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Root clustering method for a small-signal stability analysis of power systems

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Abstract: - We present a new method of a small-signal stability analysis for power systems with parametric uncertainties in load characteristics. These uncertainties must always be considered in a power system stability assessment since a power demand changes constantly and a precise composition of the electrical loads is usually unknown. The proposed method is based on an iterative algorithm which determines root clustering of a polytope of polynomials in a simply connected domain. It is suitable for an assessment of the electro-mechanical oscillations and dynamical voltage stability of power systems. According to our method stability verification at each iteration step is analytical and requires significantly shorter computation time than other methods available in literature.

Key-Words: Electro-mechanical oscillations, Parametric uncertainty, Power system analysis, Root clustering, Voltage stability.

1 Introduction

One of the dominant factors defining a post-disturbance behavior of power system and its voltage stability limits is electrical loads characteristics. Therefore an adequate load representation is of a primary importance in the power system stability assessment. In the traditional stability analysis load characteristics are assumed to be known [3]. However, it should be noted that this assumption is generally not valid. This is due to the following fact: Power demand changes permanently while precise information on a complex load composition is usually absent. As a result, there exist large uncertainties in the parameters of the load characteristics and, consequently, robust control approach is required in the voltage stability analysis. An early attempt to incorporate robust control theory in power system stability studies was carried out in [4]. In this paper static exponential load characteristics with unknown values of the exponents are assumed together with synchronous generator dynamics and algebraic system equations. A particular linearization technique [5], treating the load’s active and reactive powers as input, load voltage phase angle and magnitude as output, and load characteristic exponents as parameters of the system, leads to a state space model which is applicable to a small-signal stability analysis. A closed-loop characteristic polynomial of this model is affine in the parameters of the load characteristics. Assuming given operating conditions and nominal values of the exponents for which the closed-loop characteristic polynomial has all its zeros in a prescribed region of the complex plane, i.e. is D-stable, the maximal tolerable deviations of the load model exponents, which preserve this D-stability, are found using the testing function derived in [1]. For the robust stability application it was assumed in [4] that absolute values of the active and reactive load exponent deviations are equal. This assumption is unjustified from the engineering viewpoint. Moreover, the analysis presented in [1] and [4] suffers from even more significant drawback: the exploited testing function is not analytical. As a result, there always exists a possibility that an unstable set of parameters will be overlooked. To overcome the above mentioned drawbacks we have developed a new totally analytical method which allows one to find a complete set of stable load parameters without any preliminary assumptions on their relation [6]. This method is based on a zero set concept [2] and involves a commutative algebra to calculate boundaries of the stable parameter set. The calculations involving the commutative algebra may become very time consuming. Therefore, to achieve a fast stability assessment, we have developed a new analysis technique which is described in the present paper. This iterative technique is based on the necessary and sufficient conditions for root clustering of a polytope of polynomials in a simply connected domain [8]. At each iteration step, stability verification is completely analytical preventing an unstable set of parameters from being overlooked. The proposed technique is suitable for different types of small-signal power system stability studies, such as dynamic voltage stability and low-frequency oscillations. Its application to a three-bus power system is demonstrated by numerical examples.

2 Power System Model

In the present paper a three-bus power system shown in Figure 1 is considered. It includes synchronous generator at bus 1, large intermediate load area at bus 2 and the rest of the power system represented as a stiff power system having an infinite power capability at bus 3.

The nonlinear mathematical model of this system is derived according to the strategy proposed in [5]. The final equations, which define the relations between system variables, are shown below (neglecting damping windings, saturation, flux time derivatives, armature resistances). The synchronous generator is modeled by

Flux-decay model of the electromagnetic dynamics:

$$V_d = X_q I_q \tag{1}$$

$$V_q = -X_d^{'} I_d + E_q^{'} \tag{2}$$

$$T_{d0}^{'} \frac{dE_q^{'}}{dt} = -E_q^{'} - \left(X_d - X_d^{'}\right) I_d + E_{fd} \tag{3}$$

Rotor angle equation:

$$\frac{d\delta}{dt} = \omega - \omega_{syn} \tag{4}$$

Swing equation of motion dynamics:

$$\frac{2H}{\omega_{syn}} \cdot \frac{d\omega}{dt} = T_m - T_e - \frac{D_f}{\omega_{syn}} \left(\omega - \omega_{syn}\right) \tag{5}$$

