

step 1: $$g'(t) = g(t) \cdot \text{window function} \quad (8)$$

step 2: $$g_h = \mathrm{hilbert}\left(g'(t)\right) \quad (9)$$

step 3: $$G(f) = \mathrm{fft}\left(g_h(t)\right) \quad (10)$$

step 4: $$\left|G(\hat{f}_{if})\right| = \max\left(\left|G(f)\right|\right) \quad (11)$$

step 5: $$\hat{\varphi}_{if} = \mathrm{angle}\left(G(\hat{f}_{if})\right) \quad (12)$$

Using (5) and (1) two distance estimators are obtained. The coarse estimator, based on the frequency estimation, is

$$\hat{D}_{\mathrm{coarse}} = \frac{c}{2} \cdot \frac{T_c}{B_c} \cdot \hat{f}_{if} \quad (13)$$

while the fine estimator, based on the phase estimation, is

$$\hat{D}_{fine} = \frac{c}{2} \cdot \left(\frac{T_c}{B_c} f_0 - \sqrt{\frac{T_c^2}{B_c^2} f_0^2 - \frac{T_c}{\pi B_c}\left(\hat{\varphi}_{if} - 2k\pi\right)} \right) . \quad (14)$$

The coarse estimator $\hat{D}_{coarse}$ avoids the ambiguity problem and is used to determine the unknown integer k

in (14). The advantage of the fine estimator $\hat{D}_{fine}$ is the smaller variance [6].

3 System Simulations and experimental results

In the following section the data acquisition and the estimation errors are described. The data are collected from a real measurement system as well as synthetically generated. By comparing the results of the two different estimations, conclusions about the model mismatch and systematic errors in the estimation procedure are pointed out.

3.1 Data acquisition

The realized distance measurement system has the following specifications. The chirp generator is based on a fractional divider PLL [7]. Its parameters are $T_c = 3$ ms, $B_c = 100$ MHz and $f_0 = 750$ MHz. The chirp signal is split in a 3-dB coupler. A laser diode with a wavelength of $\lambda = 635$ nm is used in the transmitter. The receiver consists of an avalanche photo diode. The transmitted and the received signal are mixed and low pass filtered. This anti-aliasing filter has a cut-off frequency of 100 kHz. Afterwards the signal is digitalized with an analog-to-digital converter. Its sampling rate is $f_s = 1$ MHz and it has a resolution of 12 bit. The recorded measurement data are processed on a PC. MATLAB is used for evaluating the algorithms and displaying the results. Fig. 1 shows the block diagram of this system.

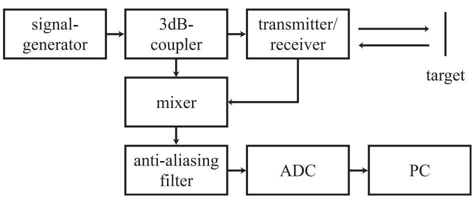

Fig. 1: Block diagram of the measurement system

Several internal errors are avoided by using an internal reference path that is measured after each distance measurement. The target is placed on a rail with a stepper motor, which has a position accuracy of 0.01 mm.

In the simulations the synthetic data are generated according to (7) using MATLAB with respect to the parameters of the realized sensor. The signal-to-noise ratio (SNR) is given by

$$SNR = \frac{A^2/2}{\sigma_n^2} \qquad (15)$$

and is chosen according to SNR values estimated from the real measurements. Fig. 2 shows the power density spectrum of the simulated IF-signal, while Fig. 3 shows the power density spectrum of the measured IF-signal. Comparing these two figures shows well accordance between the two signals in frequency domain. In both figures a single maximum is clearly visible. We conclude that the measured signal is well described by (7) and the synthetic data.

