Robust Absolute Stability of Lur'e Systems with Polynomic Parametric Uncertainty

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Abstract— **This paper presents new results to verify the robust absolute stability property of the Lur'e systems. As it is known, to solve the problem of robust absolute stability, the strict positive realness property (SPR) of a fictitious transfer function needs to be verified. In the present work, a fictitious transfer function with polynomic parametric uncertainty is considered. To verify the robust SPR property of this type of functions, an algorithm, based on the sign decomposition technique, is designed. This algorithm is codified in C language obtaining some advantages compared to others.**

Keywords— **Nonlinear systems, Lur'e systems, robust stability, absolute stability, polynomic parametric uncertainty.**

I. INTRODUCTION

The study of nonlinear systems represented by the well known Lur'e systems has endured for many years. This is due to the fact that the nonlinearities covered by this type of systems appear in many practical processes, see [15], [24], [3]. The Lur'e problem consists in determining the asymptotic stability property of a nonlinear system which is a feedback connection of a linear time invariant (LTI) system and a nonlinear element that belongs to a sector $[0, k]$. The asymptotic stability property of the equilibrium point for the previous system is called absolute stability, [15].

Recently, uncertainty has been added to the linear part of the Lur'e system. This was done because, in practice, all systems present uncertainty in some of their parameters. One of the first papers that deals with this problem is [7], where the authors provide herein some results to guarantee the robust absolute stability property by verifying the strict positive realness of eight well selected transfer functions from the uncertain linear system. Another paper related to this topic is [9], here, they generalize the Popov criterion for Lur'e systems with an polynomic plant and a controller in the linear part; also see [21] for more comments on this issue.

However, others authors prefer to transform a nonlinear control problem into a Lur'e problem to apply the existing tools to solve this type of problem, see [8]. Also, there exist others papers that deal with this problem, see [18],[26], [22], [20], [6], [16].

The present work addresses the analysis of absolute stability of Lur'e systems with uncertainty in the linear part as shown in the next figure:
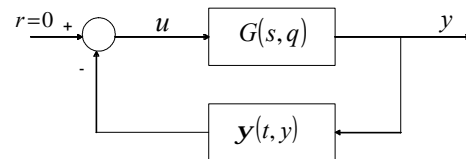

Figure 1. Uncertain Lur'e system.

where $\psi(t, y)$ is a memoryless nonlinear function that satisfies the following condition:
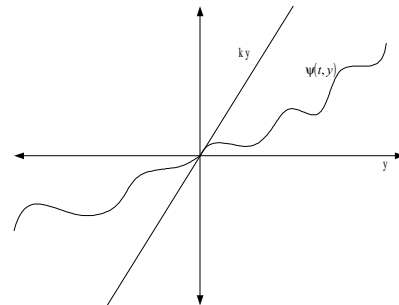

Figure 2. Nonlinear function inside a sector $[0, k]$.

The nonlinear function must be contained in a region called sector $[0, k]$. $G(s, q)$ represents a transfer function with polynomic parametric uncertainty that is defined as follows:

Definition 1 ([2]) A polynomic plant is a transfer function with parametric uncertainty that has the following structure:

$$G(s,q) = \frac{n(s,q)}{d(s,q)} = \frac{\sum_{i=0}^{m} a_i\,(s,q)\,s^i}{\sum_{i=0}^{n} b_i\,(s,q)\,s^i} \qquad (1)$$

$$\forall\, q \in Q$$

Where $a_i\,(q)$ and $b_i\,(q)$ are polynomic functions of vectors q; Q is a set that represent the parametric uncertainty and it is defined as follows:

$$Q \triangleq \left\{ q = \begin{bmatrix} q_1 & \cdots & q_n \end{bmatrix} : \mathbf{q_i^-} \leq \mathbf{q_i} \leq \mathbf{q_i^+} \right\}$$

Without loss of generality, it will be assumed that $q_i^- \geq 0$. Nevertheless, if this condition is not satisfied, it is always possible to make a linear transformation to make this condition hold. This type of set is known as box by the way it is defined; the name of polynomic plants is used because the coefficients of the transfer function are uncertain values that have polynomic structures. The absolute stability property when the linear part has uncertainty is called robust absolute stability and is satisfied if the system, shown in the figure 1, is absolutely stable for all members of the interval plant $G(s,q)$. Therefore, the aim of this work is to determine sufficient conditions to verify the robust absolute stability property of the Lur'e systems.

