# Adaptive Fading Channel Estimator For MMSE Receiver Based CDMA Systems *

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## Abstract

In this paper, An LMS adaptive channel estimator algorithm is proposed to improve the practical implementation of MMSE receiver based CDMA systems. The desired user's fading process is estimated to provide the receiver with a reference phase and amplitude to demodulate the desired user signal. Results show that the proposed adaptive coefficients estimator outperforms the fixed coefficients estimator. Furthermore, the estimator is found to be unbias and the estimation error can be modeled as a complex Gaussian random process.

**Key Words**: CDMA, MMSE Receiver, LMS algorithm, fading process estimation.

## 1 Introduction

In the last two decades, Code Division Multiple Access (CDMA) technology have been the focus of attention by many researchers in digital communication systems. The adaptation of CDMA as the prime multiple access technique for third generation 3G wireless systems and many other recent wireless networks has brought more attention to this technology. To improve the performance of the CDMA system in the presence of Multiple Access Interference (MAI), and to mitigate the near-far problem, several receivers with different degrees of complexity and performance have been developed. For example, an optimum multi-user receiver is presented in [1], however, the complexity of this receiver increases exponentially with the number of users. A suboptimal class of detectors with linear complexity are presented in [2]. Although these suboptimum receivers show linear complexity, they still require a great deal of side information.

The MMSE receiver is a suboptimum receiver which is known to be near-far resistant. In addition, the MMSE receiver does not need to know certain side information like the code sequence and the carrier frequency of the desired user. This information can be obtained through adequate training if the MMSE is implemented in its adaptive form. Adaptive algorithms such as the least-mean-square (LMS) and recursive least-square (RLS) are used to obtain the filter coefficients. The performance of the MMSE receiver in Additive White Gaussian Noise (AWGN) is presented in [3], [4], [5], [6],and [7] and its performance in a fading channel is presented in [8],[9], and [10] to mention a few. The performance of the MMSE receiver with convolutional coding and trellis coded modulation in a fading channel has been investigated in [11]. Many of the presented works in the literature have assumed known channel information available at the receiver and there is no need to estimate the fading channel.

In [8], it is shown that phase variation further de-

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grades the performance of the MMSE receiver compared to amplitude variation due to the fading experienced in a wireless channel. The degradation occurs because the errors caused by phase variations are often not localized to the deep fade period but rather propagate due to the loss of lock on the desired signal phase by the MMSE receiver. These errors dramatically degrade the MMSE receiver performance.

When operating in a fading environment, the receiver estimates the fading process to perform coherent modulation. When the fading process estimates are obtained, they are fed to the front end of the MMSE receiver to improve the performance. This technique is proposed in [8] and applied for binary phase shift keying (BPSK) modulation system. The estimation technique proposed in [8] is based on a fixed coefficients linear predictor. This technique results in a degradation of the performance of the system when compared to the known fading case. In this paper, we propose a fading estimation technique based on the Least Mean Square (LMS) adaptive algorithm. The performance of this technique is compared to the fixed coefficients estimator technique proposed in [8].

This paper is organized as follows, Section 2 describes the system model. Section 3 presents the proposed fading estimator. Section 4 shows the performance evaluation results and section 5, gives our conclusion.

## 2 System Model

In this section, an MMSE receiver based CDMA system is described. There are K active users in the system. Each user is assumed to have a unique spreading waveform $c_j(t)$. It is assumed that all users transmit asynchronously over the same channel with carrier frequency $w_o$. The modulated signal of the jth user can be written as

$$\begin{aligned} s_j(t) &= \text{Re}\big\{\sqrt{2p_j}d_j(t)c_j(t)e^{jw_ot}\big\} \\ &= \text{Re}\big\{g_j(t)e^{jw_ot}\big\} \end{aligned} \quad (1)$$

where $g_j(t)$ is the complex envelope of $s_j(t)$, $p_j$ is the transmitted power, and $d_j(t)$ is a complex baseband signalling format with symbol interval $T_s$. The waveform $c_j(t)$ is assumed to be in the polar form with chip interval $T_c$. Therefore, the processing gain $N$ is equal to $T_s/T_c$. The bandpass received signal at the receiver is given by

$$r(t) = \text{Re}\big\{\sum_{j=1}^{K}\alpha_j(t)e^{j\theta_j(t)}g_j(t-\tau_j)e^{jw_ot}\big\} + n(t) \quad (2)$$

