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Response Characteristics of LED in POF Optical Links

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Abstract: - This paper presents an examination of the optical response of LED in applied electrical signals. LED's optical response is important to examine in order to use as cheap solution for electrical–to–optical conversion in moderate and short distances optical communication networks. The evaluation parameter of the response is the optical–to–electrical current ratio T(ω). This ratio represents the possible maximum bit rate for data transmission using LED.

Key-Words: - LED, optical-to-electrical current ratio, maximum bit rate.

1 Introduction

It is well known that the continuity equation in semiconductors, using the drift and diffusion currents with the recombination rates, describes the picture of charge transport [1]. The mathematical expression of continuity equation for electron carriers is :

$$\frac{\partial}{\partial t}\delta n = \frac{1}{q}\frac{\partial}{\partial x}Jn(x) - \frac{\delta n}{\tau_n} \qquad (1)$$

where δn is the extra electron carrier density, τ_n the recombination time of electrons including radiative and non radiative procedures and J_n is the current density of electrons due to drift and diffusion. In case of p-n junctions that we are interested in this paper, the current density is mainly of diffusion currents. In that case the diffusion part of the current density is :

$$Jn(x)_{diff} = qDn\frac{\partial}{\partial x}\delta n \qquad (2)$$

Substituting (2) to (1) we get the continuity equation for electrons due to diffusion currents:

$$\frac{\partial}{\partial t}\delta n(x,t) = Dn\frac{\partial^2}{\partial x^2}\delta n(x,t) - \frac{\delta n(x,t)}{\tau_n} \qquad (3)$$

In a p-n junction we are interested in the concentration of extra electron carriers in the boundary of the depletion layer towards the p side in case of forward bias. The concentration is calculated to be [1] :

$$\Delta n_p = n_p(e^{\frac{qV}{K_B T}} - 1) \qquad (4)$$

where n_p is the electron (minority) carrier density in the p region, and V is the external applied voltage. Figure.1 presents this idea

In order to examine the frequency response of LED in the presence of specific external voltage we need the optical–to–electrical current ratio. This ratio is an estimation of the transformed optical current according to the applied electrical current, and it is represented mathematically as: $R(\omega) = \frac{qI_{ph}(\omega)}{I_e(\omega)}$.where I_{ph} is the photon current (the quantum limit of classical electromagnetic light) given by [1]:

$$I_{ph} = \frac{1}{\tau_n}\int_{w_p}^{\infty}\delta n(x,t)dx \qquad (5)$$

and I_e is the electron current given by [1]:

$$I_e = \int_{w_p}^{\infty} Jn(x)_{diff}\, dx =$$

$$= qDn \int_{w_p}^{\infty} \left(\frac{\partial}{\partial x} \delta n(x,t)\right) dx \qquad (6)$$

One of the important applications of LED is the conversion of applied electric signals into light. LED is actually a forward bias p-n junction where minority carrier is injected in an active recombination region. In order to use LED in communications we are interested in the time that is needed in order to extract the extra minority carrier from the active region. This time is compared with the recombination time τ_n of minority carriers and it results in the time or frequency response function of LED. Using a qualitative analysis it is easy to prove that the recombination time is the key to the frequency response. The recombination time is actually the mean time that a minority carrier (electron) from the conduction band of the active p region is going to be recombined with a majority carrier (hole) from the valence band. As frequency of the applied electric signal increases, the p-n junction is continuously reverse and forward biased. Every time the junction is forward biased the minority carrier concentration (electrons) is injected in the active region and by the time the junction becomes reverse biased it is extracted from the active region. If the switching time is greater than the recombination time then some minority carriers are recombined to produce photons. As the switching time approaches the recombination time (the frequency of the applied electric signal is increased) then there is no time left for the recombination process and the LED is not producing any more discrete pulses of light, hence it is not useful in communications. It is therefore interesting to examine the frequency response of the LED in to different applied electric signals and to compare the results. The examination is conducted by solving the continuity equation (3) for minority carriers (electrons) in the active p region of the LED [5,6]. In this approach it is supposed that the electrons in the p region and the holes in the n region have the same mean velocity, although this is not true. LED is supposed to be a one-dimensional system (see figure 1).

