Constraint-based Fuzzy Models for an Environment with Heterogeneous Information Granules

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Abstract:- A novel constraint-based fuzzy modeling approach is proposed. Features of the proposed model are enumerated as below. 1)The knowledge base of a constraint-based fuzzy model can incorporate information with various types of fuzzy predicates. Under this perspective, rule-based fuzzy models can be recognized as a special case of the proposed fuzzy models. 2) A corresponding inference mechanism for the proposed model can deal with heterogeneous information granules. 3) Both numerical and linguistic inputs can be accepted for predicting new outputs.

Key-Words:- Computing with words; Constraint-based problem solving; Fuzzy modeling; Granular computing; Information granulation.

1 Introduction

System modeling has the capability of discovering nonlinear, non-obvious, and potentially useful information and knowledge from data sets. For a given set of input-output data, conventional system modeling can be used to describe a given data set by mathematical equations that provide potential mapping knowledge between input and output data, and the model obtained is applied to predict output data for new inputs. In terms of computational efficiency, conventional system modeling approach is a useful tool for knowledge discovery and data prediction. For dealing with uncertainties, fuzzy modeling involving fuzzy predicates for describing unknown systems, has also been applied to notable applications, including system simulation, industrial control, and financial analysis, etc. Originally, fuzzy modeling approach proposed by Mamdani [2], [3] utilizes linguistic "IF-THEN" rules to establish qualitative relationships among the variables in a system . The linguistic models allow the usage of information granules expressed in the form of natural language statements and consequently make models transparent to interpretation. However, owing to the existence of a large gap between human knowledge and linguistic rules in complex and uncertain process, the model is hard to be optimized. On the other hand, a great deal of research activities have focused on the development of methods to build fuzzy models from numerical (raw) data. One of the most renowned data-driven fuzzy models, proposed by Sugeno [4], [5], is based on fuzzy partition of input space and least squares method. Since this type of fuzzy model uses linear equations as building granules, it is efficient in computation, but it is weak in intuitive explanation. To have both computational efficiency and interpretation transparency, a hybrid model that represents the consequent part with linguistic granules and has a modified inference mechanism similar to the one given by Sugeno's model in [4], [5] is suggested by Sugeno and Yasukawa in [6].

While the aforementioned methods have been widely applied, there are still some limitations need to be pointed out. First, each model uses homogeneous information granules as the basic building blocks for fuzzy models. That is, Mamdani's approach uses linguistics statements of human and Sugeno's method utilizes linear equations as consequent part that are derived from raw data. However, in real world problems, linguistic statements cannot exist for every system, especially for a highly dynamic and complex plant; on the other hand, the functional consequents have a drawback that the number of rules in the model may grow very fast. Therefore, a heterogeneous model may be needed for approaching the data set more precisely and providing more flexibility. The second characteristic is that not all the existing fuzzy models can treat both the numerical and linguistic values as inputs. Unfortunately, environments mixed numerical and linguistic data together are frequently occurred since the inputs can be derived from human experts and artifacts. Thus, building a fuzzy model that is capable of dealing with numerical-linguistic data becomes an important issue.

In this paper, a novel constraint-based fuzzy modeling approach is proposed. Features of the proposed model is concluded as follows: 1) The knowledge base of a constraint-based fuzzy model can incorporate information with different types of fuzzy predicates. Under this perspective, the rule-based fuzzy model can be recognized as a special case of the proposed fuzzy models. 2) A corresponding inference mechanism for the proposed model can deal with heterogeneous information granules. 3) Both numerical and linguistic inputs can be accepted for predicting new outputs. Notably, features (2) and (3) are consequents of constraints-based inference engine.

The remainder of this paper is organized as follows. In Section 2, constraint-based fuzzy models are introduced. In Section 3, the main concept of a design methodology for the constraint-based fuzzy model is presented. The design can be done via a two-stage process, including fuzzy clustering and fuzzy regression. Then, simulation results are given in Section 4 followed by some concluding remarks in Section 5.

2 Constraint-based Fuzzy Models

Constraint-based fuzzy models use fuzzy constraints as the knowledge components to model the real world. The kernel of the fuzzy model is a fuzzy constraint network (FCN), a formal definition of FCN and an associated reasoning method are shown in Sec. 2.1. Then, based on these basic concepts, the architecture and the inference mechanism of the proposed fuzzy model is presented in Sec. 2.2.

