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TWO DIMENSIONAL CHAOTIC MODULATOR

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Abstract: -In this paper a two dimensional chaotic modulator is proposed, based on an electronic controlled Colpitts Oscillator. The modulator is capable of producing constellations of distinct chaotic patterns in a bifurcation diagram, by creating pairs of specific characteristic values.

Key-Words: - Chaos, Colpitts Oscillator, Modulator, Voltage Controled Oscilator.

1 Introduction

Chaos in the Colpitts Oscillator, first reported in [1], has attracted a lot of interest due to its applications in encryption and modulation methods applied to communication systems. The main premise in these studies is that broad-band signals generated by simple deterministic systems with chaotic dynamics can replace pseudo-random carrier signals widely used in spread spectrum communication systems.

In the last few years, research effort has been devoted towards the development of efficient chaos-based modulation techniques [2]. Among several proposed, one of the best bit-error rate performances has been achieved by the differential chaos shift keying (DCSK) scheme and its variation utilizing frequency modulation (FM-DCSK) [3]. In order to achieve higher data rates, more sophisticated chaos-based communication schemes were developed, such as quadrature chaos shift keying (QCSK), having the same BER performance as DCSK. In a chaos shift key communication system, M digital symbols are represented by chaotic signals generated from M dynamic systems or from one system with M different parameter values [4], [5].

In the binary case i.e., $M=2$, the transmitted signal essentially switches between two chaotic signals which are generated from two dynamic systems, or from one dynamic system having a parameter switched between two values, according to the digital symbol to be represented [6]. Detection can take either a coherent form or a noncoherent form.

In this paper, a controlled version of a chaotic Colpitts oscillator is proposed. The important parameters here are the gain (g) and quality factor (Q) of the oscillator. The proposed circuit adds functionality to the classical Chaos Colpitts Oscillator idea that produces two or more chaotic and non chaotic signals in a wide operational regime of the active component. The electronic control of the oscillator provides the modulator with a very efficient way to generate large numbers of chaotic signals in a short settling time.

The outline of this paper is as follows. Section 2 describes the circuit design, which is at first based on a voltage controlled Colpitts Oscillator. Section 3 presents the simulation results and section 4 concludes the paper.

2 Circuit Design

Colpitts oscillator is a single-transistor implementation of a sinusoidal oscillator which is widely used in electronic devices and communication systems. The single ended Colpitts oscillator is shown in Fig.1. It is a

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combination of a transistor amplifier consisting of a single bipolar junction transistor (BJT), and an LC circuit used to feedback the output signal as it is depicted in Fig. 1. The fundamental oscillation frequency is given by:

$$f = \frac{1}{2\pi\sqrt{L_1 \frac{C_1 C_2}{C_1 + C_2}}} \qquad (1)$$

The intrinsic nonlinearity of Colpitts oscillator, which depends by the exponential characteristic of the active device, easily leads to obvious indications of chaotic behavior such as positive values of Lyapunov exponents. For a specific set of parameters values, the oscillator exhibits a complex dynamic behavior which is described by the system's attractor (Fig.2).

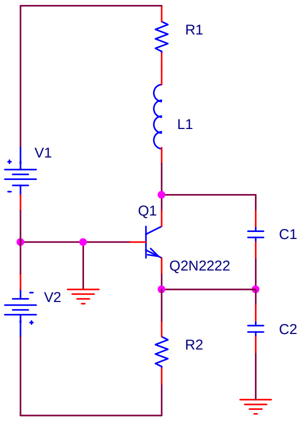

Fig.1. Circuit Layout of the Single Ended Chaos Colpitts Oscillator.

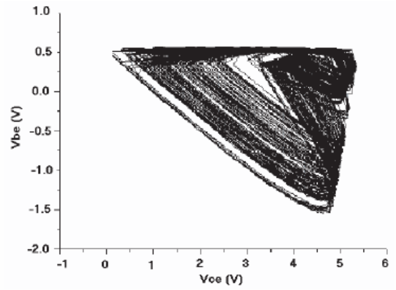

Fig.2. Colpitts Oscillator's Attractor.

