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Estimating LED's Bandwidth for POF Optical Links

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Abstract: - Light emitting diodes (LED) are usually the alternative optoelectronic devices for optical communications with moderate bandwidth efficiency and bit rates, compared to semiconductor lasers. Laser applications dominate the long distance communication networks. However LED seems to be a very good solution, especially in very short distance communication networks, where Plastic Optical Fibers, (POF) are used as the optical transmission medium. This paper investigates a qualitative technique of calculating the bandwidth, based on first principles of semiconductor physics. This technique is more convenient for a first approach of the subject on optical communication networks and its simplicity provides a very useful analytical tool. Finally these results are compared with the existing experimental ones.

Key-Words: - LED, Polymer Optical Fiber, Optical bandwidth.

1 Introduction

Optical communication links, based on Plastic Optical Fibers, POF, are under investigation nowadays. The behavior of POF differs from usual silicon fibers because the attenuation of different modes is extremely high and the Differential Mode Delay contributes to the dispersion of optical pulses. These results restrict the applications of POF links in very short distance optical links, covering in-house internet applications to business-on-floor indoor network architectures [9]. The benefit of large core to cladding ratio in POF enables future applications of LED as the transmitter, reducing the cost per link and allowing easy coupling of radiation in POF [7],[8]. The large bandwidth of LED is the only drawback for future applications and its prediction is of great importance. So far in technical papers and books the calculation of LED's bandwidth was not trivial at all, since this is the area of time-dependent quantum perturbation theory. On the contrary, this paper investigates a qualitative technique of calculating LED's bandwidth, using first principles of semiconductor physics like doping and charge transport in a p-n junction during operation. A model has to be constructed for a Ga-As LED and special considerations have to be made in order to simplify the calculations without loosing the generality. LED during operation is a forward-bias p-n junction. Electrons and holes are injected as minority carriers along the junction's p and n doped regions respectively and they are recombined either with radiative or with non-radiative transitions. LED has to be properly designed to support mostly the radiative transitions in order to be efficient. In forward-bias operation the total current is dominated by the diffusion part due to minority carrier concentrations. The diffusion current consists mainly of two parts: the electron minority diffusion current in the p-region and the hole minority diffusion current in the n-region. In this model LED

is designed with a p-region lightly doped with acceptors and an n-region highly doped with donors, implementing a $p\text{-}n^+$ junction. The reason is that photons that are emitted in the hidden substrate are most probably absorbed by the substrate's material, contributing almost nothing to the output power. The n^+-region is hidden in the substrate, producing a small hole minority diffusion current and the p-region is near the surface producing a high electron minority diffusion current. Hence the total diffusion current consists mainly of the electron current.

In the p-region there are electron minority carriers in the conduction band from two different resources, the Δn_p injected carriers from n^+-region during forward-bias and the pre-existing excited n_p carriers from the p-region's valence band due to nonzero temperature. The injected Δn_p carriers depend on the applied forward voltage and they are calculated from the following formula[1]:

$$\Delta n_p = n_p \left(e^{\frac{qV}{K_B T}} - 1 \right) \tag{1}$$

where K_B is the Boltzmann constant, T is the temperature of operation, q is the electron charge, V the applied forward bias voltage and n_p is the pre-existing excited carriers from the p-region's valence band.

The n_p carriers from the p-region's valence band are calculated using the mass-action law[4]:

$$n_p \cdot p_p = n_i^2 \Rightarrow n_p = \frac{n_i^2}{N_a} \tag{2}$$

where N_α is the acceptor density in the p-region and n_i is the intrinsic carrier concentration. During the calculation of LED's optical bandwidth in this paper, only the injected carriers Δn_p from n^+-region are considered since their concentration in forward bias operation is much higher than pre-existing excited n_p carriers.

