The Analysis and Characterization of Spurious Products in Software Radio Architectures

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Abstract: - With the advance of software defined radio (SDR) technology increasing attention has been paid to the performance of analog-to-digital converters (ADCs) with respect to not only their linearity, but the nature of their nonlinear characteristics as well. It is often difficult to compare the performance of a fully analog system with that based upon ADCs because the methodology of characterizing nonlinear phenomena differ for the two different classes of systems. Central to this characterization is the estimation of the spurious products impact on the overall system characteristics. This paper will compare the various methods used to characterize receiver components with an eye to better understand the relationship between these test results and actual receiver performance.
Key words: -Software radio, spurious products, system performance, analog-to-digital converters

1 Introduction

The fragmentation of telecommunication markets based on systems with different standards and frequency bands -- each with its own set of signal bandwidths, dynamic range, and intermodulation requirements -- has made it impossible for interaction between systems even in the most critical emergencies. This situation created an urgent need in developing a transceiver that is both frequency agile and standard independent. This capability becomes technically achievable with the introduction of True Software definable Radio systems (TSR) [1]. In such a system, all signal processing is done in software by converting the received signals into digital form immediately after the antenna. However, several issues related to the development of software radio systems have to be clarified and resolved before we can successfully compare their architectures to select the best one. This paper addresses the issue related to the definition of the SDR receiver sensitivity to the nonlinearity of its components.

It is well known that a receiver's sensitivity is determined by the RF front-end components while its selectivity by the channel frequency response. Since channel selection in this case is performed at baseband after signal sampling the discussions in subsequent sections will emphasize the effect of the ADC's performance on the receiver selectivity. The limited response time of the ADC feedback loop might cause time dependency of the ADC characteristics. That might cause the spurious tones arising at random frequencies. The amplitude of spurs might be substantially higher the amplitude of channel thermal noise. As a result, the spurious free dynamic range (SFDR), not the signal-to-noise (SNR) ratio limits the receiver sensitivity. The wide dynamic range of a receiver is necessitated by demand of acceptable operation in the near-far region of communicating entities, when interferers are at the close proximity while the source of the desired signal is at the maximum range from the receiver. SFDR has to be large to prevent blocking weak signals from desirable sources. In addition, baseband circuits of the SDR receiver need to adapt to the different dynamic range, linearity and signal bandwidth requirements of multiple communications standards. This programmability can be easily achieved by performing channel selection in the digital domain. Delta-sigma (DS) modulators are uniquely suited to this application because the high-pass-shaped quantization noise falls into the same frequency band as the adjacent-channel interferers. DS A/D converters show some advantages over the traditional Nyquist converter architectures that make them more desirable for this kind of applications where high resolution and large dynamic range is needed. The reasons for this are following[2- 8]:

- DS ADCs relax the requirements on the analog circuitry. The accuracy with which analog components have to be matched in Nyquist converters grows fast with the resolution and dynamic range.

- They are less vulnerable to noise and interferers.

- DS modulators shape the quantization noise pushing it away from the band of interest. They also shape the interferers that especially in multistandard environment might still be present in front of the ADC together with the signal. Thus, the ADC can be part of the channel selection circuitry playing a role in the selectivity of the receiver chain without affecting its sensitivity and relaxing the requirements of other selection blocks without any extra effort.

- They are easily programmable by choosing a different oversampling ratio (OSR) and bandwidth, what leads to different resolution and dynamic range.

The drawback of using DS ADCs instead of Nyquist ADCs is trading resolution with OSR, thus having a working frequency much higher compared to a Nyquist ADCs. This can potentially lead to signal integrity problems and also makes their power consumption higher. Even so, they have been considered over the years the most suitable solution for most multistandard receivers. The different types of ADCs can be classified according to their noise shaping order, their topology (single loop or cascaded), their internal quantizer resolution (single bit or multibit), their implementation (continuous time or discrete time), and the type of feedback they use. Even if they are highly linear, single bit quantizers yield a much more reduced dynamic range than multibit quantizers for the same modulator order. Moreover, their additive noise could be highly correlated with the input signal. This would make spurious tones appear at the output of the modulator [2, 5, 7]. The main drawback of using multibit quantizers is that they can cause non-linear distortion. This distortion is generally produced by the mismatch among components in the DAC that ideally should be identical. Any error at the output of the DAC is transmitted to the DS modulator input degrading the A/D conversion quality. The TechnoConcepts's DS A/D converter uses cascaded lower order modulators so it is more stable than an interpolative structure of the same order, but finite opamp gain and mismatches between the chip analog and digital circuitry components reduce the dynamic range.