Terminal voltage algebraic equations:

$$V_g = \sqrt{V_d^2 + V_q^2} \tag{6}$$

$$\theta_g = \arctan\left(\frac{V_q}{V_d}\right) + \delta - \frac{\pi}{2} \tag{7}$$

First-order automatic voltage regulator (AVR) dynamics:

$$T_E \frac{dE_{fd}}{dt} = -E_{fd} + K_A \left(V_{ref} - V_g\right) \tag{8}$$

Active and reactive power equations:

$$P_g = V_d I_d + V_q I_q = \left(X_q I_q\right) I_d + \left(-X_d^{'} I_d + E_q^{'}\right) I_q \tag{9}$$

$$\begin{aligned} Q_g &= V_q I_d - V_d I_q = \left(-X_d^{'} I_d + E_q^{'}\right) I_d + \left(X_q I_q\right) I_q \\ &= \left(-X_d^{'} I_d + E_q^{'}\right) I_d - X_q I_q^2 \end{aligned} \tag{10}$$

The transmission network is represented using the power flow equations

$$P_g = V_g^2 Y_{11} \cos\alpha_{11} + V_g V_l Y_{12} \cos\left(\theta_g - \theta_l - \alpha_{12}\right) \tag{11}$$

$$Q_g = -V_g^2 Y_{11} \sin\alpha_{11} + V_g V_l Y_{12} \sin\left(\theta_g - \theta_l - \alpha_{12}\right) \tag{12}$$

$$\begin{aligned} P_l &= -V_l V_g Y_{21} \cos\left(\theta_l - \theta_g - \alpha_{12}\right) \\ &\quad -V_l^2 Y_{22} \cos\alpha_{22} - V_l V_s Y_{23} \cos\left(\theta_l - \alpha_{23}\right) \end{aligned} \tag{13}$$

$$\begin{aligned} Q_l &= -V_l V_g Y_{21} \sin\left(\theta_l - \theta_g - \alpha_{12}\right) \\ &\quad +V_l^2 Y_{22} \sin\alpha_{22} - V_l V_s Y_{23} \sin\left(\theta_l - \alpha_{23}\right) \end{aligned} \tag{14}$$

The load power-voltage relations are described by the static exponential model:

$$P_l = P_{l0} \left(\frac{V_l}{V_{l0}}\right)^{n_p} \qquad Q_l = Q_{l0} \left(\frac{V_l}{V_{l0}}\right)^{n_q} \tag{15}$$

After a systematic linearization [5] around the operating point and some matrix manipulations, the state-space representation of the power system is obtained:

$$\begin{aligned} \frac{d}{dt}\mathbf{\Delta X} &= A\mathbf{\Delta X} + B\mathbf{\Delta S_l}, \\ \mathbf{\Delta Z_l} &= C\mathbf{\Delta X} + D\mathbf{\Delta S_l}, \\ \mathbf{\Delta S_l} &= H\left(n_p, n_q\right)\mathbf{\Delta Z_l} \end{aligned} \tag{16}$$

where

$\mathbf{\Delta X} = \begin{bmatrix} \Delta\delta & \Delta\omega & \Delta E_q^{'} & \Delta E_{fd} \end{bmatrix}^T$ is a deviation vector of the generator state variables,

$\mathbf{\Delta S_l} = \begin{bmatrix} \Delta P_l & \Delta Q_l \end{bmatrix}^T$ is a vector of active and

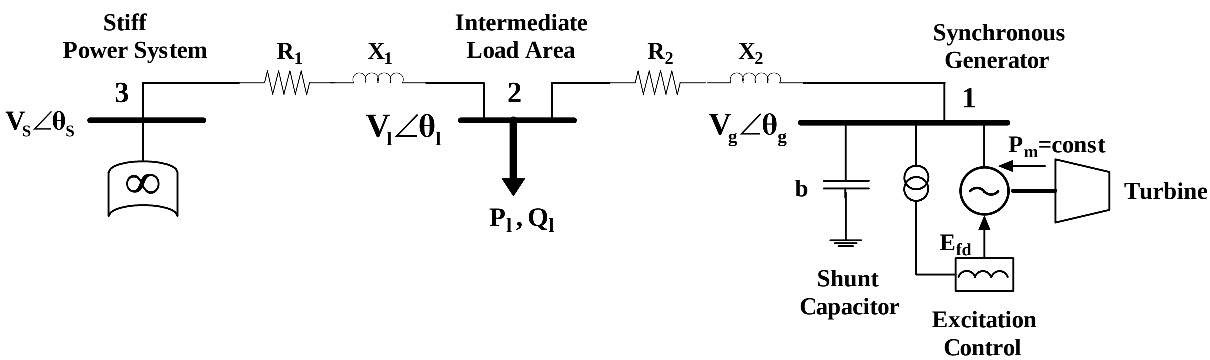

Fig. 1. A three-bus power system.