2 The Qualitative Approach

The basic idea behind this qualitative approach is the analytical calculation of maximum occupied energy level at the conduction band of the p-region of LED, after the injection of the Δn_p carriers from n^+-region in forward bias operational condition. The density of states per unit volume in the conduction band of p-region is [1]:

$$g_e(E) = \frac{\sqrt{2}\left(m_e^*\right)^{3/2}}{\pi^2 \hbar^3} \left(E - E_C\right)^{1/2} \tag{3}$$

where m_e^* is the effective mass of electron carriers in the conduction band of p-region of LED, and E_C is the energy level at the bottom of conduction band. Since LED is operating in nonzero temperatures the Fermi-Dirac distribution is used and the number of electron carriers per unit volume in the conduction band of p-region is given using the equation[1]:

$$N_e = \frac{\sqrt{2}(m_e^*)^{3/2}}{\pi^2 \hbar^3} \int_{E_C}^{\alpha} \frac{\left(E - E_C\right)^{1/2} dE}{1 + e^{\frac{\left(E - E_{f_n}\right)}{K_B T}}} \tag{4}$$

where α is the maximum occupied energy level and E_{f_n} is the quasi-Fermi level in the non-equilibrium condition of injection of excess carriers in the p-region. The time the system is in equilibrium after the injection of excess carriers is almost picoseconds while the recombination e-h time is nanoseconds to microseconds. Hence the Δn_p carriers can be represented, in non-equilibrium condition, satisfactory with a quasi-Fermi distribution[3], $f^e(E) = \frac{1}{1 + e^{\frac{\left(E - E_{f_n}\right)}{K_B T}}}$. The quasi-Fermi level E_{f_n} is calculated using the approximate formula of Joyce-Dixon[1]:

$$E_{f_n} = E_C + K_B T \left(\ln \frac{\Delta n_p}{N_C} + \frac{\Delta n_p}{\sqrt{8} N_C} \right) \tag{5}$$

where N_C is the active density of states in the boundary of conduction band, is a function of temperature and is given by $N_C = 2\left(\frac{m_e^* K_B T}{2\pi\hbar^2} \right)^{3/2}$.

Transforming equation (4) using the active density of states (appendix A) it becomes:

$$N_e = \frac{2N_C}{\sqrt{\pi}} \int_0^{\frac{\alpha - E_C}{K_B T}} \frac{(x)^{1/2} dx}{1 + e^{x - n_F}} \tag{6}$$

where $\chi = \frac{E - E_C}{K_B T}$ and using equation (5) parameter:

$$n_F = \frac{E_{f_n} - E_C}{K_B T} = \ln\left(\frac{\Delta n_p}{N_C} \right) + \frac{\Delta n_p}{\sqrt{8} N_C} \tag{7}$$

3 Theoretical obtained Results and Comments

The qualitative calculation of maximum occupied energy level α at the conduction band of the p-region of LED, is based on the solution of equation (6). The number of electron carriers per unit volume N_e in the conduction band of p-region is assumed to be equal to Δn_p and n_F is given from equation (7). The solution depends on temperature and on the applied forward bias voltage at p-n Ga-As LED diode. In order to keep a standard model, the operating temperature is assumed to be 300 °K and the parameter values of equations (6),(7) are presented for a typical p-n Ga-As LED[1]in tabel.1:

$E_c =$	1.43 eV
$N_c =$	$4.45 \cdot 10^{17}\ \text{cm}^{-3}$
$N_\alpha =$	$5 \cdot 10^{16}\ \text{cm}^{-3}$
$n_i =$	$1.84 \cdot 10^{6}\ \text{cm}^{-3}$
$n_p = \dfrac{n_i^2}{N_\alpha} =$	$67.7 \cdot 10^{-6}\ \text{cm}^{-3}$
$K_B T =$	0.026 eV

Table.1: *Typical values for the parameters of GaAs LED diode*

Substituting these values in equation (6), it becomes

$$\Delta n_p \cdot 2 * 10^{-18} = \int_0^{38,46\alpha - 55} \frac{(\chi)^{1/2} d\chi}{1 + e^{\chi - n_F}}$$

where Δn_p and n_F depends on the applied voltage. In table.2 the maximum occupied energy level α is calculated for typical values of voltages, ranging from 1 to 1.4 Volts and for the values of table.1 These values are typical because in a Ga-As p-n junction the threshold voltage is almost 1.1 Volts.

It is clear that the maximum occupied energy level α at the conduction band of the p-region of LED depends on the applied forward bias voltage. This is expected since the number of minority carriers increases at higher values of bias voltage. The mean value of the calculated top-energy occupied level is $\langle\alpha\rangle = 1.658$ eV. The energy gap of the GaAs LED is almost[1],[3] Eg = 1.43 eV. Assuming that the valence band is on zero energy level, then the lowest level of the conduction band is on 1.43 eV. The lower occupied energy level of holes in the valence band of the p-side is near 0 eV since the majority carriers are injected in the n-side and the E-k distribution is wider than the conduction band[2].