This paper is organized as follows: section II presents some preliminary definitions and results. Then, in section III, the main result is presented. And finally, the conclusions are provided in section IV.

II. Mathematical preliminaries and problem statement

The concepts of robust strict positive realness and robust absolute stability are the basis of this paper, thus, we elaborate on them next.

A. Strict positive realness

First of all, it is worth to mention that the strictly positive real (SPR) transfer functions are an old concept derived from electric network theory. These are related to the network's transfer functions when it has dissipative elements like resistors, lossy inductors and lossy capacitors. In order to introduce the definition of SPR functions it is necessary to firstly present the definition of positive real (PR) functions:

Definition 2 ([14]) A rational function G(s) of the complex variable $s = \sigma + j\omega$ is called positive real (PR) if:
(*i*) $G(s)$ is real for real s.
(*ii*) $\mathrm{Re}\,[G(s)] \geq 0 \quad$ for all $\quad \mathrm{Re}\,[s] > 0$

Now, it is possible to present the next definition:

Definition 3 ([14]) Assume that $G(s)$ is not identically zero for all s. Then $G(s)$ is SPR if $G(s - \epsilon)$ is PR for some $\epsilon > 0$.

The later is the definition of a SPR function. There are some results to verify this definition, the most common is provided in the next theorem:

Theorem 4 ([14]) Assume that a rational function $G(s)$ of a complex variable $s = \sigma + j\omega$ is real for all real s and is not identically zero for all s. Let n^* be the relative degree of $G(s) = n(s)/d(s)$ with $\mid n^* \mid \leq 1$. Then, $G(s)$ is SPR if and only if:
(*i*) $G(s)$ is analytic in $\mathrm{Re}\,[s] \geq 0$.
(*ii*) $\mathrm{Re}\,[G(j\omega)] > 0 \quad \forall \omega \in (-\infty, \infty)$.
(*iii*) (a) When $n^* = 1$, $\lim_{|\omega| \to \infty} \omega^2\, \mathrm{Re}\,[G(j\omega)] > 0$.
    (b) When $n^* = -1$, $\lim_{|\omega| \to \infty} \frac{G(j\omega)}{j\omega} > 0$.

In the previous theorem it is clear that if the transfer function $G(s)$ has a relative degree (n^*) equal to zero then the condition (iii) is not used. However, if the relative degree is 1 or -1 then the (iii) condition must be used. Nevertheless, in this regard, some comments establish that, even if this condition is not satisfied, this does not imply that the network does not dissipate energy, see [17].

Also, it is possible to verify the (*ii*) condition from theorem 4 using the following ratio:

$$\mathrm{Re}\,[G\,(j\omega)] = \frac{1}{2} \frac{n(j\omega)d(-j\omega) + n\,(-j\omega)\,d\,(j\omega)}{\mid d\,(j\omega)\mid^2}$$

It is clear that the positivity of $\mathrm{Re}\,[G\,(j\omega)]$ can be guaranteed with the following ratio:

$$R_p\left(\omega^2\right) = n(j\omega)d(-j\omega) + n\,(-j\omega)\,d\,(j\omega) \qquad (2)$$

Thus, theorem 4 can be transformed into the following theorem, see [25].

Theorem 5: A real rational function $G\,(s)$ is SPR if and only if:
(*i*) $D\,(s)$ is stable.