2.1 Main concepts of fuzzy constraints

Much of what we know about many real-world problems can be represented as sets of constraints. For example, in simulation, constraints serve not only as descriptions of the system to be simulated, but also as commands to the system telling it that certain conditions must be satisfied. By asserting additional constraints, the user can prod the simulated system and observe its response. Similarly, in engineering design, constraints represent the requirements that the artifact being designed must satisfy. The task of designing then becomes that of exploring design alternatives in a solution space bounded by these constraints.

A constraint is a construct describing a relationship among one or more objects. A constraint network is a collection of objects interlinked by a set of constraints that specify relationships which must be satisfied by the values that are assumed by these objects. Formally, a fuzzy constraint network and its intent may be defined as follows [1].

Definition 1. A *fuzzy constraint network* is a triple $(\mathcal{U}, \mathbf{X}, \mathbf{C})$ where $\mathcal{U}$ is a universe of discourse, $\mathbf{X}$ is a tuple of n non-recurring objects $X_1, \ldots, X_n$, and $\mathbf{C}$ is a set of $m \geq n$ constraints $C_1(T_1), \ldots, C_m(T_m)$. In a constraint $C_j(T_j)$, T_j is a subtuple of $\mathbf{X}$, of arity a_j, and $C_j(T_j)$ is a (possibly fuzzy) subset of the Cartesian product $\mathcal{U}^{a_j}$. Of the $m \geq n$ constraints, there are at least n unary constraints, each object $X_i \in \mathbf{X}$ being subject to its own unary constraint $C_i(X_i)$.

Definition 2. The *intent* of a fuzzy constraint network $(\mathcal{U}, \mathbf{X}, \mathbf{C})$, written $\Pi_{\mathcal{U},\mathbf{X},\mathbf{C}}$, is an n-ary possibility distribution for the objects involved in the network. That is

$$\Pi_{\mathcal{U},\mathbf{X},\mathbf{C}} = \overline{C}_1(T_1) \cap \ldots \cap \overline{C}_m(T_m) \tag{1}$$

where, for each constraint $C_j(T_j) \in \mathbf{C}$, $\overline{C}_j(T_j)$ is its cylindrical extension in the space $\mathbf{X} = (X_1, \ldots, X_n)$.

Here, the network intent is a fuzzy set of n-tuples, each tuple giving a valuation for the n objects in $\mathbf{X}$, the membership of the tuple in the intent being the degree to which the valuation satisfies all the constraints in $\mathbf{C}$.

Once a real world problem is modeled as a set of constraints, physical parameters maybe interpreted as objects of the constraint network. Finding the possible values of objects is the main task of a reasoning process. In the case of fuzzy modeling, owing to the fuzzy constraint network being considered is single directional, a fuzzy inference method can be performed by the *marginal particularized possibility distribution* that is defined as below.

Definition 3. Given a fuzzy constraint C_l and a set Π of fuzzy possibility distributions $\Pi_{X_{l,1}}, \ldots, \Pi_{X_{l,a_l}}$, associated with the objects $X_{l,1}, \ldots, X_{l,a_l}$ in $\Theta(C_l)$, respectively, the marginal particularized possibility distribution for any variable $X_{l,i}$ in $\Theta(C_l)$ that is allowed by the constraint C_l and the possibility distribution in Π is defined by

$$Q\left(C_l, \Pi, X_{l,i}\right) = \mathrm{Proj}_{X_{l,i}}(C_l \cap \overline{\Pi}_{X_{l,1}} \cap \cdots \cap \overline{\Pi}_{X_{l,i-1}} \cap \overline{\Pi}_{X_{l,i+1}} \cap \cdots \cap \overline{\Pi}_{X_{l,a_l}}), \tag{2}$$

where, $\Theta(C_l)$ means the set of objects referenced by the fuzzy constraint C_l, for all $X_{l,k}$ in $\Theta(C_l)$, $\overline{\Pi}_{X_{l,k}}$ is the cylindrical extension of $\overline{\Pi}_{X_{l,k}}$ in the space $(X_{l,1}, \ldots, X_{l,a_l})$.