The variables $\tau_j$, $\alpha_j$, $\theta_j$ are the propagation delay, amplitude, and phase of the fading process for the $j^{th}$ user. The process $n(t)$ is a real AWGN process with a spectral density of $N_o/2$. The fading amplitude is Rayleigh-distributed while the fading phase is uniformly-distributed.

Let user 1 be the intended user, and assume the receiver has knowledge of its propagation delay. Without loss of generality, we assume $\tau_1$ is set to zero. After converting the received signal to complex baseband, it is passed through a filter matched to the chip pulse shape. Furthermore, we assume the matched filter has a scale factor of $\sqrt{2p_1}T_c$ associated with it. Based on these assumptions, The equalizer contents of the MMSE receiver can be written as

$$\mathbf{r}(m) = d_1(m)\alpha_1(m)e^{j\theta_1(m)}\mathbf{c}_1 + \tilde{\mathbf{r}}(m). \quad (3)$$

Where the first term on the right hand side is the desired signal component and the other term contains the MAI and noise components. Let the autocorrelation matrix, $\mathbf{R}$, of the equalizer contents be defined as $\mathbf{R} = E\left[\mathbf{r}(m)\mathbf{r}(m)^H\right]$, The autocorrelation matrix of the MAI and noise part of the equalizer contents is given as $\tilde{\mathbf{R}} = E\left[\tilde{\mathbf{r}}(m)\tilde{\mathbf{r}}(m)^H\right]$ and the correlation between the desired user response and the received signal is given by $\mathbf{P}_i = E\left[d_i^*(m)\mathbf{r}(m)\right]$.

## 3 The Proposed Fading Estimator

In [8], it is shown that phase compensation is an effective method for improving the MMSE receiver performance in a fading channels. The phase estimate is obtained by using a fixed coefficient linear predictor.

In our case, we are dealing with multilevel modulation (16-QAM). Hence, amplitude and phase compensation are needed to improve the performance of the MMSE receiver.

In this paper, we propose a tracking technique based on adaptive linear prediction. The technique uses the decision out of the MMSE to form an estimate of the desired user's fading parameters using LMS algorithm based adaptation linear prediction. Given an input signal $\mathbf{r}(m)$, the tracking of the desired user's fading process involves the following. First, the filter output $z(m)$, given by

$$z(m) = d_1(m)\alpha_{1,m}e^{j\theta_{1,m}}\mathbf{a}^T\mathbf{c}_1 + \tilde{n} \tag{4}$$

A noisy estimate of the fading process is given by

$$\begin{aligned}\beta(m) &= \frac{z(m)}{d_1(m)\mathbf{a}^T\mathbf{c}_1} \\ &\approx \hat{\alpha}_{1,m}e^{j\hat{\theta}_{1,m}}\end{aligned} \tag{5}$$

In a decision-directed mode, $d_1(m)$ is replaced by $\hat{d_1}(m)$. The linear prediction can be formulated by the following.

It is shown in [8], that the $L^{th}$ order linear predictor of the fading channel is given by

$$\hat{\beta}(m) = \sum_{i=1}^{L}\hat{\mathbf{a}}_i\beta(m-i) \tag{6}$$

One approach which can be used to compute the predictor coefficients is based on matrix inversion [8]. The results obtained using such method show major degradations when compared to the known fading case. This degradation can be attributed to the fact that the matrix inversion technique used to obtain the predictor coefficients assumes a single user system. To improve the prediction and to account to the presents of the MAI in the input to the predictor, we have used the LMS algorithm to obtain these coefficients. The coefficients of the predictor, $\hat{\mathbf{a_m}}$, is obtained as follows

$$\hat{\mathbf{a_{m+1}}} = \hat{\mathbf{a_m}} + \mu\hat{\beta}(\mathbf{m})\mathbf{e}(\mathbf{m}) \tag{7}$$

where $\mathbf{e}(\mathbf{m})$is the estimation error. Next, the estimates of the fading process out of the linear predictor are used to remove the phase of the desired user fading from the input of the MMSE receiver and to scale the decisions in the MMSE receiver, respectively.