The theoretical result for energy bandwidth of LED is [α-Eg] = 0.2 eV or 130 nm and the calculated energy bandwidth from international bibliography [5],[1] is almost $K_B T =$ .0.03 eV. or 20 nm. This difference exists because not all of the electrons in the conduction band, up to 1.658 eV, are recombined directly with the holes in the valence band producing photons. There are also indirect recombinations[3] due to traps and impurity levels within the band gap[4] and the band gap energy is consumed in thermal absorptions and phonons[2]. Moreover the majority of the electrons injected in the p region are considered to be hot carriers and their energy is consumed in interband recombinations conrtibuting to the increase of temperature of the bulk semiconductor. All these effects are not considered in first approach. In order to correct the calculation, the [α-Eg] has to be multiplied with a coefficient, k, a correction coefficient extracted from experiments.

Voltage (Volt)	n_F	Δn_P (cm^{-3})	α (eV)
1	-11,77574779	$3,4215 \cdot 10^{12}$	1.6583
1.05	-9,852641072	$2,341 \cdot 10^{13}$	1.6821
1.1	-7,929455491	$1,6017 \cdot 10^{14}$	1.6776
1.15	-6,005635137	$1,096 \cdot 10^{15}$	1.6798
1.2	-4,077471672	$7.498 \cdot 10^{15}$	1.6814
1.25	-2,119592739	$5,130 \cdot 10^{16}$	1.6743
1.3	0,041581622	$3,51 \cdot 10^{17}$	1.6228
1.35	3.593850568	$2,40156 \cdot 10^{18}$	1.5865
1.4	16.6637067	$1.64314 \cdot 10^{19}$	1.6597

Table.2: *Calculation of maximum occupied energy level α*

4 Conlusions

In this paper a qualitative technique of calculation of LED bandwidth, using the basic theory of semiconductors, doping and charge transport in a p-n junction during operation is considered. The model was simplified and there was assumed no indirect transitions and phonon-electron interactions. Hence the theoretical calculated results are deviated from the experimental ones within a power of ten. This deviation was expected and can

be corrected by adapting a correction coefficient, including all the non-radiative recombinations and interband recombinations. However this technique is more convenient for a first approach of the subject since it contributes to an understanding of the basic theory underlying the inerband transitions and its simplicity provides a very useful analytical tool

APPENDIX A

The equation to calculate the number of electron carriers per unit volume in the conduction band of p-region is

$$N_e = \frac{\sqrt{2}(m_e^*)^{3/2}}{\pi^2\hbar^3}\int_{E_C}^{\alpha}\frac{(E-E_C)^{1/2}dE}{1+e^{\frac{(E-E_{f_n})}{K_B T}}} \quad \text{A.1}$$

Changing the variables $x = \frac{E - E_c}{K_B T}$ and $n_F = \frac{E_{f_n} - E_c}{K_B T}$, equation (A.1) becomes:

$$N_e = \frac{\sqrt{2}(m_e^*)^{3/2}}{\pi^2\hbar^3}\int_0^{\frac{a-E_c}{K_B T}}\frac{(x\cdot(K_B T))^{1/2}(K_B T)dx}{1+e^{(x-n_F)}} \Rightarrow$$

$$N_e = 2\left(\frac{m_e^*(K_B T)}{2\pi\hbar^2}\right)^{3/2}\frac{2}{\pi}\int_0^{\frac{a-E_c}{K_B T}}\frac{(x)^{1/2}dx}{1+e^{(x-n_F)}} \quad \text{A.2}$$

Recalling from equation (4) the definition of N_C, the active density of states in the boundary of conduction band as $N_C = 2\left(\frac{m_e^* K_B T}{2\pi\hbar^2}\right)^{3/2}$, equation (A.2) becomes:

$$N_e = \frac{2N_C}{\sqrt{\pi}}\int_0^{\frac{\alpha-E_C}{K_B T}}\frac{(x)^{1/2}dx}{1+e^{x-n_F}}$$

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