The projection operation used in (2) is defined as:

Definition 4. The projection of an n-ary possibility distribution $\Pi_{\mathbf{X}}$ onto a subset $\mathbf{Y} = (Y_1, \ldots, Y_k)$ of the objects in $\mathbf{X} = (X_1, \ldots, X_n)$ is a k-ary possibility distribution which is denoted by

$$\Pi_{\mathbf{Y}} = \mathrm{Proj}_Y \Pi_{\mathbf{X}},$$

and defined by

$$\pi_{\mathbf{Y}}\left(y_1, \ldots, y_k\right) = \max_z \pi_{\mathbf{X}}\left(x_1, \ldots, x_k\right),$$

where $\pi_{\mathbf{Y}}$ is the possibility distribution function of $\Pi_{\mathbf{Y}}$ and the maximum is taken over a subset $\mathbf{Z} = \{Z_1, , Z_l\}$ of objects $\mathbf{X}$ which is complementary to the subset $\mathbf{Y}$.

The marginal particularized possibility distribution plays a key role in an inference process, that will be elaborated in the next section.

2.2 A novel constraint-based fuzzy model

A constraint-based fuzzy model incorporates fuzzy constraint network in the knowledge base and uses constraint-based inference engine to infer output while inputs are given. Consider a MISO system: $\mathbf{x} = (x_1, \ldots, x_n)$ is an n-dimensional input vector; y is the output variable. Then, a constraint-based fuzzy model can be defined as below.

Definition 5. A constraint-based fuzzy model can be represented as a fuzzy constraint network $(\mathcal{U}, \mathbf{X}, \mathbf{C})$, where $\mathcal{U}$ is a universe of discourse, $\mathbf{X} = \mathbf{x} \times y$, and $\mathbf{C}$ is a set of $m \geq n+1$ constraints $C_1(T_1, y)$, ..., $C_m(T_m, y)$. In a constraint $C_j(T_j, y)$, T_j is a subtuple of $\mathbf{x}$. Furthermore, for each $C_j(T_j, y)$ in $\mathbf{C}$, it can be divided into a amalgamate (denoted as $\tilde{\cup}$) of a set of granules:

$$\begin{aligned} C_j(T_j, y) &= \mathcal{G}_{j1} \tilde{\cup} \mathcal{G}_{j2} \tilde{\cup} \cdots \tilde{\cup} \mathcal{G}_{jk} \\ &= (\mathcal{G}_{j1}|_{\tilde{\mathbf{A}}_1}) \cup (\mathcal{G}_{j2}|_{\tilde{\mathbf{A}}_2}) \cup \cdots \cup (\mathcal{G}_{jk}|_{\tilde{\mathbf{A}}_k}) \end{aligned}$$

where $\mathcal{G}_{jh}$ is a fuzzy relation defined on $\mathbf{X}$;

$$\begin{aligned} (\mathcal{G}_{ji}|_{\tilde{\mathbf{A}}_i}) = \{((\bar{x}_i, y_i), \mu) \in \mathbf{x} \times y \mid \\ \mu = \min(\mathcal{G}_{ji}(\bar{x}_i, y_i), \tilde{\mathbf{A}}_i(\bar{x}_i))\} \end{aligned}$$

is a reduced fuzzy set. The fuzzy set $\tilde{\mathbf{A}}_h$ is called a corresponding region of $\mathcal{G}_{jh}$.

Notably, the constraint-based fuzzy model has some features. The architecture of the constraint-based fuzzy model can incorporate information with different types of fuzzy predicates. That is, the corresponding inference mechanism for the proposed model can deal with such a model. Then, both numerical and linguistic inputs can be accepted for predicting new outputs. The first feature can be proved by the following proposition.

Proposition 1. The architecture of fuzzy model can incorporate information with heterogeneous types.

Proof. Without loss of generality, we consider a fuzzy system consists of fuzzy predicates with linguistic and functional types. A linguistic fuzzy model can be represented as:

$$\mathbf{S} = \{(\tilde{\mathbf{A}}_i, \tilde{B}_i) \mid i = 1 \ldots m\}, \tag{3}$$

where $\tilde{\mathbf{A}}_i$ is an $n-$dimensional fuzzy set defined on $\mathbf{x} = (x_1, \ldots, x_n)$ and $\tilde{B}_i$ is a one dimensional fuzzy set on y. For each fuzzy pair $(\tilde{\mathbf{A}}_k, \tilde{B}_k)$, which represents a fuzzy granule $\mathcal{G}_k = (\tilde{\mathbf{A}}_k, \tilde{B}_k)$ with corresponding region $\tilde{\mathbf{A}}_k$. Specifically, the details of the fuzzy granule is:

$$\begin{aligned} \mathcal{G}_k = \{((a_1, a_2, \ldots, a_n, b), \mu) \mid \mu = \\ \min(\mu_{\tilde{\mathbf{A}}_k}(a_1, a_2, \ldots, a_n), \mu_{\tilde{B}_k}(b))\}. \end{aligned}$$