reactive load power deviations, $\mathbf{\Delta Z_l} = \left[\Delta\theta_l \quad \Delta V_l\right]^T$ is a vector of load bus voltage magnitude and phase angle deviations.

As can be seen, the above equations define MIMO dynamic system shown in Fig. 2 with $\mathbf{\Delta S_l}$ as an input and $\mathbf{\Delta Z_l}$ as an output. The transfer function matrix of this system is

$$G(s) = C(sI - A)^{-1} B + D \tag{17}$$

and, consequently, the closed-loop characteristic polynomial (CLCP) is obtained according to [4] as

$$CLCP(s, n_p, n_q) = \det\left[I - G(s) H(n_p, n_q)\right] \times \times \Delta\left[G(s)\right]\Delta\left[H(n_p, n_q)\right] \tag{18}$$

where $\Delta\left[G(s)\right]$ and $\Delta\left[H(n_p, n_q)\right]$ are characteristic polynomials of $G(s)$ and $H(n_p, n_q)$ respectively.

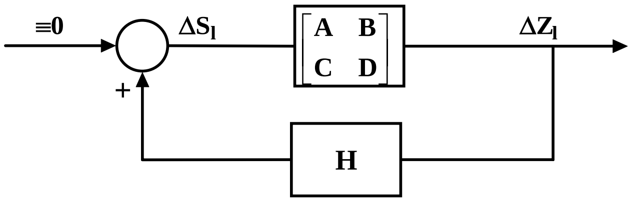

Fig. 2. MIMO model of the power system.

3 Problem Formulation

As it was shown in [6], the closed-loop characteristic polynomial of the linearized power system model derived in the previous section depends affinely on the parameters of the exponential load characteristics. Therefore our analysis is restricted only to the case when the characteristic polynomial

$$p(s, q) = s^n + \sum_{i=0}^{n-1} a_i(q) s^i \tag{20}$$

has coefficients $a_i(q)$ which depend affinely on underlying physical parameters $q_1, q_2, ..., q_l$, while each of these parameters is known only within given bounds $\left[q_k^-, q_k^+\right]$.

The resulting set of polynomials turns out to be a polytope. That is, this set is the convex hull of the 2^l polynomials obtained by setting q to an extreme point.

The main theorem presented in [8] and given below provides an effective method of the robust multidimensional stability check up.

Theorem 1 [8]: *All zeros of the polynomials in the given polytopic family lie in a simply connected domain D (i.e. the polytope of the polynomials is D-stable), if and only if*

i). one arbitrary vertex polynomial of the polytope has all its zeros inside D, and

ii). For every two vertices of the polytope (corresponding to polynomials P_i and P_j) which are the end-points of the exposed edge, if

$$R_i(\delta) X_j(\delta) - R_j(\delta) X_i(\delta) = 0 \tag{21}$$

for some real parameter $\delta = \delta_0 \in \Delta$ ($\Delta \in \mathbb{R}$) then, unless $R_i(\delta_0) = R_j(\delta_0) = 0$, the condition is

$$R_i(\delta_0) R_j(\delta_0) > 0 \tag{22}$$

If $R_i(\delta_0) = R_j(\delta_0) = 0$ the above condition should be replaced by

$$X_i(\delta_0) X_j(\delta_0) > 0 \tag{23}$$

where

$$R_i(\delta) = \mathrm{Re}\{P_i\ [\varphi\ (\delta)]\},$$

$$R_j(\delta) = \mathrm{Re}\{P_j\ [\varphi\ (\delta)]\},$$

$$X_i(\delta) = \frac{1}{j}\mathrm{Im}\{P_i\ [\varphi\ (\delta)]\},$$

$$R_j(\delta) = \frac{1}{j}\mathrm{Im}\{P_j\ [\varphi\ (\delta)]\}$$

The additional conditions needed to avoid a possibility of a zero moving from the domain D in the complex plane to its complementary $\bar{D}$ through infinity are

$$a_n^{(i)} \neq 0 \quad \forall i \tag{24}$$

$$\frac{a_n^{(j)}}{a_n^{(j)} - a_n^{(i)}} \notin [0,1] \tag{25}$$

where $a_n^{(i)}$ denotes a leading coefficient of the polynomial P_i.