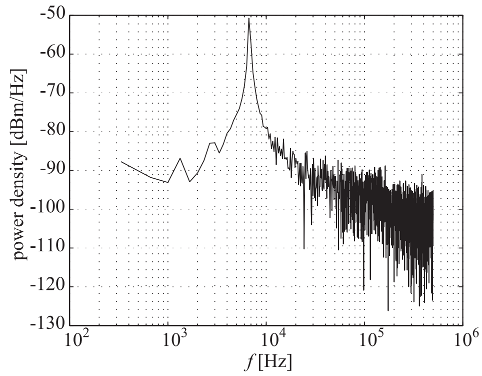

Fig. 2: Power density spectrum of the simulated IF-signal with $SNR = 20$ dB, $A = 20$ mV and $f_s = 1$ MHz

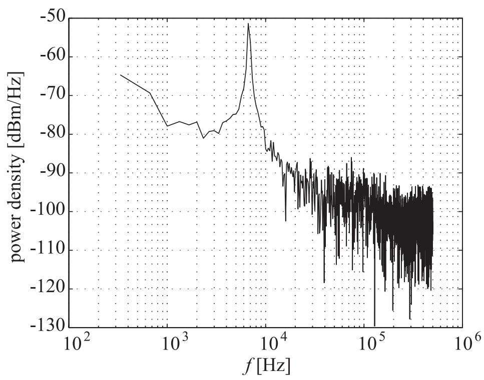

Fig. 3: Power density spectrum of the the measured IF-signal with $SNR = 20$ dB, $A = 20$ mV and $f_s = 1$ MHz

However it remains unclear at this point weather there is a model mismatch that results in different positions of the maximum or deviating corresponding phases for a given distance.

3.2 Error comparison

To analyze the performance of the measurement system two kinds of errors are distinguished. Systematic errors, also called deterministic errors, are represented by the estimation bias

$$e_{\hat{D}} = \mathrm{E}(\hat{D}) - D \qquad (16)$$

and stochastic errors are represented by the variance of the distance estimation

$$\sigma_{\hat{D}}^2 = \mathrm{E}(\hat{D} - \mathrm{E}(\hat{D}))^2 \qquad (17)$$

where E(.) denotes the statistical expectation, which is calculated by the mean value $\bar{D}$ of N measurements at one distance:

$$\bar{D} = \frac{1}{N}\sum_{i=1}^{N}\hat{D}_i \ . \qquad (18)$$

Thus in the following the bias is calculated with

$$\hat{e}_{\hat{D}} = \bar{D} - D \qquad (19)$$

The true distance D is known from the position of the stepper motor. Therefore (19) gives the absolute distance error. The variance of a series of measurements at M different distances is calculated with

$$\hat{\sigma}_{\hat{D}}^2 = \frac{1}{NM}\sum_{j=1}^{M}\sum_{i=1}^{N}(\hat{D}_i - \bar{D})^2 \ . \qquad (20)$$

(20) shows, that the variance does not depend on the true distance D, but is given by the deviation of the single measurements from their mean value.

Stochastic errors occur due to noise. For a given SNR the stochastic error is limited by the Cramer-Rao bound (CRB), which is a lower bound for the variance of unbiased estimators [8]. The performance of the measurement system concerning stochastic errors is investigated in [2]. The CRB of the distance estimation for this sensor is

$$CRB(\hat{D}) = \frac{c^2}{4}\frac{1}{\pi^2 f_0{}^2 f_s \cdot T_c \cdot SNR}\,, \qquad (21)$$

which is not dependent on the distance itself. It is shown that the variance of the distance estimation by real measurements, as well as by simulated measurements follows the CRB very accurate, what shows that the estimation procedure is close to optimal concerning stochastic errors.

While we showed in section 3.1 that the spectrum of the real IF-signal is well characterized by the simulated signal, here we investigate systematic errors in distance estimation. To analyze the appearance of systematic errors the distance between transmitter and target is varied in steps of 1 cm between 3.2 m and 4.8 m, so that $M = 161$. At each distance 50 measurements are performed, that means $N = 50$. At first simulated measurements with synthetic data are investigated. In Fig. 4 the estimation error $\hat{e}_{\hat{D}}$ vs. target distance D is presented, obtained by the fine estimation with (14).