2 RZ Digital Signal

It is supposed that a rectangular voltage is applied to the LED. The p-n junction (LED) is forward-biased in (-T/2 to T/2) and unbiased in the rest of the period. In order to examine the frequency response of the LED, the continuity equation has to be solved. The mathematical description of the signal is:

$$V(t) = \begin{cases} A, & -\frac{T}{2} \prec t \prec \frac{T}{2} \\ 0, & \frac{T}{2} \prec t \prec T_0 \end{cases} \qquad (7)$$

In order to solve the continuity equation we need to represent V(t) in Fourier series, since it is a periodic signal. The coefficients c_n are calculated to be *(see Appendix A):*

$$c_n = \frac{A}{\pi n} \sin\left(\frac{n\pi T}{T_0}\right)$$

and the representation is

$$V(t) = \sum_{n=-\infty}^{n=+\infty} \frac{A}{\pi n} \sin\left(\frac{n\pi T}{T_0}\right) e^{\frac{j2\pi nt}{T_0}} =$$

$$= \frac{AT}{T_0} + \sum_{n=1}^{+\infty} \frac{2A}{n\pi} \sin\frac{n\pi T}{T_0} \cos\frac{2n\pi t}{T_0}$$

The d.c. term AT/T_0 is of no interest for the frequency response of LED. The other term is the superposition of n the number a.c. terms $e^{jn\omega t}$. As n increases (higher harmonics) the amplitude $2A/n\pi$ of the n terms decreases. The diffusion equation is solved and the total optical and electron currents are :

$$I_e^{TOT}(\omega) = -qn_p \frac{D_n}{\sqrt{D_n \tau_n}} \sum_{n=1}^{\infty} \left(e^{\frac{q\left(\frac{2A}{n\pi}\sin\left(\frac{n\pi T}{T_0}\right)\right)}{k_B T}} - 1\right)$$

$$I_{ph}^{TOT}(\omega) = \frac{qn_p \sqrt{D_n \tau_n}}{\tau_n} \sum_{n=1}^{\infty} \left(e^{\frac{q\left(\frac{2A}{n\pi}\sin\left(n\pi\frac{T}{T_0}\right)\right)}{k_B T}} - 1\right)\left(\frac{1}{\sqrt{1 + jn\omega\tau_n}}\right)$$

Hence the ratio is :

$$R(\omega) = -\frac{\sum_{n=1}^{\infty} \left(e^{\frac{q\left(\frac{2A}{n\pi}\sin\left(n\pi\frac{T}{T_0}\right)\right)}{K_B T}} - 1\right)\left(\frac{1}{\sqrt{1 + jn\omega\tau_n}}\right)}{\sum_{n=1}^{\infty} \left(e^{\frac{q\left(\frac{2A}{n\pi}\sin\left(n\pi\frac{T}{T_0}\right)\right)}{K_B T}} - 1\right)}$$

and $T(\omega) = \left|R(\omega)\right|^2$

3 Performance Analysis Results

The evaluation parameter is the optical-to-electrical current ratio amplitude T(ω). From this function of frequency it is possible to extract the maximum transmission bit rate. Figure.1 presents the plot of the function T(ω).The three different plots (a,b,c) stand for three different values of the recombination time
τ_n:

$a \Rightarrow \tau_n = 10^{-9}$,

$b \Rightarrow \tau_n = 10^{-8}$,

$c \Rightarrow \tau_n = 10^{-7}$

It is obvious that the function T(ω) is the characteristic function of a low-pass filter, meaning that the LED when transforming the electron-hole pairs (current due to applied voltage) into photons it allows the frequency of the applied voltage to vary between zero and a maximum cut frequency ω_c. This behavior has a physical meaning. The allowed zero frequency is the DC voltage that keeps the LED in forward bias during its operation. This permanent forward bias maintains an extra electron carrier density in the boundary of the depletion layer towards the p side, and any delays due to turn-on turn-off are eliminated. For higher frequencies above ω_c , there is no time left for the recombination process (the minority electron carriers in the p-side are injected and withdrew veryfast) and the LED is not producing any more discrete pulses of light. The optical current decreases rapidly compared to the injected electrical current. As a consequence the LED, for higher frequencies of operation, is continually turned-on due only to the DC component and no information is transmitted. Hence ω_c represents the maximum available transmission bit rate. Moreover from the graph it is obvious that the lower the recombination time the higher the ω_c. The physical meaning of this result is again the recombination process. Since frequency is ω = 1/T then as frequency increases, time T that the minority carriers exist in the p-side decreases and becomes comparable to the recombination time. Hence the lower the recombination time the more the frequency can be increased, the higher the ω_c and the higher the allowed transmission bit rate.