2 Analysis of a Nonlinear Amplifier

Central to the distortion model used in characterizing the generation of spurious signals is the assumption that the component is "static", i.e., that the equation governing the nonlinear response of the component is constant with frequency and over all time. Implicit with this assumption is that the nonlinear component has no memory and operates only on the immediately current value of the input. Under this assumption any nonlinear function, F(x), can be expressed as a Taylor series provided that enough terms are used to reduce the series error to an arbitrary and acceptably small value:

$$F(x) \approx F(0) + x \cdot F^{'}(0) + \frac{x^2}{2!}F^{''}(0) + \bullet\bullet\bullet + \frac{x^n}{n!}F^{[n]}(0) + \bullet\bullet\bullet \qquad (1)$$

It could be easily shown [2] that if $x \equiv A\cos(\omega t)$ then:

$$\cos^{2n}\omega t = \frac{1}{2^{2n}}\left\{\sum_{k=0}^{n-1} 2\binom{2n}{k}\cos 2(n-k)\omega t + \binom{2n}{n}\right\}$$
$$\cos^{2n-1}\omega t = \frac{1}{2^{2n}}\left\{\sum_{k=0}^{n-1} 2\binom{2n-1}{k}\cos(2n-2k-1)\omega t\right\} \qquad (2)$$

Applying (2) to (1) yields

$$x^2 = \frac{A^2}{2}\left(\cos 2\omega t + 1\right)$$
$$x^3 = \frac{A^3}{4}\left(\cos 3\omega t + 3\cos\omega t\right)$$
$$x^4 = \frac{A^4}{8}\left(\cos 4\omega t + 4\cos 2\omega t + 3\right) \qquad (3)$$

and so on. Using (2),(3) Equation (1) can be rewritten as

$$F(x) \approx a_0 + a_1\cos\omega t + a_2\cos 2\omega t + \bullet\bullet\bullet \qquad (4)$$

An important observation from the above set of equations (3) is that the n^{th} term of the Taylor series contains not only the n^{th} harmonic of the incoming sine wave, but all odd or even subharmonics as well. Thus conventional distortion analysis requires that the nonlinearity of the function would be sufficiently weak. That requirement would be met if

$$F^{'}(0)\frac{A}{1!} > F^{'''}(0)\frac{1}{3!}\frac{3A^3}{4} > \bullet\bullet\bullet$$
$$F^{''}(0)\frac{1}{2!}\frac{A^2}{2} > F^{iv}(0)\frac{1}{4!}\frac{4A^4}{8} > \bullet\bullet\bullet \qquad (5)$$

In other words that $F^{[n]}(0)$ decreases with increasing n sufficiently that the exponent with which the amplitude increases for the n^{th} harmonic is defined solely by the n^{th} order derivative.

For a nonlinear function operating on the sum of two sine waves ($x = A_1\cos\omega_1 t + A_2\cos\omega_2 t$) such as in a two-tone test, we note that

$$
\begin{aligned}
x^2 &= A_1^2 \cos^2 \omega_1 t + 2A_1 A_2 \cos \omega_1 t \cos \omega_2 t \\
&\quad + A_2^2 \cos^2 \omega_2 t \\
x^3 &= A_1^3 \cos^3 \omega_1 t + 3A_1^2 A_2 \cos^2 \omega_1 t \cos \omega_2 t \\
&\quad + 3A_2 A_1^2 \cos \omega_1 t \cos^2 \omega_2 t + A_2^3 \cos^3 \omega_2 t \\
x^4 &= A_1^4 \cos^4 \omega_1 t + 4A_1^3 A_2 \cos^3 \omega_1 t \cos \omega_2 t \\
&\quad + 64A_1^2 A_2^2 \cos^2 \omega_1 t \cos^2 \omega_2 t + \\
&\quad + 4A_2 A_1^3 \cos \omega_1 t \cos^3 \omega_2 t + A_2^4 \cos^4 \omega_2 t
\end{aligned}
$$

An inspection of the above equation reveals that if the amplitude of the second sine wave having frequency ω_2 is kept constant, then the amplitude of the cross-product terms will only vary as the power of the ω_1 term contained therein – we must therefore conclude that not all n^{th} order components will vary in amplitude with a slope that is n times that of the fundamental.