2.1 Voltage Control Chaotic Colpitts Oscillator

The system in Fig. 3. is described by the following set of equations [7],[8]:

$$C_3 \frac{dV_{CE}}{dt} = I_{L1} - I_{C1}$$

$$L_1 \frac{dI_{L1}}{dt} = V_3 - R_1 I_{L1} - V_{CE} - V_{BE} - VQ_{4on} \quad (2)$$

$$C_1/2 \frac{dV_{BE}}{dt} = I_{L1} - I_{C1} + I_{E1}(V_{CE}, Q_1) - I_1$$

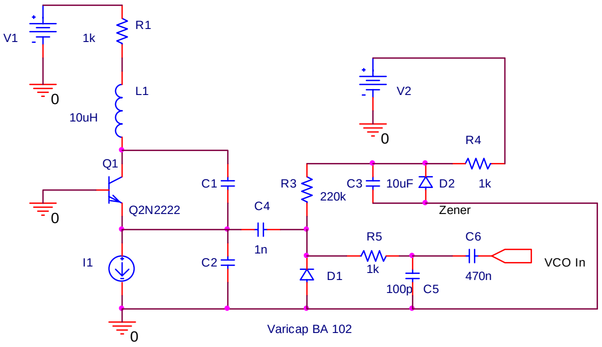

Fig. 3. Circuit Layout of the Voltage control Colpitts Oscillator.

The collector current I_C is proportional to the emitter current:

$$I_{c1} = \alpha I_{e1} \qquad (3)$$

The non-linear current-voltage characteristic of the emitter-base junction can be approximated by a piecewise linear function as follows:

$$I_{E1}(V_{BE}) = \begin{cases} \frac{-V_{BE1} - V_{th}}{r}, V_{BE1} < -V_{th} \\ 0, V_{BE1} \geq -V_{th} \end{cases} \qquad (4)$$

With r being the small signal ON resistance of the emitter base junction and Vth the break point voltage (approximately 0.7 Volt).
The state equations (2) of the Colpitts oscillator can then be rewritten in the form:

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \frac{g}{Q(1-k)}[-n(x_2)+x_3] \\ \frac{g}{Qk}x_3 \\ -\frac{k(1-k)Q}{g}[x_1+x_2]-\frac{1}{Q}x_3 \end{pmatrix} \quad (5)$$

where $n(x_2) = e^{-x_2} - 1$

$$Q = \frac{2\pi f_o L}{R} \quad (7) \qquad g = \frac{I_0 L}{V_T R(C_1 + C_2)} \quad (8)$$

and

$$k = \frac{C_2}{C_1 + C_2} \quad (9)$$

In system (5), only the first equation contains the nonlinear term $n(x_2)$ which, in turn, depends upon only one state variable, namely x_2. Also, it should be noted that the dynamic behavior of the system (5) depends on the following two parameters specifically:

1. g, the loop gain of the oscillator (8).
2. Q the quality factor of the unloaded tank circuit (7).

Note that k has only a scaling effect on the state variables.

2.2 Voltage Controlled Colpitts Oscillator

The circuit (Fig.3) consists of a single bipolar transistor (Q1) which is biased in its active region by the use of V1,

R1, and I1. The feedback network consists of L1, C1, and C2. Parallel operation between C2 and varicap diode D1 can be achieved, through the use of C4. The varicap diode is reversely biased by the use of R3, V2, and R4. As the inversely polarized varicap does not conduct a d.c. current, R3 reaches a very high value thus not charging the tank circuit (L1,C1,C2). C3 parallels the operation of zener diode D2 diminishing any white noise generated by the brake-down of zener diode D2 [9]. This particular white noise affects the varicap diode D1 in such a way that modulation noise will be created. R4 and D2 can create a voltage stabilization circuit. R5 and C5 form a low pass circuit that prevents R.F. leakage from VCO In. Finally, C6 is a d.c. blocking capacitor.

By varying the capacitance of varicap diode D1 the value of g is inversely affected, thus changing the dynamic behavior of the VCCO. In effect, such a capacitance value of varicap diode D1 can be achieves, that the chaotic dynamics of the system change from the operational point Pa (g_1, Q) to another Pb (g_2, Q), in a bifurcation diagram,

2.3 Modulator's Circuit

The full circuit layout of the two-dimension modulator is presented in figure 5. The first part of the circuit is which concerns the control over the gain of the oscillator is presented in the previous paragraph. Here, a second sub-circuit is added, which aims at controlling the quality factor of the unloaded tank circuit. This can be achieved by controlling the collector's resistance R, according to the equation (7).

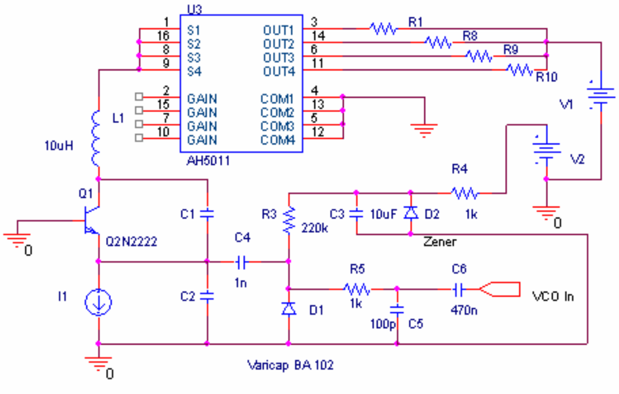

Fig. 4. Circuit Layout of the Modulator.