It is possible to calculate ω_c considering the frequency which corresponds to half the maximum value of T(ω). The maximum value of T(ω) occurs obviously at zero frequency :

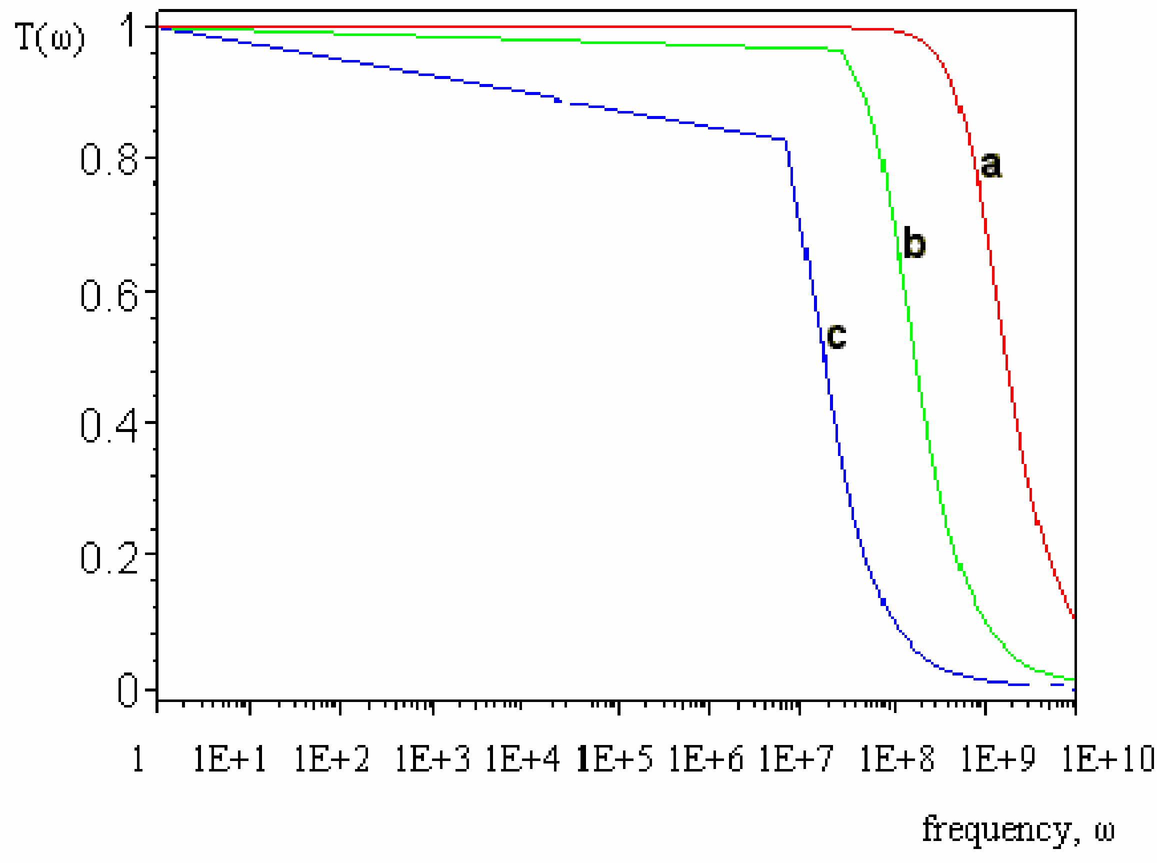

Figure 1: Plot of T(ω) when the applied signal is a RZ digital

$$T(\omega_c) = \left(\left|R(\omega_c)\right|\right)^2 = \frac{1}{\sqrt{1+\omega_c{}^2\tau_n{}^2}} = \frac{1}{2}T(\omega)\Big|_{\omega=0} = \frac{1}{2} \Rightarrow \omega_c = \frac{\sqrt{3}}{\tau_n}$$

The plot of function T(ω) follows and it is possible to calculate ω_c

$$a \Rightarrow \tau_n = 10^{-9} \rightarrow \omega_c = 1{,}72 \quad GHz$$

$$b \Rightarrow \tau_n = 10^{-8} \rightarrow \omega_c = 172 \quad MHz$$

$$c \Rightarrow \tau_n = 10^{-7} \rightarrow \omega_c = 17{,}2 \quad MHz$$

The results in that case are similar to the previous cases. The best response (higher cut frequency) takes place for the less recombination time. It is important to mention again that although there is no apparent DC component, the Fourier analysis indicates an indirect DC component