Analyzing the D-stability conditions, which were described earlier in the present section, one may conclude that they fit to the case when the parameter perturbations are known. At the same time our goal is to find a complete set of the load parameters n_p and n_q for which D-stability of the power system is preserved. That is we would like to determine all complex loads which, been connected in the intermediate area at bus 2, provide the required system performance. In order to make the above mentioned conditions applicable to our case, we have developed the iterative algorithm described in the next section.

4 Problem Solution

First of all we create a proper ray partition in the parameter space $n_p - n_q$ as shown in Fig. 3 [7]. Recall that a ray partition in $\mathbb{R}^n$ is a set $\Re = \{r_i,\ i = 1, 2, ..., N\}$ of rays, where $r_i = \{x \in \mathbb{R}^n : x = \lambda_i e_i, \lambda_i \geq 0, e_i \in \mathbb{R}^n, e_i \neq 0\}$ with the unit ray vectors e_i specifying the rays. Hence any point on the ray r_i is uniquely determined by the non-negative scalar λ_i which is called a scaling factor. The proper ray partition is a subclass of the ray partitions with all the rays intersecting in only one point – origin, i. e. $r_1 \cap r_2 \cap ... \cap r_N = \{\mathbf{0}\}$.

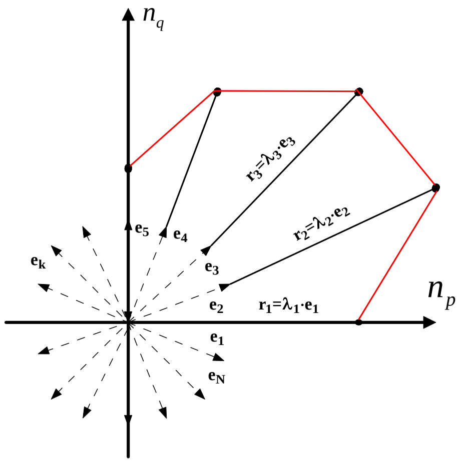

Fig. 3. A ray-partition of the parameter space.

The origin of the parameter space ($n_p = n_q = 0$) corresponds to the stable operating conditions. Noting that the required set of the parameters is convex due to the affine dependence of the characteristic polynomial, it can be approximated by the ray-polytope [7]. Hence, our problem reduces to a search of the vertices of this ray-polytope along the ray vectors e_i. That is we need to find a maximal scaling factor λ_i along the ray vector e_i such that a point $r_i = \lambda_i \cdot e_i$ in the parameter space is stable. The problem can be solved using a simple bisection algorithm visualized in Fig. 4. In this algorithm, stability verification is based on the theorem presented in the previous section. Using the ray partition and the bisection algorithm, a stable region in the parameter space can be determined with an arbitrary accuracy depending on a number of rays used for the space partition. Note that the construction of this region can be speeded up significantly if its convexity is taken into consideration. Indeed, the initial population of N_0 ray vectors forms a polytope. The scaling factor of the ray r_k passing through the mid-point of a segment connecting two vertices of the polytope r_i and r_j which belong to the same face is not smaller than

$$\lambda_k \geq \lambda_{\min}^k = \left\| \frac{r_i + r_j}{2} \right\|_2 \qquad (26)$$