$(ii)\ R_p(\omega) > 0 \qquad \forall \omega \in (0, \infty).$

Remark 6: It is worth to bring upon our attention the following three things. First, the $R_p(\omega)$ is the same that $R_p(\omega^2)$ substituting ω^2 by ω; and then the degree of $R_p(\omega)$ is n. Second, the ω interval in (ii) condition from theorem 5 is reduced because the $R_p(\omega)$ is a symmetric function. And third, the (iii) condition from theorem 4 is included in the (ii) condition from theorem 5, for more details see [25].

So far, we have settled the basis of SPR, now we may continue to talk about robust SPR.

B. Robust strict positive realness

The previous definitions and results were obtained for transfer functions without uncertainty. Nevertheless, the interest of this paper is on transfer function with parametric uncertainty; specially, polynomic uncertainty. The robust version of SPR property is defined as follows:

Definition 7: A polynomic plant is robustly SPR if $G(s, q)$ is SPR for all $q \in Q$.

It is clear that the polynomic plant from (1) represents an infinite number of transfer functions, and therefore the robust SPR property results impossible to be verified directly. However, the objective of this paper is to find a method to verify this property using finite tests. Before introducing a finding that helps to build our main result, it is necessary to present the following definition:

$$
\begin{aligned}
h(\omega^2, q) &= |d(j\omega, q)|^2 \\
g(\omega^2, q) &= R_p(\omega^2, q)
\end{aligned}
\qquad (3)
$$

where:

$$R_p(\omega^2, q) = n(j\omega, q)d(-j\omega, q) + n(-j\omega, q)d(j\omega, r)$$

It can be seen that the function $h(\omega^2, q)$ is a polynomic function of ω with uncertain coefficients which are again polynomic functions of q_i in the vector q; this is also true for $R_p(\omega^2, q)$. Hence the next theorem presents necessary and sufficient conditions to verify the robust SPR property, see [25].

Theorem 8 ([25]) A polynomic plant is robustly SPR if and only if $G(s, \overline{q})$ is stable for some $\overline{q} \in Q$ and:

$(i)\ h(\omega, q)$ is positive for all $q \in Q$ and $\omega \in (0, \infty)$.
$(ii)\ g(\omega, q)$ is positive for all $q \in Q$ and $\omega \in (0, \infty)$.

As it can be noted, the problem of robust strict positive realness has been transformed into a problem where the positivity of two multivariable polynomic functions is verified using the previous theorem. Hereupon our goal is to verify the positivity of these functions using some procedure, e.g. the sign decomposition technique.

C. Sign decomposition of multivariable polynomic functions

This method analyzes the positivity of a multivariable real polynomic function by its decomposition into his positive and negative parts, see [10]. The sign decomposition is defined as follows:

Definition 9 ([10]) Let $f : R^n \to R$ be a continuos function and $Q \subset P \subset R^n$ a convex subset, $f(\cdot)$ has sign decomposition in Q if there exist two bounded nongrowing functions $f_n(\cdot) \geq 0$, $f_p(\cdot) \geq 0$, that $f(q) = f_p(q) - f_n(q)$ for all $q \in Q$. These functions will be called positive part of the function $f_p(\cdot)$, and negative part of the function $f_n(\cdot)$.

Here, the P set is considered to be a positive convex cone; see [10]. Now, the maximum (v_{max}) and minimum (v_{min}) vertices of the uncertainty set Q will be defined:

$$\| v_{min} \|_2 = \min_{q \in Q} \| q \|_2 \qquad (4)$$

$$\| v_{max} \|_2 = \max_{q \in Q} \| q \|_2$$

We can see that the v_{min} and v_{max} vertices are the minimum and maximum Euclidean of the uncertain set Q. The functions with sign decomposition have some properties related to the v_{min} and v_{max} vertices; these are presented in the next result:

Lemma 10 ([10]) Let $f : R^n \to R$ be a continuos nongrowing function and let $Q \subset P \subset R^n$ be a box with minimum and maximum Euclidian vertices v_{min}, v_{max}, then:

$$\min_{q \in Q} f(q) = f(v_{min}), \ \max_{q \in Q} f(q) = f(v_{max}) \qquad (5)$$

Based on this previous lemma it is now possible to present the following relevant result:

Theorem 11 ([10]) Let $f : R^n \to R$ be a continuos function with sign decomposition in Q such $Q \subset P \subset R^n$ is a box with minimum and maximum Euclidian vertices v_{min}, v_{max}, then $f(q)$ is lower and upper bounded

by $f_p(v_{\min}) - f_n(v_{\max})$ and $f_p(v_{\max}) - f_n(v_{\min})$, respectively.