Then, $C(\mathbf{x}, y) = \mathcal{G}_1 \cup \mathcal{G}_2 \cup \cdots \cup \mathcal{G}_m$. For any input vector $\mathbf{x} = (a_1, \ldots, a_n)$, the output $\tilde{y}$ can be calculated as

$$\begin{aligned} \tilde{y} &= Q(C, \Pi, y) \\ &= \mathrm{Proj}_y\left(C \cap \overline{\Pi}_{x_1} \cap \overline{\Pi}_{x_2} \cap \cdots \cap \overline{\Pi}_{x_n}\right) \\ &= \pi_{\mathbf{Y}}\left(y_1, \ldots, y_k\right) = \max_z \pi_{\mathbf{X}}\left(x_1, \ldots, x_k\right). \end{aligned}$$

Next, we consider a fuzzy system consists of functional fuzzy constraints, that can be represented as a set of pairs such as

$$\mathbf{S} = \{(\tilde{\mathbf{A}}_i, f_i(x)) \mid i = 1 \ldots m\}, \tag{4}$$

where $\tilde{\mathbf{A}}_i$ is an $n-$dimensional fuzzy set defined on $\mathbf{x} = (x_1, \ldots, x_n)$ and $f_i(x)$ is a crisp equation. Specifically, (4) can be rewritten as

$$\mathbf{C} = \{(x, \frac{\sum_i \tilde{\mathbf{A}}_i(\mathbf{x}) f_i(\mathbf{x})}{\sum \tilde{\mathbf{A}}_i(\mathbf{x})}) \mid i = 1 \ldots m\}.$$

For the considered fuzzy constraints, the marginal particularized possibility distribution can be derived. Therefore, a constraint-based fuzzy model can deal with heterogeneous fuzzy predicates. □

Consequently, we have the following:

Corollary 1. *In a constraint-based fuzzy model, both numerical and linguistic inputs can be accepted for predicting new outputs.*

3 A Design Methodology

Based on granular computing, the main idea of a design methodology for constraint-based fuzzy model is provided in this section. According to Zadeh's original paper on granular computing [7], [8], human cognition may be understood to be based on and structured by processes of *granulation*, *organization*, and *causation*. Granulation is a process that decomposes the whole concept space into parts, conversely, organization is the process that integrates parts into wholes, and then, causation is a process that associates causes with effects. Fuzzy modeling is a process that can be mapped to organization. During the process of organization, some characteristics could be induced:

- The level of abstraction is risen;
- the purpose of organization of fuzzy granules is to integrate some smaller granules into fewer but larger ones;
- the decision of which granules should be drawn together is based on some predefined similarity criterion; and
- there is no direct relation between the granularity of a granule and its levels of abstraction.

Thus, based on these properties, *organization* can be formally defined as below.

Definition 6 (Organization). Given a set of granules $\mathfrak{G}$ and a similarity criterion $\mathcal{S}$, *organization* is a process that integrates the elements of $\mathfrak{G}$ in order to form a new set of granules $\mathfrak{G}'$ such that the following conditions hold.

- The number of elements of $\mathfrak{G}'$ is fewer than that of elements of $\mathfrak{G}$; and
- the similarity criterion $\mathcal{S}$ is satisfied with $\mathfrak{G}$ and $\mathfrak{G}'$.

Fuzzy modeling mainly deals with the construction of quasi-structured knowledge from unstructured data. In other words, with fuzzy modeling, knowledge at a higher-level of abstraction is obtained from a lower-level one. From a granular perspective, fuzzy modeling is the process of seeking fuzzy granules at a higher level of knowledge abstraction that are appropriate to represent the lower-level knowledge. In this sense, the task of fuzzy modeling is actually a process of information organization. Specifically, a fuzzy-constraint-descriptive model can be derived from a two-stage organization process, that is, a granule-prototype fuzzy clustering method (GFCM) followed by a fuzzy regression method for granules (FRG). A granular-prototype fuzzy clustering is a fuzzy clustering technique that uses fuzzy granules as predefined prototypes. Then, a fuzzy regression method for granules is fuzzy regression method for manipulating fuzzy granules as inputs. The details of these two processes are elaborated in the next section.

4 Experimental Results

In this example, a set of 2-dimensional samples is generated randomly to serve as the initial raw data needed by the GFCM; then, with those granules acquired through the GFCM acting as the input data, the FRG is employed to obtain the final results.