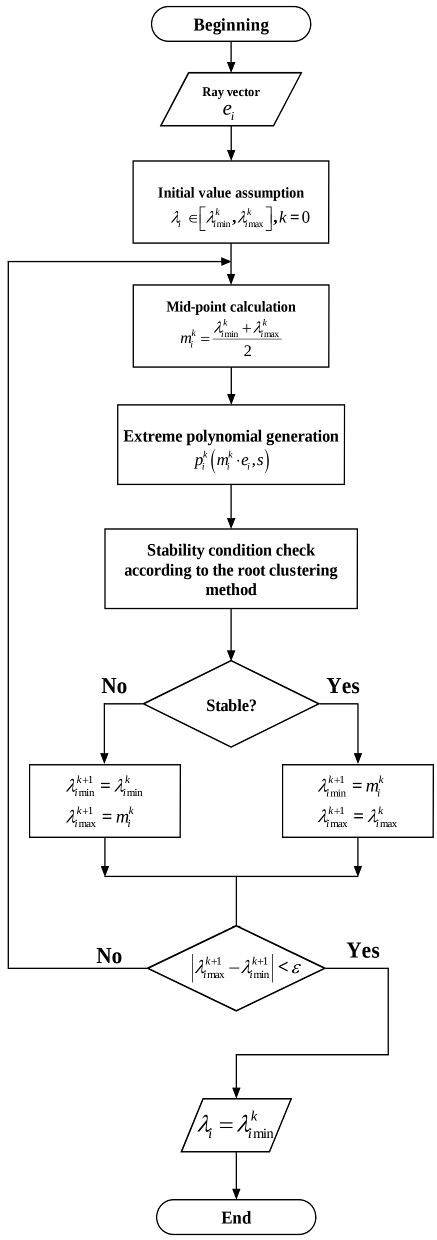

Fig. 4. A bisection algorithm.

This fact allows decreasing of the initial interval which has to be assumed in the bisection algorithm shown in Fig. 4 to find a scaling factor λ_k of the ray r_k passing trough the midpoint $\left(r_i + r_j\right)/2$, i.e. along the direction

$$e_k = \frac{r_k}{\|r_k\|_2} = \frac{r_i + r_j}{\|r_i + r_j\|_2} \quad (27)$$

Generating successively next populations $N_k \ (k = 1, 2, ..., n)$ of the rays along the directions defined in (27) one may find a stable region in the parameter space with arbitrary precision.

5 Numerical example

To visualize an application of the described above iterative technique, we consider the power system described in Fig. 1 which operates at the operating point defined by the system parameters and loading conditions given in Appendix. The closed loop characteristic polynomial (CLCP) of this system derived according to Section 2 is

$$\begin{aligned} CLCP(s, n_p, n_q) &= \left(1 + 0.0206n_p + 0.0834n_q\right)s^4 \\ &+ \left(36.1 + 0.7679n_p + 3.0336n_q\right)s^3 \\ &+ \left(291.4 - 1.4821n_p + 22.0498n_q\right)s^2 \\ &+ \left(2293.8 - 152.5494n_p + 272.9624n_q\right)s \\ &+ \left(9269.2 - 749.2177n_p + 736.0847n_q\right) \end{aligned} \quad (26)$$

Note that the same polynomial was considered in [4] and [6]. The roots of the CLCP for the voltage independent load ($n_p = n_q = 0$) are

$$s_1 = -28.2861, \quad s_2 = -5.7161,$$
$$s_{3,4} = -1.0654 \pm j7.4987$$

We assume a region of the D- stability, that is a desired region of the CLCP roots location in the complex plane, to be [6]

$$D = \bigcup D_i \qquad i = 1, 2, 3 \quad (27)$$

where D_1 is a half-plane with real part $\sigma \le -5$,
D_2 is a disk of radius $\varepsilon = 1$ centered on the CLCP root $s_3 = -1.0654 + j7.4987$,
D_3 is a disk of radius $\varepsilon = 1$ centered on the CLCP root $s_4 = -1.0654 - j7.4987$.

This region is depicted in Fig. 5 and corresponds to the desired behavior of the power system during electro-mechanical oscillations.

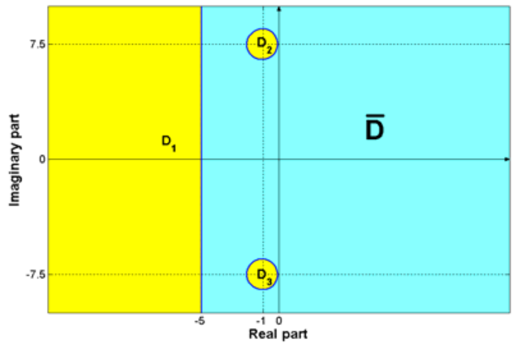

Fig. 5. The *D*-stable region in the complex plane.