From this result it is possible to determine if a function with sign decomposition is positive. This can be verified if $\mu = f_p(v_{\min}) - f_n(v_{\max})$ is greater than zero, this is only a sufficient condition, it is relaxed with the following theorem.

Theorem 12 ([11]) Let $f : R^n \to R$ be a continuos function with sign decomposition in Q such $Q \subset P \subset R^n$ is a box with minimum and maximum Euclidian vertices $v_{\min}$, $v_{\max}$, then the function $f(q) > 0$ in Q if and only if there exist some Γ^j box sets, such that $Q = \bigcup_j \Gamma^j$ and $\mu_j = f_p(v_{\min}^j) - f_n(v_{\max}^j)$ are greater than zero for each one Γ^j.

A more general result of this theorem is presented in [10], where a graphical version is included. However, for the purpose of this paper we only need this simplified version.

D. Robust absolute stability

The result to verify the robust absolute stability of the Lur'e system is presented in the following lemma:

Lemma 13 ([15],[25]) Consider the system from figure 1, where $\psi(t, y)$ satisfies the sector $[0, k]$ condition. Then, the system presented in figure 1 is robustly absolutely stable if $z(s, q) = 1 + kG(s, q)$ is SPR for all $q \in Q$.

It is important to note that this condition implies verifying the robust SPR condition of the fictitious transfer function $z(s, q)$. Hence, the robust absolute stability problem is transformed to determine if $z(s, q)$ is robustly SPR.

Many research results related to SPR transfer functions and robust SPR transfer functions have been published. While some of these results discusses the design and synthesis of robust SPR transfer function, see [4], [5], [13], [1], others are related to the analysis of robust SPR functions, see [19], [23]. The present work uses the sign decomposition to analyze the robust SPR property in order to get the results to verify the robust absolute stability property of Lur'e systems.

III. Main Result

The main result is divided in three sections. The first one contains some results to verify the robust SPR

property. The second part contains the sufficient conditions to verify the robust absolute property. And, the last one, is an illustrative example where the results are applied.

A. Robust SPR property

It was mentioned that the robust SPR condition of transfer functions will be verified using the sign decomposition approach. However, it is important to mention that this approach needs full knowledge of the low and high boundaries of the uncertainty, and as it was seen in theorem 8, ω is an unbounded parameter. Therefore, it is necessary to make a previous special operation before applying the sign decomposition method. This operation consists in determining the limits for the bound of ω. To do so we will define the next function:

$$
\begin{aligned}
h_{\min}(\omega) &= h_p(\omega, q^-) - h_n(\omega, q^+) \qquad (6) \\
g_{\min}(\omega) &= g_p(\omega, q^-) - g_n(\omega, q^+)
\end{aligned}
$$

where $h_p(\cdot)$, $h_n(\cdot)$, $g_p(\cdot)$ and $g_n(\cdot)$ are the negative and positive parts of $h(\omega, q)$ and $g(\omega, q)$ respectively, as it was defined in the definition 9; $q^- = \begin{bmatrix} q_1^- & \cdots & q_m^- \end{bmatrix}^T$ and $q^+ = \begin{bmatrix} q_1^+ & \cdots & q_m^+ \end{bmatrix}^T$. In the previous definition it is clear that the next conditions are satisfied:

$$
\begin{aligned}
h_{\min}(\omega) &\leq h(\omega, q) \\
g_{\min}(\omega) &\leq g(\omega, q)
\end{aligned}
$$