3 Analysis of a Non-ideal Mixer

The analysis of a non-ideal mixer is somewhat similar to the nonlinear amplifier analysis just elaborated but additionally introduces cross-products due to the mixing action itself as shown below, assuming the mixer inputs are "x" and "y":

$$
\begin{aligned}
F(x) &\approx a_0 + a_1 \cos \omega_1 t + a_2 \cos 2\omega_1 t + \bullet\bullet\bullet \\
G(y) &\approx b_0 + b_1 \cos \omega_2 t + b_2 \cos 2\omega_2 t + \bullet\bullet\bullet \\
F(x)G(y) &\approx a_0 b_0 + a_1 b_0 \cos \omega_1 t + a_2 b_0 \cos 2\omega_1 t + \bullet\bullet\bullet \\
&\quad + a_0 b_1 \cos \omega_2 t + a_1 b_1 \cos \omega_1 t \cos \omega_2 t \\
&\quad + a_2 b_1 \cos 2\omega_1 t \cos \omega_2 t + \bullet\bullet\bullet \\
&\quad + a_0 b_2 \cos 2\omega_2 t + a_1 b_2 \cos \omega_1 t \cos 2\omega_2 t \\
&\quad + a_2 b_2 \cos 2\omega_1 t \cos 2\omega_2 t + \bullet\bullet\bullet \\
&\quad \bullet \\
&\quad \bullet \qquad (6) \\
&\quad \bullet
\end{aligned}
$$

If we assume that the amplitude of the LO is constant (therefore all b_n are constant) and that all of the amplitude dependent terms (a_n) vary with exponent n, then this mixer model would suggest the following:

- If the LO power were varied, the feedthrough of n^{th} harmonic of the LO ($n \cdot \omega_2$) increases in proportion to the n^{th} power of the LO input amplitude.
- The intermodulation products (or cross-products) at frequencies $m\omega_1 \pm n\omega_2$ will vary in amplitude according to the RF input amplitude with exponent m.

Note however, that these observations are dependent on the assumption of weak nonlinearity as discussed in the paragraph on nonlinear amplifier analysis.

4 Spurious Free Dynamic Range – A Comparison of Definitions

One of the challenges in comparing the characterization of conventional analog receivers with those based upon A/D converters is in the understanding differences in the way spurious free dynamic range (SFDR) is defined and the impact this difference in definition may have in the measured results.

In analog receivers, SFDR is defined in terms of the difference between the input power that causes the desired signal to rise above a prescribed noise floor and that power that causes a prescribed spurious product to exceed this same threshold. This definition is graphically demonstrated in Figure 1.

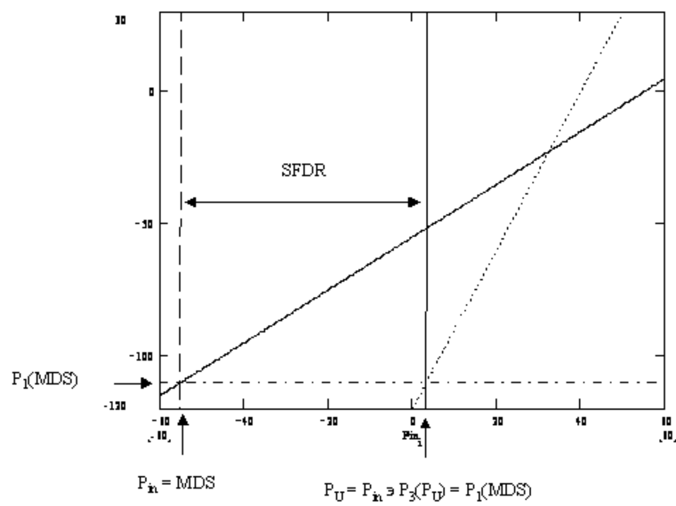

Fig 1 Graphical depiction of how SFDR is computed from the fundamental and intermodulation curves. Once a minimum detectable signal (MDS) threshold is defined, the resulting output power (P_1(MDS))in response to an input power of MDS is computed. A second input power (P_U) is computed representing the point at which the intermodulation transfer function produces a power equal to P_1(MDS). The difference between these two power levels is the SFDR.