The integration of this circuit is the addition of U3, which is an electronic controlled switching element, among four different resistors. The value of the selected resistor affect the quality factor Q, according to (7), thus changing the dynamic behavior of the oscillator. Now it can also be said that the behavior of the oscillator can change from the operational point $p_1(g_{1,}Q_1)$ to another $p_2(g_2,Q_2)$ (Fig.4).

3 Simulation and Results

The simulation verifies that the sub-circuits used to expand the operation of the Colpitts oscillator can, in a controlled manner affect the operational point of the oscillator. Note also that the chaotic signals created by the modulation of the two parameters (g, Q), have distinct chaotic dynamical properties (Fig. 6 and 7).

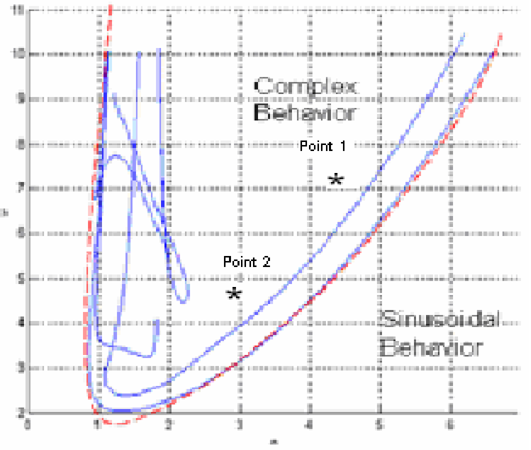

Fig. 5 Bifurcation Diagram of the Colpitts Oscillator.
(Horizontal axis log(Q), Verical axis log(g)).

However, an extra care should be taken so that the dynamics of the system remain in the chaotic region, as marked in the bifurcation diagram (Fig.5).

Circuit Elements	V1	I1	R	L1	C1/C2	D1
Elements' Values P. 1	12 V	22 mA	36 Ω	48 µH	2 nF	100 pF
Elements' Values P. 2 2	12 V	22 mA	34 Ω	48 µH	2 nF	10 pF

Table 1. Circuit Parameters Values used in Simulation and in Experiments

So, by controlling both parameters (g,Q) of the operational point, distinct chaotic states are created in the two-dimensional plane of the bifurcation diagram, thus modulating the chaotic carrier. In effect, a large number of signals can be created, forming a constellation, also having orthogonal properties and a short settling time.

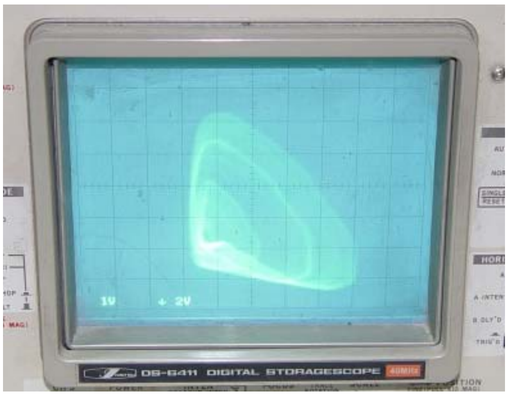

Fig. 6. Lissajou output – Operational Point 1

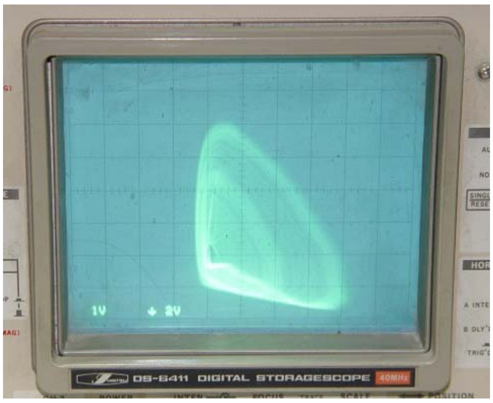

Fig. 7. Lissajou output – Operational Point 2.

4 Conclusion

The proposed modulator uses electronic control to create chaotic signals with different dynamical properties. Modulation of the chaotic carrier can be achieved by changing one, or both of the values of gain (g) and quality factor (Q), which define the operational point of the oscillator. Like in conventional modulation methods (i.e. QCSK), a constellation of points in the bifurcation diagram can be constructed. The modulator's main advantage is the relative ease of its actual implementation, comparing to similar modulation methods. Moreover, the use of electronic control of the Colpitts oscillator can produce chaotic signals in a short settling time.

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