The proposed technique in addition to using linear prediction, uses pilot symbols. In this technique, pilot symbols, known by the receiver, are sent periodically, in this work, every 10 symbols. The MMSE receiver uses these pilots to obtain an estimate for the fading process in the same manner as in Eqn. 5. The use of pilot symbol can help in improving the MMSE receiver performance in two ways. First, pilot symbols can be used to periodically train the MMSE and prevent the MMSE filter from feeding back wrong decisions. Second, pilot symbols are used to aid the receiver in estimating the channel fading condition. This is especially helpful when the fading channel conditions are bad and unreliable decisions are made by the receiver.

Channel estimates are made by feeding back a linear predictor of the previous channel estimates. The good performance of the proposed technique can be attributed to three reasons. First, the use of pilot symbols provides the MMSE receiver with a reference that helps the receiver not to lose lock on the desired user. Second, using adaptive linear predictor, reliable estimates are made for every received symbol. This gives the linear predictor recent past channel estimates to predict the channel conditions. Third, pilot symbols can help the linear predictor not to lose track of the fading process by interrupting the propagation of decision errors.

# 4 Numerical Results

In Figures 1 and 2 the performance of the adaptive coefficients proposed estimator compared to performance of the fixed coefficients estimator proposed in [8]. The results for the known fading case is presented for comparison and to serve as a lower bound in the BER performance. In this case, the fading process of the desired user is assumed to be known for the receiver. To generate these figures, the following simulation environment has been used. The mobile speed was 5 mph, the mobile operates at the 900 MHZ band, the bit rate was 9600 bps, a pilot symbol was sent every 10th symbol. This corresponds to $f_sT_s$ of 0.0028 for 16-QAM. The received powers were modeled as a lognormal distribution with zero mean and 1.5 dB standard deviation.

The bit error rate (BER) performance of a single user system as a function of $\frac{E_b}{N_o}$ is shown in Fig. 1. The figure indicates that the proposed adaptive estimator outperforms the fixed estimator. For example, at $\frac{E_b}{N_o}$= 30 dB, the BER for the adaptive estimator is $1.3 \times 10^{-3}$ while the BER is $4.6 \times 10^{-3}$ for the fixed estimator case. For higher $\frac{E_b}{N_o}$ values, the BER difference becomes even greater. In terms of $\frac{E_b}{N_o}$ gain, the proposed adaptive estimator can achieve about 5 dB gain when compared to the fixed estimator. Furthermore, when the system is loaded by 30 users, the adaptive estimator still outperforms the fixed estimator. For example at $\frac{E_b}{N_o}$= 30 dB, the BER when the proposed adaptive estimator is used is $3.7 \times 10^{-3}$ while for the fixed estimator is $9.5 \times 10^{-3}$.

By comparing the results in Figures 1 and 2, it can be noticed that the performance gap between the lower bound, the case where the fading process of the desired user is known at the receiver, and the adaptive estimator is smaller for the 30 users case when compared to the single user case. This is expected, since for the lower bound results are based only on the exact knowledge of the desired user fading process. The MAI contribution is not taken into consideration.

Another criterion that is interesting to investigate is the fading estimation error. Let $\gamma_(m)$ be the exact desired user fading process. Then fading estimation error is defined as

$$e(m) = \gamma(m) - \hat{\beta}(m) = X + jY \qquad (8)$$

Since $\gamma_(m)$ was modeled as a complex zero mean Gaussian random process, we assume the estimate of the fading follows a Gaussian process since it is produced by a linear operation on a Gaussian process. Therefore, the estimation error is a complex Gaussian process. If the estimator is unbiased, the mean of the estimation error is zero. Figure 3 shows the probability density functions (pdf) of the real and imaginary parts, X and Y, of the estimated error. The means of X and Y are very close to zero hence, the estimation error can be modelled as a zero mean complex Gaussian process.