Let us consider a set of input-output patterns generated by

$$\begin{aligned} x_k &= -1.5 + 0.05(k-1), \qquad k = 1, 2, \ldots, 61, \\ y_k &= x_k^3 - x_k - 0.5 + \mathrm{rand}[-1, 1], \end{aligned} \tag{5}$$

where $\mathrm{rand}[-1, 1]$ represents a real number randomly generated in the interval $[-1, 1]$. By this mechanism, we can generate 61 points between $y = x^3 - x - 0.5$ and $y = x^3 - x + 0.5$.

Applying GFCM to these data, granule-prototype constraints and similarity constraints should be defined at first. The granule-prototype constraints can be chosen from any symmetrical membership functions. In this example, we utilize the following Gaussian type membership function,

$$\mathcal{G}(\sigma, \mathbf{c}) = e^{-\frac{1}{2}(\frac{\|\mathbf{x}-\mathbf{c}\|}{\sigma})^2}, \tag{6}$$

as the prototype of the upper-level granules.

A possible choice of the similarity constraints, then, is defined as

$$\text{maximize} \qquad J(g) = \sum_{i=1}^{c} \sum_{j=1}^{n} e^{-\frac{1}{2}(\frac{\left\|\mathbf{x}_j - \mathbf{c}_i\right\|}{\sigma})^2}. \tag{7}$$

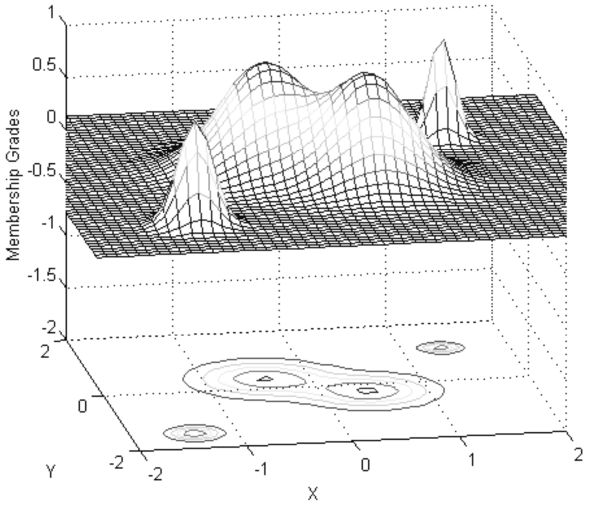

Figure 1: The organized granules: $\mathcal{G}_1([(-1.42, -1.40), 0.5])$, $\mathcal{G}_2([(-0.53, 0.46), 0.5])$, $\mathcal{G}_3([(0.52, -0.25), 0.5])$, and $\mathcal{G}_4([(1.27, 0.92), 0.5])$.

Then, we can organize the sample data as the following four Gaussian-type fuzzy granules,

$$\begin{aligned}
\mathcal{G}_1(\mathbf{c},\sigma) &= \mathcal{G}_1([(-1.42, -1.40), 0.5]),\\
\mathcal{G}_2(\mathbf{c},\sigma) &= \mathcal{G}_2([(-0.53, 0.46), 0.5]),\\
\mathcal{G}_3(\mathbf{c},\sigma) &= \mathcal{G}_3([(0.52, -0.25), 0.5]),\\
\mathcal{G}_4(\mathbf{c},\sigma) &= \mathcal{G}_4([(1.27, 0.92), 0.5]),
\end{aligned} \tag{8}$$

as shown in Figure 1. Figure 2 shows the contour plot of the organized granules with original sample data.

The resultant granules (8) will be considered as the lower-level granules of FRG. After that, a nonlinear fuzzy equation will be derived through a generalized TSK model.

The final nonlinear fuzzy equation is shown in Figure 3. Figure 4 is the contour of the final fuzzy constraints.