To define the overall SFDR of a receiver, the SFDR would be computed in accordance with Figure 1 each order of spurious product and the worst result would be used. In contrast, most A/D converter application notes define spurious free dynamic range as the ratio between the amplitude of the desired signal and the largest undesired signal. This ratio may vary with different input amplitudes and must therefore be evaluated at different input levels.

The challenge in reconciling these two apparently divergent paradigms of measurement lies in understanding the dominant mechanisms behind distortion encountered in each case. In the case of analog systems, the transfer curve is usually monotonic and free of discontinuous “kinks”. For flash or cascaded A/D converters, however, nonlinearities may be extremely localized due to, for example, a handful of comparators with inaccurate threshold voltages. In this case, the transfer function may be considered strongly nonlinear in a localized region. The effect of this nonlinearity may be insignificant for large signals, but dramatic for small signals depending upon how much of the input signal traverses this nonlinear region.

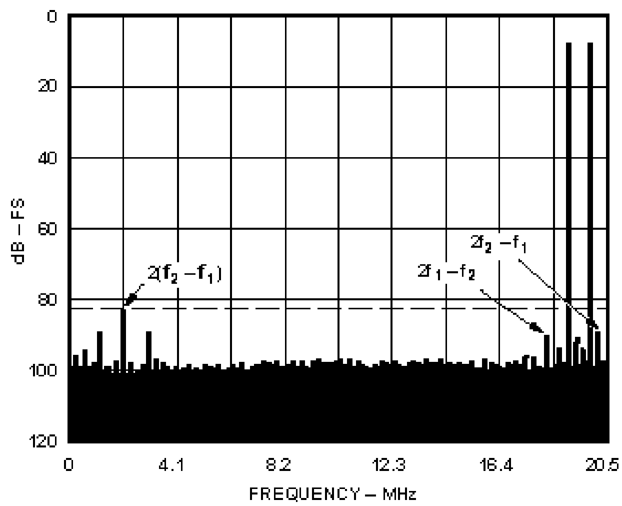

Fig 2 Diagram (extracted from a manufacturer’s application note) showing the measurement of spurious free dynamic range (SFDR) in a typical A/D converter using a two tone test. SFDR is defined as the ratio between the desired signal(s) and the largest undesired signal.

In contrast, receivers based upon delta-sigma A/D converters (in particular the single-bit type) are not prone to such localized “kinks” in the transfer characteristic. In this case the dominant nonlinearity is usually due to the distortion caused by analog interface circuitry. Nevertheless, because delta-sigma converters require large signal inputs for optimum operation, it is often necessary to drive input amplifiers fairly “hard”, resulting the moderate levels of nonlinearity compared to conventional narrowband front ends.

5 Characterization of Software Defined Radios with A/D Front End

In order to directly conduct a series of two-tone tests upon the software defined radio front end, three frequency synthesizers were reference locked to one another. One synthesizer was used to generate the clock frequency for the receiver, and the remaining two were used to generate the two tones used to characterize the receiver. A simplified equivalent block diagram of the test setup is shown in Fig 3.