A parametrical representation of the boundary of the D-stable region in the complex plane is given by

$$\partial D_1 = -5 + j\delta, \ \delta \in (-\infty, +\infty) \quad (27)$$

for the half-plane, and by

$$\begin{aligned} \partial D_{2,3} &= \left[-1.0654 + \cos(2\pi\delta)\right] \\ &\pm j\left[7.4987 + \sin(2\pi\delta)\right], \delta \in [0,1] \end{aligned} \quad (28)$$

for the two unit circles.

The physically meaningful values of the load parameters lie in the interval $[-5, 5]$. Therefore, we assume that the maximal scaling factor $\lambda_{\max}$ in the bisection algorithm is $\sqrt{5^2 + 5^2} = 5\sqrt{2}$.

Using this maximal value of the scaling factor and the parametrical representation of the D-stable region in the complex plane, which was described earlier, one may find the D-stable domain in the parameter space applying the iterative technique presented in the previous section. The D-stable domain obtained using 64 rays is shown in Figure 6. Note that it represents a fairly good approximation of the complete set of the stable load parameters found in [6].

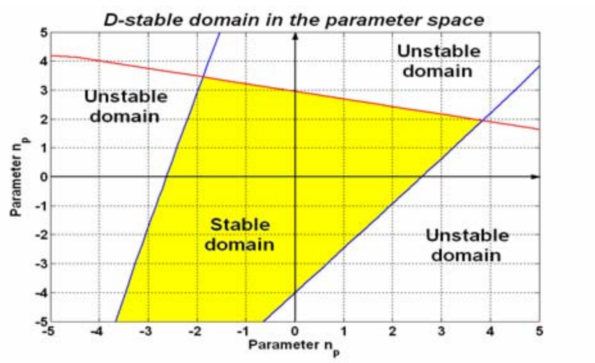

Fig. 6. The *D*-stable domain in the parameter space (electro-mechanical oscillations).

A computational efficiency of the proposed algorithm can be increased significantly if the following variable substitutions are made

$$\cos\left(2\pi\delta\right)=\frac{1-\tan^2\left(\pi\delta\right)}{1+\tan^2\left(\pi\delta\right)}=\frac{1-t^2}{1+t^2} \qquad (29)$$

$$\sin\left(2\pi\delta\right)=\frac{2\tan\left(\pi\delta\right)}{1+\tan^2\left(\pi\delta\right)}=\frac{2t}{1+t^2} \qquad (30)$$

Using these substitutions (21) is transformed into a polynomial equation, for which efficient algorithms of the solution search are available.

Obviously, the proposed technique can be used for the dynamic voltage stability assessment. In this case the D-stable region is a left half-plane and the parametric representation of its boundary is

$$\partial D = j\delta,\ \delta\in\left(-\infty,+\infty\right) \qquad (31)$$

6 Conclusion

A new iterative method of the small-signal power system stability analysis has been developed in the present paper. This method is suitable for an assessment of the electro-mechanical oscillations and dynamical voltage stability of power system and allows one to find a complete set of load parameters for which a particular system performance is preserved. The stability verification procedure used in our method is totally analytical. It is based on root clustering of a polytope of polynomials in a simply connected domain. The proposed method has a number of advantages over the other techniques available in literature: it does not require a testing function which is grid-sensitive as in [4], the computational time and computer memory capability are significantly lower than those needed in [4] and [6].

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Appendix

Table A.1. Synchronous generator data.

$X_d,[p.u.]$	$X_q,[p.u.]$	$X_d^{'},[p.u.]$	$D_f/\omega_{syn},[p.u.]$
1.72	0.45	0.45	0.05

$T_{d0}^{'},[\sec]$	$H,[\sec]$	$\omega_{syn},[rad/\sec]$
6.3	4.0	377

Table A.2. Exciter data.

$K_A,[p.u.]$	$T_E,[\sec]$
20	0.03

Table A.3. Transmission line and capacitor data.

$R_1=R_2,[p.u.]$	$X_1=X_2,[p.u.]$	$b,[p.u.]$
0.012	0.3	0.066

Table A.4. Operating point data

$V_g,[p.u.]$	$\theta_g,[el.\deg.]$	$V_S,[p.u.]$
1.0	24.177	1.0

$P_g,[p.u.]$	$Q_g,[p.u.]$	$P_l,[p.u.]$	$Q_l,[p.u.]$
0.9	0.286	0.5	0.3

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