$$
\forall\, \omega \in (0, \infty)\,;\ q \in Q
$$

and therefore if $h_{\min}(\omega)$ and $g_{\min}(\omega)$ are greater than zero, also $h(\omega, q)$ and $g(\omega, q)$ will be greater than zero. It is important to note that due to the shape of $h_{\min}(\omega)$ and $g(\omega, q)$ it is possible to set a minimum value ω^- and a maximum value ω^+ so that the functions may take negative values an thus, the functions only have the possibility of being negative for the value range of $\omega \in [\omega^-, \omega^+]$ for $h(\omega, q)$ and $[\tau^-, \tau^+]$ for $g(\omega, q)$. This in turn implies that $h(\omega, q)$ and $g(\omega, q)$ will only have the possibility to be negative only when they are inside these intervals. This allows us to obtain the limits we were looking for the ω parameter in both functions. These values usually correspond to some roots of $h_{\min}(\omega)$ and $g_{\min}(\omega)$ respectively, and can be gotten graphically. With the limits of ω it is possible to define the following sets in order to use the sign decomposition approach:

$$V = [\omega^-, \omega^+] \text{ x } Q \qquad (7)$$
$$U = [\tau^-, \tau^+] \text{ x } Q$$

Applying theorems 8 and 12, we can present the next result.

Theorem 14: The polynomic plant from (1) is robustly SPR if and only if:

1. There exist some Γ^j box sets, such that $V = \bigcup_j \Gamma^j$ and $\gamma_j = h_p(v_{\min}^j) - h_n(v_{\max}^j)$ are greater than zero for each one γ_j.
2. There exist some Σ^j box sets, such that $U = \bigcup_j \Sigma^j$ and $\sigma_j = g_p(u_{\min}^j) - g_n(u_{\max}^j)$ are greater than zero for each one σ_j.

for all $v \in V$ and $u \in U$ where V and U are the sets defined in (7). This theorem can be applied to obtain the robust SPR property using the following algorithm:

Step 1.- Set $j = 1$.
Step 2.- Let $v_{\min}^j = \begin{bmatrix} q_1^- & \cdots & q_n^- \end{bmatrix}^T$ and $v_{\max}^j = \begin{bmatrix} q_1^+ & \cdots & q_n^+ \end{bmatrix}^T$ for each j.
Steo 3.- Compute γ_j for all j.
Step 4.- if $\gamma_j > 0$, then stop.
Step 5.- Divide the Q box into j smaller boxes and go to step 2.

This procedure is simpler than that presented in [25].

B. Robust absolute stability

From lemma 13 it is seen that the condition to ensure the robust absolute stability consists in determining if the $z(s, q)$ is a robust SPR transfer function. Additionally, when the relative degree of the polynomic plant $G(s, q)$ is known to be equal to 1, we assume that the $z(s, q)$ is also a polynomic plant with relative degree equal to zero. Then, it is possible to use the previous result presented in theorem 14. Before presenting the result, it is necessary to introduce the following definitions:

$$z(s, q) = \frac{\eta(s, q)}{\delta(s, q)} \qquad (8)$$

$$\alpha(\omega, q) = |\delta(j\omega, q)|^2 \qquad (9)$$
$$\beta(\omega, q) = \eta(j\omega, q)\delta(-j\omega, q) + \eta(-j\omega, q)\delta(j\omega, q)$$

The sets V and U are defined as in (7). Now, the result of robust absolute stability with polynomic uncertainty will be presented in the next theorem.