5 Conclusions

This paper has presented a novel constraint-based fuzzy model. In the proposed framework, knowledge transformation process is viewed as the organization process in granular computing. A fuzzy-constraint-descriptive model is thus attained from two-stage operations: fuzzy clustering for granules and fuzzy regression. In contrast to other fuzzy modeling approaches, the approach presented in this paper has five important aspects:

- *General:* The proposed model is a generalized model. The building blocks of the fuzzy model

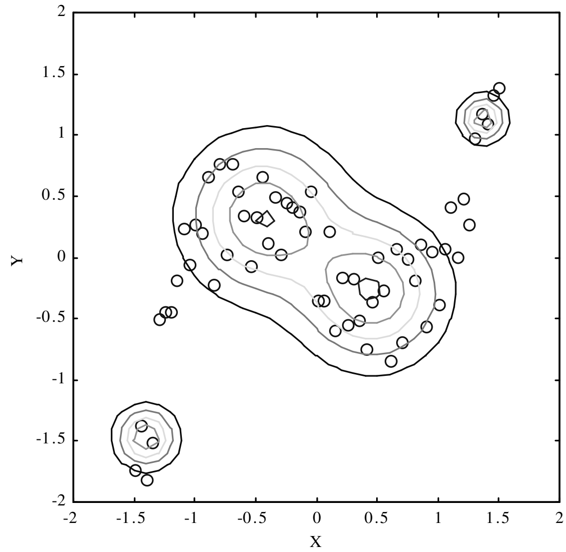

Figure 2: The contour plot of organized granules along with input-output data.

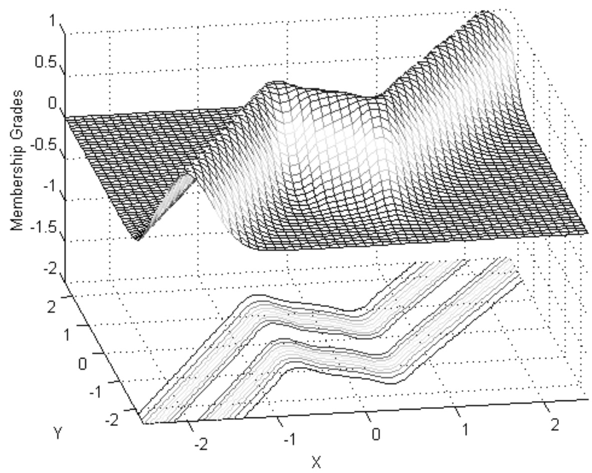

Figure 3: The final fuzzy nonlinear equation. Local constraints are: $\tilde{y} = ([2, 0.6])x + ([1.6, 0.6])$, $\tilde{y} = ([-0.5, 0.6])x + ([0, 0.6])$, and $\tilde{y} = ([1.7, 0.6])x + ([-1.2, 0.6])$.

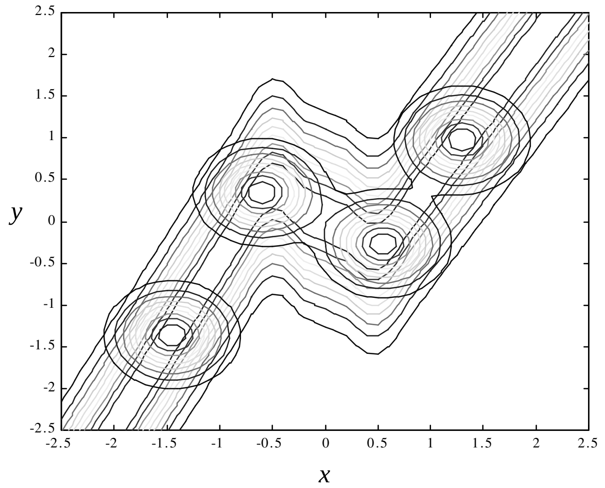

Figure 4: The contour plot of the organized nonlinear fuzzy equation and the lower-level granules.

are fuzzy constraints, which provide more expressive power than fuzzy rules. The rule-based fuzzy models are a special case of the proposed model is proved.

- *Flexible:* Constraint-based fuzzy model is hetrogeneous, and the types of constraints are not restricted to linear equations or fuzzy singletons. Actually, fuzzy constraints are relations among system variables, types including fuzzy equations, inequations, fuzzy points, etc. These different types of fuzzy constraints can be viewed as granules with different granularity.

- *Intuitive:* Granular computing is highly related to human cognition. In granular computing, we operate on information with different levels of granularity. Fuzzy modeling can be recognized as organization from finer granules to coarser ones. As a consequent, the deriving process for constraint-based fuzzy model is intuitive.

- *Unified Framework:* There are two folds that the proposed framework is unified. In one sense, constraint-based fuzzy model can be viewed as a generalized model of rule-based one. On the other hand, constraint-based granular computing is also a unified formalism of various proposed methods.

While the proposed approach has yielded some promising results, considerable work remains to be done, such as the development of a methodology for asymmetrical fuzzy granules and the examination of the proposed approach for practical scenarios.

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