4 Conlusions

As it was mentioned on the introduction, the evaluation parameter of the response is the optical-to-electrical current ratio T(ω). According to a previous discussion if the switching time is greater than the recombination time then some minority carriers are recombined to produce photons; otherwise there is no switching at all but LED is constantly turned on, producing a continuous stream of photons. This can be translated to the electrical–to–optical current ratio as: "if the switching time is greater than the recombination time" means that the optical current is comparable to the electron current. "if the switching time is equal or less than the recombination time" means that the optical current is too small compared to the injected electron current. Hence calculating the optical-to-electrical current ratio we can examine the behavior of the LED. This ratio was calculated for three different signals. The important outcome was the cut frequency, ω_c, which represents the maximum allowed transmission bit rate. The LED response to a sinusoidal signal is more effective when there is also a D.C. component. The response to a rectangular pulse is also similar to a sinusoidal response with a DC component. This allows digital data transmission to be sent by LED with satisfactory bit rate, 1 Gb/s, using well known line codes as NRZ, AMI, HDB3. But this bit rate is idealized. The reason is that the used model for LED didn't take into account indirect transitions, non radiative recombinations (Auger recombination) and multiband transitions. Taking into account the mentioned procedures, bit rate is less than the calculated one. For a first presentation though, the model used in this paper represents the underline phenomena of LED transmission satisfactory.

APPENDIX A

In the main body of paper, calculations like Fourier representation coefficient are boring and space consuming. In this appendix the necessary calculations for Fourier coefficients are executed and reader can follow them as a supplementary information

The fourier representation of a signal is

$$V(t) = \sum_{n=-\infty}^{+\infty} c_n e^{j\frac{2\pi nt}{T}} \qquad \text{A.1}$$

where $c_n = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} V(t) e^{-j\frac{2\pi nt}{T}}, n = 0, \pm1, \pm2.......$ A.2

Fourier representation of the RZ digital signal

Applying for the digital signal V(t):

$$c_n = \frac{1}{T_0}\int_{-\frac{T}{2}}^{\frac{T}{2}} V(t) e^{-j\frac{2\pi nt}{T_0}} dt = \frac{1}{T_0}\int_{-\frac{T}{2}}^{\frac{T}{2}} V_0 e^{\frac{-j2n\pi t}{T_0}} dt = \frac{A}{n\pi}\sin(\frac{n\pi T}{T_0}), \quad n = 0, \pm1, \pm2, \pm3, ...$$

A.3

Replacing c_n into equation A.1:

$$V(t) = \sum_{n=-\infty}^{+\infty} c_n e^{j\frac{2\pi nt}{T}} = \sum_{n=-\infty}^{+\infty} \frac{A}{\pi n}\sin(\frac{n\pi T}{T_0}) e^{j\frac{2\pi nt}{T}} =$$

$$= \sum_{n=-\infty}^{-1} \frac{A}{\pi n}\sin(\frac{n\pi T}{T_0}) e^{j\frac{2\pi nt}{T}} + \frac{A}{n\pi}\sin(\frac{n\pi T}{T_0}) e^{j\frac{2n\pi t}{T_0}}\Big|_{n=0} + \sum_{n=1}^{+\infty} \frac{A}{n\pi}\sin(\frac{n\pi T}{T_0}) e^{j\frac{2n\pi t}{T_0}} =$$

$$= \sum_{n=1}^{+\infty} \frac{A}{\pi(-n)}\sin(\frac{(-n)\pi T}{T_0}) e^{j\frac{-2\pi nt}{T}} + \frac{AT}{T_0} + \sum_{n=1}^{+\infty} \frac{A}{n\pi}\sin(\frac{n\pi T}{T_0}) e^{j\frac{2n\pi t}{T_0}} =$$

$$= \sum_{n=1}^{+\infty} \frac{A}{\pi n}\sin(\frac{n\pi T}{T_0}) e^{j\frac{-2\pi nt}{T}} + \frac{AT}{T_0} + \sum_{n=1}^{+\infty} \frac{A}{n\pi}\sin(\frac{n\pi T}{T_0}) e^{j\frac{2n\pi t}{T_0}} =$$

$$= \sum_{n=1}^{+\infty} \frac{A}{\pi n}\sin(\frac{n\pi T}{T_0})[e^{j\frac{-2\pi nt}{T}} + e^{j\frac{2n\pi t}{T_0}}] + \frac{AT}{T_0} =$$

$$\sum_{n=1}^{+\infty} \frac{A}{\pi n}\sin(\frac{n\pi T}{T_0})\cos\frac{2n\pi t}{T_0} + \frac{AT}{T_0}$$

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