Theorem 15: Consider the nonlinear system from figure 1, where $\psi(t, y)$ satisfies the sector $[0, k]$ condition and $z(s, q) = 1 + kG(s, q) = \frac{\eta(s,q)}{v(s,q)}$ is analytic in $\text{Re}[s] \geq 0$. Then, the Lur'e system is robustly absolutely stable if:

1. There exist some Λ^j box sets, such that $V = \bigcup_j \Lambda^j$ and $\lambda_j = \alpha_p(v_{\min}^j) - \alpha_n(v_{\max}^j)$ are greater than zero for each one λ_j.
2. There exist some Θ^j box sets, such that $U = \bigcup_j \Theta^j$ and $\theta_j = \beta_p(u_{\min}^j) - \beta_n(u_{\max}^j)$ are greater than zero for each one θ_j.

Now, we may use the previous algorithm to apply this theorem.

C. Example

To illustrate the previous result, let us consider the following polynomic plant, that was considered in [25]:

$$G(s, q) = \frac{n(s, q)}{d(s, q)}$$

where:

$$n(s, q) = s^3 + (q_1^2 + 2)s^2 + (q_2^2 q_1 + 4)s + q_1 + 3$$
$$d(s, q) = s^3 + (q_1 + 2)s^2 + (q_2^2 + 2)s + 1$$

$$Q = \{q \in Q : q_1 \in [0, 1], q_2 \in [0, 1]\}$$

The $h(\omega, q)$ and $g(\omega, q)$ functions are as follows:

$$h(\omega, q) = h_p(\omega, q) - h_n(\omega, q)$$
$$g(\omega, q) = g_p(\omega, q) - g_n(\omega, q)$$

where the positive and negative parts of the $h(\omega, q)$ and $g(\omega, q)$ functions are the following:

$$h_p(\omega, q) = \omega^3 + (q_1^4 + 4q_1^2)\omega^2 + (q_1^2 q_2^4 + 8q_1 q_2^2 + 4)\omega$$
$$+ q_1^2 + 6q_1 + 9$$
$$h_n(\omega, q) = (2q_1 q_2^2 + 4)\omega^2 + (2q_1^3 + 6q_1^2 + 4q_1)\omega$$
$$g_p(\omega, q) = 2\omega^3 + (4q_1 + 4q_1^2 + 2q_1^3)\omega^2$$
$$+ (8q_2^2 + 4q_1 q_2^2 + 2q_1 q_2^4)\omega + 2q_1 + 6$$
$$g_n(\omega, q) = (2q_2^2 + 2q_1 q_2^2 + 4)\omega^2 + (4q_1^2 + 10q_1)\omega$$

Using the equations defined in (6) we get:

$$
\begin{aligned}
h_{\min}(\omega) &= \omega^3 - 6\omega^2 - 8\omega + 9 \\
g_{\min}(\omega) &= 2\omega^3 - 6\omega^2 - 14\omega + 6
\end{aligned}
$$

From these functions it is possible to get the intervals $[\omega^-, \omega^+] = [0.7531, 6.9633]$ and $[\tau^-, \tau^+] = [0.3757, 4.4279]$. And then the next sets.

$$
\begin{aligned}
V &= [0.7531, 6.9633] \text{ x } [0,1] \text{ x } [0,1] \\
U &= [0.3757, 4.4279] \text{ x } [0,1] \text{ x } [0,1]
\end{aligned}
$$

Now, applying the theorem 14 through the algorithm presented in section A, we can get the following result:

$$
\begin{aligned}
h(\omega, q) &> 6.8673 \\
g(\omega, q) &> 0.8658
\end{aligned}
$$

With this, we can conclude that the $G(s,q)$ is robustly SPR. This result was gotten running the algorithm presented in section A after the 8th iteration. It is important to mention that the obtained values in [25] are: 4.6532 and 0.8284 for $h(\omega, q)$ and $g(\omega, q)$ respectively, which represents a more conservative result.

IV. Conclusions

This paper presented some results to verify the robust absolute stability property for polynomic Lur'e systems. These results were based on the sign decomposition concept which enables to obtain a simpler test to verify the robust stability property of a nonlinear system than other previous results. Moreover, these results can be easily programmed into a computer to get the conditions. A future research problem can be to consider a more general representation of the uncertainty like nonlinear uncertainty.

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