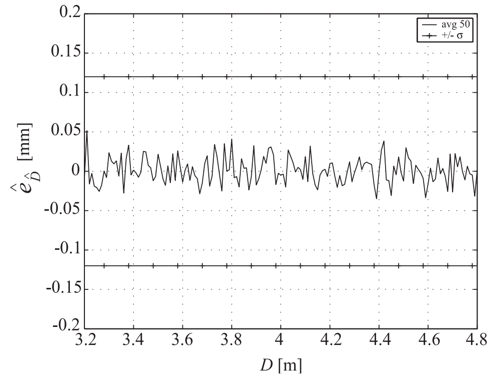

Fig. 4: Distance error versus target distance for simulated measurements with synthetic data with SNR = 20 dB, 50 averages per distance;
-+- line is the respective standard deviation

The average distance error for the simulation of 50 measurements is approximately +/- 0.05 mm. The standard deviation of the entire series of measurements, calculated with the square root of (20), is 0.12 mm.

For an equal scenario a real measurement is accomplished. The result is shown in Fig. 5.

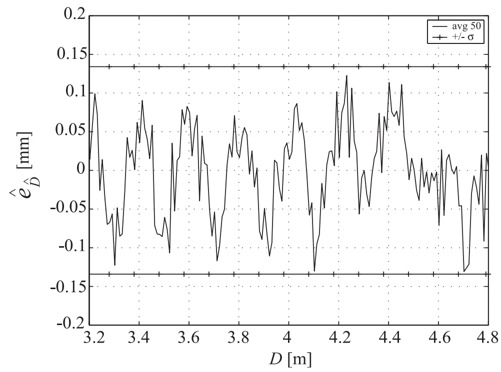

Fig. 5: Distance error versus target distance for real measurements with SNR = 20 dB, 50 averages per distance;
-+- line is the respective standard deviation

For the real measurements the bias is about +/- 0.12 mm and the standard deviation is 0.134 mm. The bias has a periodic nature caused by systematic errors. With the results of Fig. 4 we conclude, that the algorithm does not produce a significant bias. The estimation error decreases with an increasing number of averages. Therefore, the bias in Fig. 5 must be an effect of the system hardware. A possible reason is a slightly nonlinear chirp.

Otherwise the standard deviations of both simulated and real measurements yields good agreement with the corresponding CRB as demonstrated in Tab. 1.

	simulated measurements	real measurements	Cramer-Rao bound
standard deviation	0.120 mm	0.134 mm	0.116 mm

Tab. 1: Standard deviation of the distance measurements

As a result of these studies the estimation bias of the real measurements is smaller than the respective standard deviation. Thus the bias does not influence the measurement accuracy, if no averaging is applied. In consequence a model mismatch exists between (7) and the IF-signal measured with the real system. On the other hand it is negligible for the considered settings, especially for $SNR = 20$ dB. Therefore, the simple signal model introduced in section 2.1 well describes the measured signal. In addition the proposed distance estimation procedure does not affect the estimation accuracy. Only for higher SNR a significant model mismatch exists, so that in this case the model must be adjusted or the system hardware must be revised in order to reduce the systematic error even more.

4 Conclusion

In this paper a FMCW system is developed as a possible alternative solution to the most common systems for optical distance measurements. A sensor based on the FMCW-method has been investigated both from simulated and real measurements. The paper describes the simple signal model for which a distance estimation procedure is proposed. The FMCW-method extracts the distance information from the instantaneous frequency signal in frequency domain. Simulation results obtained from synthetic data generated according to the signal model are compared to measurements recorded from a real FMCW distance sensor. For both cases, the measurement accuracy is analyzed. It is shown that the used signal model accurately represents the system for the settings in consideration. In this distance sensor systematic errors are negligible due to proper design of hardware and algorithms and stochastic errors yield measurement variance close to optimal.

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