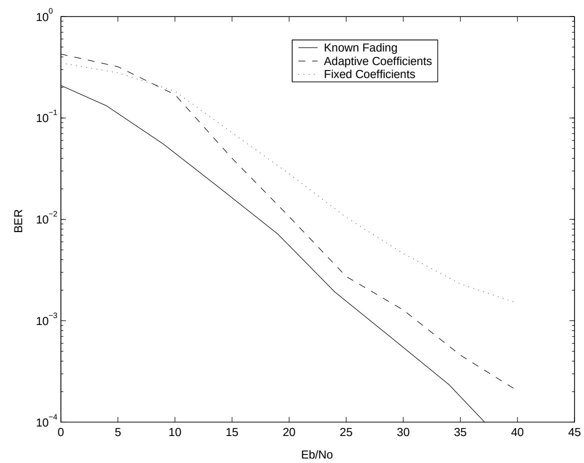


Figure 1: BER 16-QAM for a single user in a slow fading channel; PSAM rate = 0.1, pg = 124, $f_dT_s$ = 0.0028

# 5 Conclusion

In this paper, an LMS adaptive channel estimator algorithm is proposed to improve the practical implementation of MMSE received based CDMA system. The proposed adaptive estimator has been found to outperforms the fixed coefficients fading estimator by a wide margin. Significant improvements on the BER performance for different system loadings were obtained. Furthermore, the proposed estimator is found to be unbiased and the estimation error can be modelled as a complex Gaussian random process.

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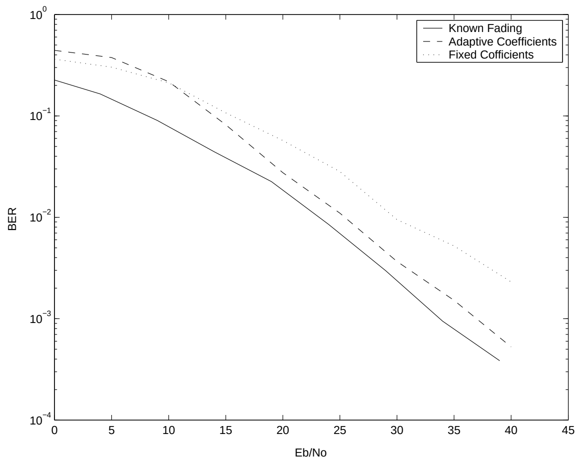


Figure 2: BER 16-QAM for 30 users in a slow fading channel; PSAM rate = 0.1, pg = 124, $f_d T_s = 0.0028$

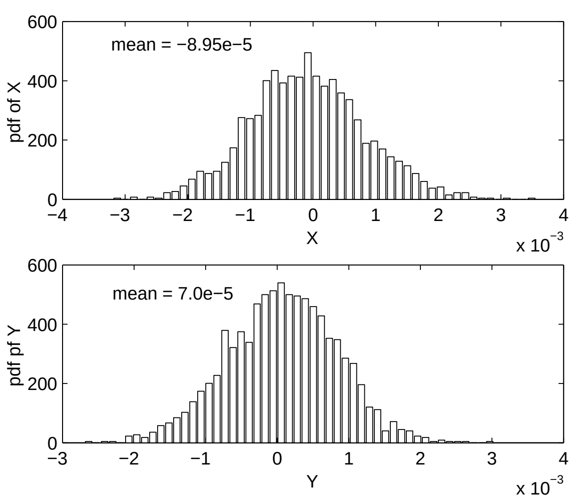


Figure 3: the pdf of the real and imaginary parts of the estimation error for a 16-QAM 30 users system;PSAM rate = 0.1, pg = 124, $f_d T_s = 0.0028$

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