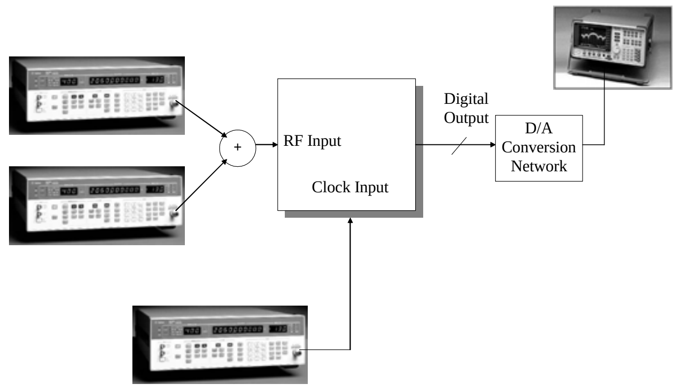

Fig 3 Block diagram of the two-tone characterization test setup. Individual synthesizers are used for each of the two tones and the input clock for the digital radio front end. The synthesizers are reference locked in order to minimize the effects of phase noise. The front end of the device under test includes the function of frequency translation, so that tests similar to those generally used to characterize superheterodyne architectures (which include mixing and amplification) are readily adapted.

A typical plot from the tests that were run is shown in Fig 445. In the example, the two input tones are at 747 and 751 MHz, the clock frequency is 1500 MHz, which results in a frequency translation by 750 MHz. Therefore (as expected), the 1 and 3 MHz outputs have a slope of 1 dB output power per 1 dB of input power (unity slope).

The two intermodulation products that were “closest in” (2 MHz and 4 MHz, respectively) had slopes surprisingly close to unity as well. Given that only a fourth and eighth order intermodulation product (from 751 MHz) could produce these frequencies, we would have expect the slope to be two and four, respectively. The fact that these slopes are unity leads us to conclude that these intermods are spurious products of the frequency translation process, but are due to a direct intermodulation between the 1 and 3 MHz outputs of the receiver front end. In this scenario, both processes have unity slope and therefore results in an overall, unity slope as observed. The likely “culprit” in this case is distortion in the D/A circuit used to drive the spectrum analyzer rather than distortion in the receiver front end.

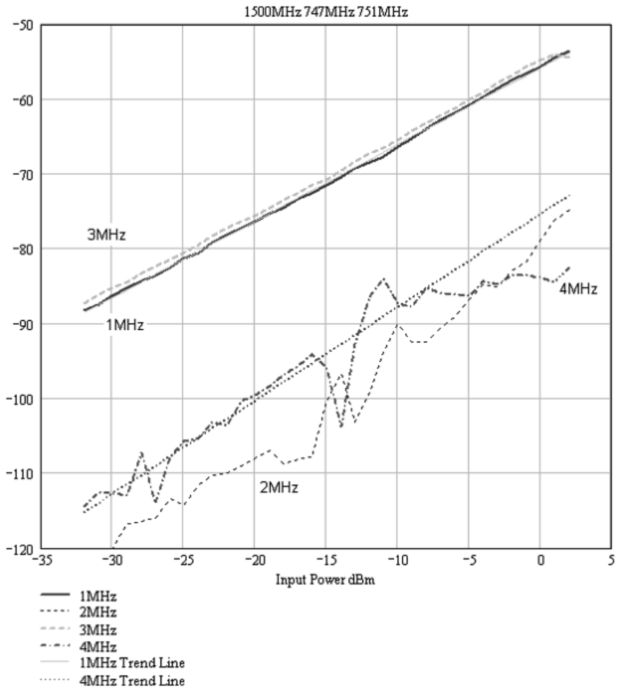

Fig 4 Typical plot of input versus output power for a two-tone test previously described. As expected, the two main tones (1 and 3 MHz) have a 1 dB for 1 dB dependence upon input power (a slope of 1) and represent the translation of 747 and 751 MHz down by 750 MHz. The 2 MHz and 4 MHz intermodulation products are apparently from a secondary intermodulation between the 1 and 3 MHz outputs rather than being produced during the frequency translation process.

6 Conclusion

Signal distortion caused by nonlinearities is recognized as the performance limitation of channel receivers. Even if it is pronounced in a substantially lesser degree than for the analog receivers it still limits sensitivity of direct downconversion Software Radio digital receivers. The analysis of the spurious intermodulation products on the overall performance of the software radio systems has shown that interferers lying in the passband can create undesirable spectral components making detection of a targeted signal more difficult. The main way to deal with the problem is by careful design of the receiver components and optimizing their characteristics to bring inevitable signal distortion to the acceptable level. The paper analyzed differences in the way spurious free dynamic range (SFDR) was defined for analog and digital receivers and the impact this difference in definition could have on the measured results.

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